The Non-Transparency of Timeless Rules [*]

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Abstract

We address the usefulness of both fully optimal and optimal rules in a timeless perspective. We prove the existence of a determinate representation, which complements the Dennis (2010) observation that the optimal rule may be written in multiple ways, and note that it cannot therefore be estimated by econometricians. We show that this multiplicity problem cannot be resolved by appeal to robustly optimal policy as claimed by Woodford, nor by providing the private sector with the optimal conditions, as the latter may lead to loss of reputation if these are interpreted, as they are likely to be, as optimal policy. We relate this to the transparency of central banks, which is now regarded as the main guarantee of commitment by these bodies.

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1 Introduction and Literature Review

This paper criticises the timeless approach to optimal policy largely associated with Woodford (2003) due to its problems of observational equivalence. Several authors, most notably Dennis (2010) have noted that there are multiple ways of representing the timeless (and indeed the fully optimal) policy when the system incorporates forward-looking variables. This feature, in turn, has implications for transparency of such policies. Although the multiplicity of representations is discussed by Woodford (2003), he contends that it can be handled by the notion of *robustly optimal policy*, by which the representation of the rule remains the same no matter what the form of shocks to the system. While this leads to a unique representation for the optimal rule when the system is purely forward-looking, we argue that such robustness may not exist when some of the variables are backward-looking.

The main contributions of the paper are that it proves the existence of a determinate representation of the timeless optimal policy when the policy variable is expressed in terms of current and past values of the economic variables, and also draws out its implications for the transparency of the resulting optimal and timeless optimal policy. The proof of existence also contains a means of constructing a determinate representation, which Dennis (2010) does not provide. Its other contributions are to show that even for the simplest New Keynesian model the optimal policy has multiple representations, and that the robustly optimal policy is more questionable than hitherto appreciated. Thus the paper may be seen as complementary to that of Dennis (2010).

The key to the existence of multiple representations is that optimal policy under rational expectations depends on past history, whereas on the equilibrium path of the system under optimal policy, the instrument is linearly dependent on current variables. Thus in principle any weighted average of the two representations also represents optimal policy. In the light of this, an alternative to announcing optimal policy has been proposed: inform the private sector of the optimal conditions only, which they can then verify. A direct consequence of this is that agents are likely to interpret the latter as the optimal policy, which in many cases would imply instability. How agents would react to this is not clear from economic theory, but would certainly lead to loss of reputation of the policymaker.

The possibility of multiple representations demands further investigation particularly in the context of Woodford's claim that a policy is timeless "if and only if the rule in question, when adjoined to the other equations of the structural model, is consistent with the existence of a rational-expectations equilibrium, and implies a determinate solution for the state-contingent path of the policy instrument" (Woodford (2003), pages 544-5). Adherence to this requires a proof of existence, but surprisingly such a proof does not appear anywhere in the literature. The closest to it, and which contains the only other mention of multiplicity that we are aware of, is in the work of Dennis (2010). This uses the description of the optimal rule due to Currie and Levine (1993), and manipulates it to obtain a representation of the timeless rule for a system which contains both forward and backward looking variables. Since it requires the pseudo-inverse of a rectangular matrix, which is not unique, this implies that this representation of the timeless policy is also not unique.

We argue that due to the problem of multiple representations it will be difficult, if not impossible, to monitor the 'timeless rule'. Furthermore, if agents suspect that it may be indeterminate (in the sense of the system under the rule being over-stable) this must completely undermine its transparency, as is required for it to be credible. Our argument also applies to the *universal optimal rule* as defined by Damjanovic *et al.* (2008), and in principle to the fully optimal rule of Currie and Levine (1993). However the latter is usually regarded as a policy that is unannounced, and is not therefore subject to transparency requirements.

We initially motivate our discussion with reference to the New Keynesian Policy Model. The latter is often presented as a hybrid, using standard micro-founded aggregate supply and demand equations coupled with a more or less arbitrary equation for the policy interest rate. This policy rule is not micro-founded like the rest of the model and might not reflect the implicit preference functional and objectives of the policy maker (see Svensson (2003) for a critique). This anomaly is apparently remedied in another, highly influential, branch of the monetary policy literature, where extensive reliance is made on articulated optimal conditions, which are explicitly distinct from fully optimal rules as in e.g. Woodford (2003) and Svensson and Woodford (2004). These optimal conditions are treated in a timeless form (as initiated by Woodford (1999)) but we argue later that the issue of multiplicity of solutions is crucial in the timeless approach, especially given the importance of transparency in current analyses of monetary policy. Although the timeless approach is based on the fully optimal rule, it is represented in a way that is intended to avoid the time inconsistency problem. But though many contributors to the optimal policy literature have treated the derivation of the fully optimal rule as an analytical exercise only, (see Currie and Levine (1993) for example), Woodford (1999) argues that the fully optimal rule, amended according to the timeless perspective, could realistically be followed by the policymaker in practice. Blake and Kirsanova (2004)[1] have shown that this may in some circumstances yield a higher welfare loss than the non-precommitment time-consistent policy, but our argument that there are serious defects from a transparency perspective in Woodford's analysis, takes a completely different tack to that of Blake and Kirsanova (2004). We first note that for the fully optimal rule, as exposited by Currie and Levine (1993) for example, no check is made whether this implementation leads to an overall system representation which is both stable and determinate. Other studies

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[1] In a model that incorporates a wealth effect from government debt, and therefore both forward and backward-looking variables, they utilize the timeless form of the optimal conditions to obtain a representation of the rule.

