Are we living beyond our means? A comparison of France, Italy, Spain and the United Kingdom

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ABSTRACT

This paper explores the savings needs of France, Italy, Spain and the United Kingdom on the assumption that each cohort is self-sufficient. Cross-section profiles for labour income and consumption (taking public and private consumption together) are used. They are adjusted for trend growth with the consumption profile being scaled so that the life-time budget constraint for the youngest cohort is met. Rent from land is found to be important to the calculations. The income and consumption profiles are used to calculate the required wealth holding of each older cohort. Savings and wealth holding figures are calculated by age and population data are used to aggregate them for the whole population. Savings rates are found to be lower than those required and holdings of produced wealth are inadequate to pay for current consumption patterns by the current adult population in all four countries. The United Kingdom faces particularly acute problems. The impact of rising survival rates and of longer working life is explored.

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1. Introduction

The problems of ageing and many of the questions of paying for it have been widely discussed over the last fifteen years or so. Internationally most of the concern has focused on the budgetary implications of ageing populations, with a recognition that population ageing on the one hand reduces the tax base and on the other hand leads to an increase in age-related expenditures. Generational accounts were developed by Auerbach and Kotlikoff (1987) to explore these issues.

Other work has looked at the saving behaviour of households. For example, Scholz, Seshadri and Khitatrakun (2006) solve an optimising life-cycle consumption savings model separately for individuals in a large random sample of US households and compare the assets that the individuals actually have with the levels that they would be predicted to have were they saving 'optimally'. They find that, despite the commonly held view that the US baby boomers need to save more, there is relatively little evidence of under-saving. They suggest that only 18.6% of the households have savings that are lower than optimal, and amongst these the shortfall is relatively small.

While an analysis of the public sector addresses many important issues it does not establish whether a particular country is making adequate provision for old age or not. First of all, people may expect to finance consumption in old age out of their own savings (or private pension schemes) as well as out of state pensions. The mix between these different forms of provision for old age is likely to differ from one country to another and an assessment of provision for old age needs to take account of both forms of provision. Secondly, even if the public sector appears to face strain from population ageing, the private sector might well be saving up to meet the future tax burden associated with population ageing. Conversely, cohorts currently alive may be relying on rising land values to finance retirement, with implications for inter-generational equity. Thus generational accounts focus only on the public sector while studies like that by Scholz *et al.* (2006) look only at the private sector and do not take account of land value effects. An assessment of national saving and overall saving adequacy needs to bring the public and private sectors together.

In this paper, we therefore set out a framework for assessing the adequacy of overall saving. This approach is robust to differences in the mix of public and private provision of resources for old age in different countries, and therefore allows meaningful cross-country comparisons to be made. We assess savings adequacy based on the decisions of representative individuals of different ages in France, Italy, Spain and the United Kingdom, four countries with very different pension, tax and capital market policies. Our work is closely related to the studies of savings behaviour by Gokhale, Kotlikoff and Sabelhaus (1996) and Kirsanova and Sefton (2007). As with these studies, our analysis addresses national saving rates rather than household saving. However both of these studies authors attempt to explain differences in savings rates, over time in the case of the first paper and between countries in the second paper. We focus instead on the absolute level of savings and wealth holdings so as to provide a direct answer to the question of savings adequacy.

2. Assessment of Savings Needs in the EU

There are a number of ways of identifying consumption and thus savings needs and thus building up a picture of the country's overall savings requirement. We follow the approach of generational accounts but applied to the economy as a whole rather than just to the public sector.

We identify profiles for both consumption and income and scale the consumption profiles so that people at the start of their working lives meet the life-time budget constraint on the assumption that they pay their own way during their lives. We assume that all produced capital is fully annuitised, with the implication that the produced assets of people who die are shared among the survivors of the same cohort (Yaari, 1965), while land is bequeathed rather than sold from one generation to the next and the rent on land accrues to adults on a *per capita* basis. Taking account of land in this way is of considerable importance for the results of the study.

We then identify the resulting holdings of produced wealth and net saving at each subsequent age. Information on the current population structure allows us to calculate required overall rates of saving and wealth holding for the economies in question and to explore the adequacy of current wealth holdingss. Thus our results do

not identify levels of consumption and saving which are in any sense optimal. Rather they show the levels of saving and wealth holding which follow from the consumption and income profiles on the assumption that each cohort pays its own way.

We compute four variables of interest. First of all there is the affordability ratio for young people. This is the ratio of life-time resources to life-time consumption of a current twenty-year old. Secondly we look at the savings rate of the economy on the assumption that twenty-year olds adjust their consumption in line with life-time resources and that older cohorts make similar adjustments to their consumption. Thirdly we compute ratios of produced wealth to labour income on the assumption that consumption is adjusted as above. Finally we look at the affordability ratio for the current population. This shows extent to which consumption needs to be adjusted for the consumption plans of everyone currently aged twenty and over to be affordable, taking into account the actual produced wealth of the country. This is determined by actual past saving and not solely by the past saving which would have been needed to meet the consumption paths identified. High past saving makes high current levels of consumption affordable.

Notation

For each cohort-specific variable of interest we need to identify both the cohort and the age of the cohort. We use double subscripts, with the first indicating the year of both of the cohort and the second its age at the time in question. *t* is used to indicate calendar years and and are used to indicate age. $c^{C}_{t,\tau}$ indicates the comprehensive consumption of a representative member of cohort *t* when of age and similarly $c^{C}_{t-\tau,\tau}$ is the consumption of the people aged in year *t.* We consider only adults aged 20 and over and set an upper limit of 95 since beyond this age data become very scanty. Where necessary we distinguish men from women with superscripts, so that $c^{C,m}_{t-\tau,\tau}$ is the consumption of men and $c^{C,w}_{t-\tau,\tau}$ that of women.

We denote the series of cross section observations of consumption in period t by the vector $\mathbf{c}_t^{C,x}$ $(x = m, w)$. Since we consider only people aged 20 and over, element of $\mathbf{c}_t^{C,x}$ is $c_{t-\tau,\tau}^{C,x}$, with a similar notation used for other variables.

Savings Requirements

We denote $c_{t,\tau}^{C,x}$ as the comprehensive consumption of a representative member of cohort t when aged . Comprehensive consumption is related to individual consumption, $c_{t,\tau}^{I}$ in the following way:

$$c_{t,\tau}^{C,x} = c_{t,\tau}^{I,x} + c_{t,\tau}^{G,x} - z_{t,\tau}^{C,x} \quad (1)$$

where $c_{t,\tau}^{G,x}$ is consumption provided by the government and $z_{