(usually without direct reference to Currie and Levine (1993)) are aware that the latter's representation of policy may lead to indeterminacy, and indeed does so for the basic New Keynesian model. As a consequence they use an alternative representation of either the fully optimal or the timeless policy (for example Levine and Pearlman (2008)).

Given that there are a potentially infinite number of representations of the optimal/timeless policy that all have the same structure and produce identical trajectories for the model variables, it follows that not only is there a problem of establishing this timeless policy, but also that there will be difficulties with Woodford's robustly optimal approach too. More broadly, there are serious ramifications of this for the transmission mechanisms of monetary policy as presently interpreted, as recent discussion of central bank transparency and accountability highlights (Geraats (2002), Geraats (2005)).

The issue of announcement of policy rules is one that is usually associated with 'transparency' of central banks. The most common view of transparency is that it is beneficial for the effectiveness of monetary policy (and also for democratic accountability, but this is not discussed here). It enables agents in the markets to have clarity about monetary policy objectives and strategies, and facilitates the management of expectations, viewed by some as the main channel by which monetary policy affects the economy (see for example Gali and Gertler (2007)). We shall see below that management of expectations may be problematic when adopting the timeless optimal policy, and that the non-transparency of such a policy may be a key factor in raising the volatility of inflation.

The view of transparency as generally benign is not without its critics. Thus Morris and Shin (2002), Morris and Shin (2005) show that when there are coordination externalities, individuals may place too much weight on public information as opposed to idiosyncratic private information; if this public information is imprecise then the resulting equilibrium may be inefficient in the case where private information is more precise. Other arguments suggesting that information overload may sow confusion have been made by Clare and Courtenay (2001), Goodhart (2001), Mishkin (2004) and Cruijsen *et al.* (2008). Nevertheless, recent empirical work by Dincer and Eichengreen (2009) suggests that greater transparency lowers inflation volatility, but more importantly for the purposes of this paper they conclude that greater transparency is here to stay. It is in the light of this that we present our arguments on optimal policy design.

2 Timeless and Robustly Optimal Rules for the Basic New Keynesian Model

Consider the basic New Keynesian model, with a Phillips curve that arises from firms choosing not to re-optimize prices at each period and an aggregate demand curve given by an Euler equation, with π_t, y_t and r_t representing inflation, the output gap, and the interest rate respectively.

$$\pi_t = E_t\pi_{t+1} + y_t + v_t \tag{1}$$

$$y_t = E_t y_{t+1} - \sigma(r_t - E_t\pi_{t+1}) \tag{2}$$

where v_t is associated with a taste shock, and E_t denotes expectations based on full information at time t. For convenience we have assumed that the consumers discount rate is 0, and we have also assumed for expositional purposes that the coefficient on the output gap in (1) is unity.

Now consider a policymaker whose objective is to minimize the welfare loss function (without discounting, for convenience)

$$L = \sum \frac{1}{2}(q\pi_t^2 + y_t^2) \tag{3}$$

subject to (1) and (2). Because the interest rate is not costed in the welfare loss, it turns out that we can ignore the constraint (2), and just consider the Lagrangian problem[2]

$$L = \sum [\frac{1}{2}(q\pi_t^2 + y_t^2) - \lambda_{t+1}(\pi_{t+1,t} - \pi_t + y_t - v_t)] \tag{4}$$

where we assume for convenience that the output gap is the instrument[3], although it is merely a proxy, and the interest rate may be obtained by substituting into the Euler condition (2).

First-order conditions are

$$q\pi_t - \lambda_t + \lambda_{t+1} = 0 \qquad y_t - \lambda_{t+1} = 0 \tag{5}$$

with a first-order condition $q\pi_0 + \lambda_1 = 0$ in the initial period $t = 0$, which represents the time inconsistency problem. Since our focus is on the timeless perspective, we ignore this, and make the conventional 'timeless' assumption, that this 'optimal' policy has been in place for some time.

Note that the two focs provide the dynamics of y_t as an implementable solution to the timeless problem:

$$y_t = y_{t-1} - q\pi_t \tag{6}$$

and this equation can also be described as the optimal condition for the policy problem.

Let us first consider the case with no shock, $v_t = 0$ for all t. It is useful at this stage to define the value function for the case of perfect foresight ($\pi_{t+1,t} = \pi_{t+1}$, which corresponds to a backward-looking system), which is given by $\frac{1}{2}s\pi_t^2$ where $s > 0$ is given by the Ricatti equation

$$s = q + \frac{s}{1+s} \tag{7}$$

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[2] Henceforth we use the notation $x_{t+1,t}$ for $E_t x_{t+1}$.

[3] Any change to interest rate has a first-round effect on output, followed later by the impact on prices.

It is then easy to show that along the optimal path there is an equilibrium relationship[4]

$$y_t = \frac{s}{1+s}\pi_t \tag{8}$$

Note that this cannot represent the *policy rule* under RE because that would imply a relationship $\pi_{t+1,t} = \frac{1}{1+s}\pi_t$, which has a stable root and therefore would lead to indeterminacy. However, given this equilibrium relationship we could formally divide up $y_t = \mu y_t + (1-\mu)y_t$ for *any* μ and (6) can also be written as

$$\mu y_t = y_{t-1} - \frac{(1-\mu)s}{1+s}\pi_t - q\pi_t \tag{9}$$

Thus we get the result that that there is a continuum of representations of the optimal rule that all have the same structure, with (6) as a special case of (9). Coupled with the NKPC, it produces the same behaviour of π_t, y_t[5] for all μ.[6]

Note that the eigenvalues of the system (although not the equilibrium relationship $y_t = \frac{s}{1+s}\pi_t$) depend on μ, and are given by $\frac{1}{1+s}$ and $(1+s)/\mu$. Clearly for μ between 0 and $1+s$ the system is saddlepath stable, with the stable eigenvalue equal to $\frac{1}{1+s}$ for all μ. However for $\mu > 1+s$ both eigenvalues will have modulus less than 1, implying that the system under the timeless rule is indeterminate. This, and the multiplicity of representations of the timeless rule for even the simplest of New Keynesian models, are the first crucial results of the paper.

It follows that since there is a continuum of rules that represent the timeless rule, this means that it is impossible for the econometrician and the private sector to identify this rule, and must therefore take it on trust. But if the private sector is unwilling to trust the policymaker, and is also unable to verify the rule by using data, it will therefore be unable to rule out the possibility of an implementation of a rule that has two stable eigenvalues, which introduces indeterminacy and the generation of sunspots. We note however that the optimal policy rule (9) is only non-identifiable within the context of the very basic New Keynesian model.

In Section 3, for the more general case of timeless and/or optimal rules, we shall see that with the introduction of a matrix that plays a similar role to that of μ the choice of this is not arbitrary, but essential to ensure that the system under the timeless rule is saddlepath stable and determinate.

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[4] The perfect foresight case uses the value function iteration $s\pi_t^2 = max_{y_t}(y_t^2 + q\pi_t^2 + s\pi_{t+1}^2) = max_{y_t}(y_t^2 + q\pi_t^2 + s(\pi_t - y_t)^2)$ and this generates (8) and (7).

[5] One can write the state space setup as $\begin{bmatrix} y_t \\ \pi_{t+1,t} \end{bmatrix} = \begin{bmatrix} \frac{1}{\mu} & \frac{-(1-\mu)s-s^2}{\mu(1+s)} \\ -\frac{1}{\mu} & 1+\frac{(1-\mu)s+s^2}{\mu(1+s)} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_t \end{bmatrix}$ (after substituting for q from (7)). The eigenvalues of the matrix are $\frac{1}{1+s}$ and $(1+s)/\mu$; assuming the latter is greater than 1, its left eigenvector then implies $y_{t-1} = s\pi_t$ which, when substituted into (9), yields (8).

[6] Svensson (2003) in footnote 21 briefly mentions the same idea in the context of time-consistent rules, but does not take it any further. Later on he discusses whether optimal rules are verifiable, but in the context of complexity, rather than identifiability as here.

2.1 Resolution of non-Uniqueness via the Robustly Optimal Criterion

Woodford (2003) gets round the non-uniqueness problem in a very neat way for the example of this section. To motivate his resolution of the problem, we now re-introduce the shock v_t in the New Keynesian Phillips curve (1). If this is an AR1 process, for example, then the equilibrium relationship $y_t = \frac{s}{1+s}\pi_t$ must be amended to include an additional term in v_t[7]. But this means that any alternative representation of the optimal rule other than (6) will contain v_t. Woodford (2003) very sensibly proposes that the rule should be what he terms robustly optimal, in the sense that it should be independent of any shock terms or of any parameters associated with shock process - so that it is valid for any form of shock. The implication of this is that the rule (6) is unique with respect to this criterion.

Woodford (2003) effectively obtains a similar result to this when all variables are forward-looking, which we shall refer to in more detail below.

The above approach also works when there are no forward-looking variables. In particular consider an engineering-type system with only one backward-looking variable x_t, one control variable u_t, a welfare function $\frac{1}{2}\sum_{t=0}^{\infty}(qx_t^2 + u_t^2)$ and an AR1 shock process v_t. Let us express this[8] as

$$x_{t+1} = x_t - u_t + v_t \qquad v_{t+1} = \rho v_t + \varepsilon_t \tag{10}$$

Then the forward-looking version, below, of (6), together with (10) is saddlepath stable:

$$u_t = u_{t+1,t} + qx_{t+1,t} \tag{11}$$

Although this is obviously nothing like the standard control-theory feedback solution of u_t on x_t, it provides a motivation for Woodford (2003) attempting to extend the robustly optimal criterion to the case when there are both backward and forward-looking variables.

2.2 A Problem with Stating the Optimal Conditions

Woodford (2003) and Svensson and Woodford (2004) have argued that articulating the optimal conditions is an alternative to announcing the timeless optimal rule, and that this is a means of conveying the policymaker's transparency. But if private agents interpret these as a means of calculating the optimal policy, then things could go badly wrong. Consider the backward-looking system (10) above; the more conventional way of writing the first-order condition analogous to that of (6), is not given by (11), but rather by

$$u_t = u_{t-1} - qx_t \tag{12}$$

Taken together with (10), this is a system with one stable and one unstable root. Whereas this was fine for a forward-looking system, it implies instability for a backward-looking system.

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[7] If the AR1 parameter is ρ, then the coefficient on v_t is $\frac{s}{1+s-\rho}$

[8] We can think of this as the perfect foresight version of (1), with x_t representing predetermined inflation.

A statement of the optimal condition in this case could lead agents to draw the conclusion that the policymaker was ineffective at his or her job, at least if the optimal condition were interpreted as the optimal policy. We shall see below how this impacts on a system with both backward and forward-looking variables.

3 Implementation of Timeless/Fully Optimal Rules in a General Setting

Consider now a general linear economic model and welfare loss function. The aim of this section is to show that in principle we have exactly the same problem as described earlier, that the fully optimal or timeless rule cannot be implemented in a unique way. This compounds the problem for timeless policy, given the well-documented result that it might not even be as good in welfare terms as the optimal time-consistent (or discretionary) policy (see Blake and Kirsanova (2004)).

Accordingly we consider the general linear-quadratic problem under RE. For convenience we focus on the purely deterministic problem. We start with a general model of the form

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ E_t x_{t+1} \end{bmatrix} = A \begin{bmatrix} z_t \\ x_t \end{bmatrix} + B w_t \tag{13}$$

where z_t is an $(n-m) \times 1$ vector of predetermined variables including non-stationary processed, z_0 is given, w_t is a vector of policy variables, x_t is an $m \times 1$ vector of non-predetermined variables and $E_t x_{t+1}$ denotes rational (model consistent) expectations of x_{t+1} formed at time t. Then $E_t x_{t+1} = x_{t+1}$ and letting $y_t^T = \left[z_t^T \; x_t^T \right]$ (13) becomes

$$\mathsf{y}_{t+1} = A\mathsf{y}_t + B w_t \tag{14}$$

The policy-maker's loss function at time t is given by

$$\Omega_t = \frac{1}{2} \sum_{i=0}^{\infty} \beta^t [\mathsf{y}_{t+i}^T Q \mathsf{y}_{t+i} + \mathsf{w}_{t+i}^T R \mathsf{w}_{t+i}] \tag{15}$$

where Q is a symmetric and non-negative definite matrix, R is a positive definite matrix and $\beta \in (0,1)$ is discount factor. The slightly more general case involves terms in $\mathsf{y}_{t+i}^T U w_{t+i}$, but the analysis below is barely changed with the introduction of this extension, so we ignore this term.

Consider the policy-maker's *ex-ante* optimal policy at $t = 0$. This is found by minimizing Ω_0 given by (15) subject to (14) and and given z_0. We proceed by defining the Hamiltonian

$$\mathcal{H}_t(\mathsf{y}_t, \mathsf{y}_{t+1}, \mu_{t+1}) = \frac{1}{2}\beta^t (\mathsf{y}_t^T Q \mathsf{y}_t + \mathsf{w}_t^T R \mathsf{w}_t) + \mu_{t+1}(A\mathsf{y}_t + B\mathsf{w}_t - \mathsf{y}_{t+1}) \tag{16}$$

where μ_t is a row vector of costate variables. By standard Lagrange multiplier theory we minimize

$$\mathcal{L}_0(y_0, y_1, \ldots, w_0, w_1, \ldots, \mu_1, \mu_2, \ldots) = \sum_{t=0}^{\infty} \mathcal{H}_t \tag{17}$$

with respect to the arguments of L_0 (except z_0 which is given). Then at the optimum, $\mathcal{L}_0 = \Omega_0$.

Redefining a new costate column vector $\mathsf{p}_t = \beta^{-t}\mu_t^T$, the first-order conditions lead to

$$\mathsf{w}_t = -R^{-1}\beta B^T \mathsf{p}_{t+1} \tag{18}$$

$$\beta A^T \mathsf{p}_{t+1} - \mathsf{p}_t = -Q\mathsf{y}_t \tag{19}$$

Substituting (18) into (14) we arrive at the following system under control

$$\begin{bmatrix} I & \beta BR^{-1}B^T \\ 0 & \beta A^T \end{bmatrix} \begin{bmatrix} \mathsf{y}_{t+1} \\ \mathsf{p}_{t+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -Q & I \end{bmatrix} \begin{bmatrix} \mathsf{y}_t \\ \mathsf{p}_t \end{bmatrix} \tag{20}$$

To complete the solution we require $2n$ boundary conditions for (20). Specifying z_0 gives us $n-m$ of these conditions. The remaining condition is the 'transversality condition'

$$\lim_{t\to\infty} \mu_t^T = \lim_{t\to\infty} \beta^t \mathsf{p}_t = 0 \tag{21}$$

and for the *fully optimal* rule, the initial condition

$$\mathsf{p}_{20} = 0 \tag{22}$$

where $\mathsf{p}_t^T = \left[\mathsf{p}_{1t}^T \, \mathsf{p}_{2t}^T\right]$ is partitioned so that p_{1t} is of dimension $(n-m) \times 1$. Equations (18), (20) together with the $2n$ boundary conditions constitute the system under optimal control. For the *timeless* rule, the initial value p_{20} is not 0, but is dependent on the value of y_0. However this is not the focus of our attention, so we ignore the details of this.

The first result of this section is the following proposition, which is an obvious generalization of that for the basic New Keynesian model:

Proposition 1: When all variables are forward-looking, and the discount factor β is sufficiently close to 1, the optimal rule can be written in a unique way so as to satisfy the robustly optimal criterion.

Proof: For a linear quadratic optimal control problem it is well known that the eigenvalues of the system (20), coupled with the dynamics of the Lagrange multipliers, has eigenvalue pairs of the form $\{\lambda, 1/(\beta\lambda)\}$. Thus for β close enough to 1, the system together with Lagrange multipliers is saddlepath stable. Since the optimal policy variable is a linear function of the Lagrange multipliers, which are governed by equations independent of shocks, the result follows.

3.1 Possible Implementations of the Optimal Rule

For the special case that corresponds to Proposition 1, namely that all variables are forward-looking, the timeless rule can be expressed using the lag operator L on (18) and (19), so that $w_t = R^{-1}\beta B^T(\beta A^T - LI)^{-1}Q\mathsf{y}_t$[9].

However Currie and Levine (1993) and Dennis (2010) describe the optimal rule using the following unique representation that represents the rule solely in terms of the predetermined variables and the costate variables p_2:

$$\mathsf{w}_t = -F \begin{bmatrix} I & 0 \\ -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{bmatrix} \begin{bmatrix} \mathsf{z}_t \\ \mathsf{p}_{2t} \end{bmatrix} \tag{23}$$

where

$$\begin{bmatrix} \mathsf{z}_{t+1} \\ \mathsf{p}_{2t+1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ S_{21} & S_{22} \end{bmatrix} G \begin{bmatrix} I & 0 \\ -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{bmatrix} \begin{bmatrix} \mathsf{z}_t \\ \mathsf{p}_{2t} \end{bmatrix} \tag{24}$$

$$\mathsf{x}_t = \begin{bmatrix} -S_{22}^{-1}S_{21} & S_{22}^{-1} \end{bmatrix} \begin{bmatrix} \mathsf{z}_t \\ \mathsf{p}_{2t} \end{bmatrix} \tag{25}$$

where $F = \beta(R + \beta B^T SB)^{-1}B^T SA,\ G = A - BF = (I + \beta BR^{-1}B^T S)^{-1}A$ and

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \tag{26}$$

is partitioned conformably with z, x and is the solution to the steady-state Ricatti equation

$$S = Q + \beta A^T S(I + \beta BR^{-1}B^T S)^{-1}A \tag{27}$$

Note the well-known result that G has only stable eigenvalues for β sufficiently close to 1, and that the Ricatti equation arises from the relationship $\mathsf{p} = S\mathsf{y}$.

Attractive as this unique representation is, if all variables are forward-looking, then z_t does not exist, so from (24) we see that p_{2t} only feeds back on itself. It therefore follows that the policy variable w_t is not dependent on the state variable x_t at all. Thus if the underlying system is completely forward-looking and is indeterminate under no policy rule, then the Currie-Levine implementation of the optimal rule will be indeterminate as well. Thus it is flawed as a general implementation, and fails in particular for the basic New Keynesian model.

Dennis (2010) partially gets round this problem by noting from (23) and (25) that w_t, x_t are linearly dependent on z_t, p_{2t}. By finding the pseudo-inverse of this linear relationship, he writes p_{2t}, (in a non-unique way) as a linear function of $\{z_t, x_t, w_t\}$, which generates an ARMA process for $\{w_t\}$ in terms of z_t, x_t, but provides no means of ensuring of determinacy.

We shall address the representation of the optimal/timeless rule below.

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[9] If all variables are forward-looking then A is invertible

3.2 The Implications of Publicizing the Optimal Conditions

Consider (20), for the case when not all variables are forward-looking. If A is invertible, then one can express p_t in terms of past values of y_t, from which it follows from (18) that w_t can be expressed as an ARMA process in y_t. This ARMA process can be taken as representing the optimal condition for the problem.

Now consider what happens if this is in turn regarded as the optimal rule. Then the eigenvalues of the system under this supposed rule would be the same eigenvalues as those of (20); for β close enough to 1, there are n stable and n unstable eigenvalues. But if y_t contains some backward-looking variables (e.g. the capital stock) then the number of forward-looking variables is less than n. Thus it follows that if private agents interpret the optimal conditions as constituting the optimal rule, then they would regard the system as under control as unstable, with consequent loss of reputation of the policymaker.

3.3 Implementable Rules that are Saddlepath Stable

Clearly, to ensure that the system is determinate, we must find an expression for the implementable rule that is not of the Currie-Levine form. For the example used in Section 2, we found that we could write the policy instrument y in terms of its own past value and current π, and it is easy to check that this leads to a determinate system. Thus the most obvious possibility is to ensure that the policy feedback rule for the general case involves a feedback on x as well as on z and p_2; there is no point of course in using p_1 in the feedback rule, because it is a forward-looking variable, and it would be confusing to agents to introduce yet another forward-looking variable into the system. How do we achieve this feedback? We first note (dropping the t subscript for the moment) that the $\mathsf{p} = S\mathsf{y}$ relationship can be expanded to

$$p_1 = S_{11}z + S_{12}x \qquad p_2 = S_{21}z + S_{22}x \tag{28}$$

so that we can obtain an infinite number of representations of p_1 in terms of the other variables:

$$p_1 = S_{11}z + S_{12}x + M(p_2 - S_{21}z - S_{22}x) \tag{29}$$

where M is a matrix of appropriate dimensions. The Currie-Levine implementation uses $M = S_{12}S_{22}^{-1}$ so that p_1 only depends on z and p_2. Thus, given that the policy rule is expressed in terms of p_{t+1}, our objective is to write the policy rule in a way that includes p_{2t}, but eliminates the variable p_{1t}, and replaces it by (29). The appendix indicates how to obtain an ARMA representation of the timeless rule in terms of the variables y_t.

We need to choose M in such a way that the implemented system is saddlepath stable. The following theorem, which is the main result of this section, not only shows how to do this, but is also the first ever general statement that an optimal rule exists that ensures that an RE system under control is saddlepath stable.

Theorem 1: For non-zero M, the system is determinate provided that $G_{22} + M^T G_{12}$ has stable eigenvalues, where G_{12} is the $(n-m) \times m$ top right matrix of G. Furthermore, there exist an infinite number of such matrices with the required property.
Proof: See Appendix.

There are several implications of this theorem, of which the first is :

Corollary 1: If $M = 0$, then the system is determinate provided that G_{22} has stable eigenvalues, where G_{22} is the $m \times m$ bottom diagonal matrix of the system matrix $G = (I + \beta B R^{-1} B^T S)^{-1} A$.

Note that for the basic NK model there are no predetermined variables, so $G_{22} = G$, and this implementation automatically leads to determinacy. However when there are also predetermined variables, then there is no reason at all why G_{22} should be a stable matrix.

The main implication of Theorem 1 is:

Corollary 2: There are an infinite number of representations of the optimal/timeless policy that have identical structure but with different parameter values.

The results above also apply to the 'universally optimal' rule of Damjanovic *et al.* (2008). The latter shows that when the optimal choice of instrument must be chosen over all equilibrium realizations of initial conditions, then this is equivalent to solving the problem of maximizing (15) with the discount factor β (within (15)) set equal to 1. The effect on the basic New Keynesian model, which is slightly more general than that of (1), as there is a term β multiplying $E_t \pi_{t+1}$, is that the timeless rule remains as (6), whereas the universally optimal rule becomes $y_t = \beta y_{t-1} - q\pi_t$. In general then, since the only thing that is changed from Corollary 1 is the discount factor, we have the following result:

Corollary 3 There are an infinite number of representations of the universally optimal policy, which yield determinacy, that have identical structure but with differing parameter values.

3.4 Flawed Nature of the Robustly Optimal Criterion

Whereas we have seen that the robustly optimal criterion is useful both for pure forward-looking and for pure backward-looking systems, Giannoni and Woodford (2003a) and Giannoni and Woodford (2003b), attempt to show how it can be used when the system includes both backward and forward-looking variables. Although these latter two papers are daunting, their content is very simple when the system is expressed in the form of (13)[10]. In the case addressed here, the robustly optimal criterion is merely based on the

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[10] Ellison and Pearlman (2010) show in an appendix that most RE models initially written in the form $A_0 Y_{t+1,t} + A_1 Y_t + A_2 Y_{t-1} = DW_t$, where A_0 is singular can be written in this way.

first-order conditions for the states (19). If the system is purely forward-looking then the robustly optimal rule is rewritten as

$$p_{t+1} = \frac{1}{\beta} A^{-T}(p_t - Q\mathsf{y}_t) \tag{30}$$

whereas for the purely backward-looking case it takes the form

$$p_t = Q\mathsf{y}_t - \beta A^T E_t p_{t+1} \tag{31}$$

However for the case when there are both backward and forward-looking variables, Giannoni and Woodford (2003a) focus on the eigenvalues of matrix βA^T, but are not explicit about the differing roles of the p_{1t} and p_{2t} costate variables.[11]

To understand what is going wrong in their analysis, it is useful to write (19) in a form that incorporates p_{1t} as a forward-looking variable, as it is associated with a set of backward looking equations, and conversely p_{2t} as a backward-looking variable:

$$\beta A^T \begin{bmatrix} p_{1t+1,t} \\ p_{2t+1} \end{bmatrix} - \begin{bmatrix} p_{1t} \\ p_{2t} \end{bmatrix} = -Q\mathsf{y}_t \tag{32}$$

Although one can formally write the solutions for p_{1t} and p_{2t} in lag operator form as

$$(I-\beta A_{11}^T L^{-1})p_{1t} = \beta A_{21}^T L^{-1} p_{2t} + Q_1 \mathsf{y}_t \qquad (I-\frac{1}{\beta} A_{22}^{-T} L)p_{2t} = -\frac{1}{\beta} A_{22}^{-T}(Q_2 \mathsf{y}_{t-1} + A_{12}^T p_{1t,t-1}) \tag{33}$$

the closed form representation for these solely in terms of y_t, which is required to obtain the representation for w_t, is evidently considerably more complicated than the representation of the timeless rule in Appendix B. The presence of the term $p_{1t,t-1}$ in the second of these equations also implies that the ARMA process for w_t not only contains current expectations of future variables, but also contains past expectations of future variables. This would render the robustly optimal rule opaque rather than transparent.

Giannoni and Woodford (2003a) fail to account for the forward-looking nature of p_{1t}, which is why their robustly optimal solutions would be identical for the cases of (**??**).

4 Implications for Central Bank Transparency

In their largely consensus model of monetary policy, Gali and Gertler (2007) observe that, since in the model, its structural equations depend upon forwardlooking expectations of

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[11] Without going into details, consider the following two systems, where a_t, b_t are scalars:

$$\begin{bmatrix} a_{t+1} \\ E_t b_{t+1} \end{bmatrix} = A \begin{bmatrix} a_t \\ b_t \end{bmatrix} + Bw_t \qquad \begin{bmatrix} E_t a_{t+1} \\ b_{t+1} \end{bmatrix} = A \begin{bmatrix} a_t \\ b_t \end{bmatrix} + Bw_t$$

Following the method of Giannoni and Woodford (2003a), the robustly optimal rule would then be written in an identical way for each of these, which is clearly implausible. Further evidence of confusion involving use of expectations is that in Giannoni and Woodford (2003b), equation (A.16) should be a lagged version of the first equation on p79, except that both equations use expectations at time t.

households and firms, then the current values of output and inflation depend not upon the current value of the policy rate (as was the case in traditional models) but on its expected future path. Thus

"...the policy process is as much, if not more, about communicating its future intentions of policy in a transparent way, as it is about choosing the current (value of the) policy instrument" (words in parentheses added), Gali and Gertler (2007)[12].

The work of Geraats (2005) and Eijffinger and Geraats (2006) on Central Bank transparency gives a further insight into our argument that it would be inappropriate to use the timeless rule if the private sector is unable to check that this is what is being used. As noted in the Introduction, transparency is essential to ensuring CB credibility, and thereby to manage agents' expectations. Geraats (2005) lists 13 measures of transparency, included under the five headings of Political, Economic, Procedural, Policy and Operational Transparency.

Under Economic Transparency is the question of whether the Central Bank discloses its macroeconomic model; under Procedural Transparency whether it provides an explicit policy rule or strategy, and more generally under Policy Transparency whether there is prompt disclosure of policy decisions, an explanation of the decision, and policy inclination and indication of future policy. All of these impinge on disclosure of the policy rule.

Suppose now that the Central Bank decides to implement the optimal or the timeless policy. If it is transparent on each of the aforementioned questions, then the policymaker will reveal the rule. But suppose that agents now decide to check whether the rule is being followed. As we have seen from Theorem 3, there are an uncountable number of ways to represent the timeless optimal rule. We therefore have the following result:

Theorem 4: The timeless optimal rule is not transparent, in the sense that it cannot be verified by estimation because of the identification problem.

This result is not innocuous. If the timeless policy is not transparent, it cannot be verified so will not be credible. In consequence, the authorities will not able to manage expectations so changes in the policy rate will not change the yield curve as required. Inflation will likely be more volatile following shocks than would be the case for a credible monetary policy regime. Consider the following thought experiment: suppose that agents decide that they wish to check the transparency announcement of the central bank on policy and decide to try and estimate the rule not as a system, but as a single equation,

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[12] In their Experiment 1, they contrast the effects of an aggressive (credible) versus a passive (non-credible) application of monetary policy in the aftermath of a cost-push shock due to the productivity downturn with real wage resistance. The credible policy raises short term interest rates 1.5% for a 1% increase in inflation, and is credible, so shifts the whole yield curve up. A passive policy, by contrast, even if it raises the short rate by the same amount as the aggressive one, period by period, does not succeed in shifting the yield curve in the same way because it is not credible, so incurs significantly higher inflation for the same falls in output.

notwithstanding the theorem above. Then given that there cannot be a statistically consistent estimate of the parameters, it will invariably happen that some series of shocks will lead agents to estimate a set of parameters which imply that the overall system is indeterminate. This could lead to the onset of expectational jumps and bubbles. But whereas a bubble dies out in an indeterminate system, when there is determinacy the effects of this bubble are magnified over time. Thus a bubble emerges because agents may have the wrong estimate of the rule, but given that the actual rule makes the system determinate, this bubble could have considerable effects. Presumably at some point agents would either realise that the bubble was getting out of hand or else they would have new updated estimates of the parameters implying determinacy of the system, and the bubble would burst.

5 Conclusion

We have focused our main attention on timeless policy, and have shown that there is an identifiability problem for both the timeless and the universally optimal rule as well which in general is impossible to resolve. This is because the notion of robustly optimal rules is misconceived when the system contains backward-looking variables.

We have also discussed an assertion that providing agents with the optimal conditions bypasses the need to inform agents of (one of the many representations of) the optimal policy. However we have established that this may lead to loss of reputation if these conditions are viewed as the optimal policy.

We have also established that a timeless policy exists for any linear RE system with both backward and forward-looking variables, that is both stable and determinate

The problem of timeless optimal policy is exacerbated for monetary policy rules when the central bank is transparent about the form of rule that it uses; there is a possibility that if agents attempt to estimate the rule then there could be a severe effect from time to time of expectational bubbles.

So what form should policy rules take? There are two answers to this. Firstly, if the policymaker is not transparent about the model, then the only transparent rule will be a simple one. But a more useful answer is that it could well be a timeless rule provided that the policymaker is also transparent about the model on which it is based. In the latter case, the timeless policy still cannot be estimated consistently, but at least it can be verified as displaying the required stability and determinacy properties.

Appendix

A Proof of Theorem 2

First of all write $S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}$, $A = [A_1 \; A_2]$, $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, $Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$ conformably with z and x. Also note that $G = (I + \beta BR^{-1}B^T S)^{-1}A = A - \beta BR^{-1}B^T(I + \beta SBR^{-1}B^T)^{-1}SA$, so that $G_{22} = A_{22} - \beta B_2 R^{-1}B^T(I + \beta SBR^{-1}B^T)^{-1}SA_2$, $G_{12} = A_{12} - \beta B_1 R^{-1}B^T(I + \beta SBR^{-1}B^T)^{-1}SA_2$. In addition the Ricatti equation implies that $S_2 = Q_2 + \beta A_2^T(I + \beta SBR^{-1}B^T)^{-1}SA$.

The system under control is (20), but with $p_1 = S_1 y + M(p_2 - S_2 y)$. It follows that we can then write the system as

$$\begin{aligned} \mathsf{y}_{t+1} + \beta BR^{-1}B_1^T(S_1\mathsf{y}_{t+1} + M(p_{2,t+1} - S_2\mathsf{y}_{t+1}) + \beta BR^{-1}B_2^T p_{2,t+1} &= A\mathsf{y}_t \qquad \text{(A.1)} \\ \beta A_{22}^T p_{2,t+1} + \beta A_{12}^T(S_1\mathsf{y}_{t+1} + M(p_{2,t+1} - S_2\mathsf{y}_{t+1})) &= -Q_2\mathsf{y}_t + p_{2t} \qquad \text{(A.2)} \end{aligned}$$

For the saddlepath condition to hold, we need this system to have n stable eigenvalues and m unstable eigenvalues. We first note that (A.1) and (A.2) can in turn be rewritten as

$$(I + \beta BR^{-1}B^T S)\mathsf{y}_{t+1} + \beta(BR^{-1}B_1^T M + BR^{-1}B_2^T)(p_{2,t+1} - S_2\mathsf{y}_{t+1}) = A\mathsf{y}_t \qquad \text{(A.3)}$$

$$\beta A_2^T S\mathsf{y}_{t+1} + \beta(A_{22}^T + A_{12}^T M)(p_{2,t+1} - S_2\mathsf{y}_{t+1})) = (S_2 - Q_2)\mathsf{y}_t + (p_{2t} - S_2\mathsf{y}_t) \qquad \text{(A.4)}$$

where (A.4) uses $A_{12}^T S_1 + A_{22}^T S_2 = A_2^T S$. After substituting for y_{t+1} from (A.3) it follows that (A.4) can be rewritten as

$$(G_{22}^T + G_{12}^T M)(p_{2,t+1} - S_2\mathsf{y}_{t+1}) = (p_{2t} - S_2\mathsf{y}_t) \qquad \text{(A.5)}$$

Hence the eigenvalues under this implementation of the optimal rule are the union of eigenvalues of G, all n of which are stable, and the inverse of the eigenvalues of $G_{22}^T + G_{12}^T M$; if the latter inverses are stable, then the whole system will be saddlepath stable. We first show that there is at least one value of the matrix M which ensures that $G_{22}^T + G_{12}^T M$ is stable.

Consider the set of left eigenvectors of G, each of which we write conformably with z and x as $[e_i^T \; f_i^T]$, $i = 1, ..., n$. It is a standard result that this set of eigenvectors spans n-dimensional space, so this implies that there must be at least one subset of m eigenvectors which has the property that the set $\{f_i\}$ span m-dimensional space. It follows that we may write $[N \; I]\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \Lambda[N \; I]$, where Λ is a square matrix with stable eigenvalues; but this implies in particular that $NG_{12} + G_{22} = \Lambda$, so $M = N^T$ is one particular choice of M that ensures determinacy of the system.

Finally, by continuity, there exists a neighbourhood of $M = N^T$ for which $G_{22}^T + G_{12}^T M$ is stable. This completes the proof.

We also note that substituting $M = 0$ provides the proof of Theorem 1.

B ARMA Representation of the Timeless Rule

The calculation of this rule is rather tedious, so we motivate it by appeal to the case of a backward-looking system. For the latter, the equations governing the system are

$$\mathsf{y}_{t+1} = A\mathsf{y}_t + Bw_t \qquad w_t = -\beta R^{-1} B^T p_{t+1} \qquad p_t = S\mathsf{y}_t \tag{B.6}$$

Substituting $p_{t+1} = S\mathsf{y}_{t+1}$ and for y_{t+1} yields the expression

$$w_t = -\beta R^{-1} B^T S(A\mathsf{y}_t + Bw_t) \tag{B.7}$$

which can be solved to give the standard expression $w_t = -(I+\beta R^{-1}B^T SB)^{-1}\beta R^{-1}B^T A\mathsf{y}_t$.

For the case in hand, the relevant equations are:

$$\mathsf{y}_{t+1} = A\mathsf{y}_t + Bw_t \qquad w_t = -\beta R^{-1}(B_1^T p_{1,t+1} + B_2^T p_{2,t+1}) \qquad p_{1t} = S_1\mathsf{y}_t + M(p_{2t} - S_2\mathsf{y}_t) \tag{B.8}$$

$$\beta A_2^T S\mathsf{y}_{t+1} + \beta(A_{22}^T + A_{12}^T M)(p_{2,t+1} - S_2\mathsf{y}_{t+1}) = -Q_2\mathsf{y}_t + p_{2t} \tag{B.9}$$

One can eliminate $p_{1,t+1}$ so that w_t is written in terms of $p_{2,t+1}, \mathsf{y}_t$, $p_{2,t+1}$ can be written as an ARMA process in terms of y_t, w_t, so that w_t can be written as an ARMA process in y_t.

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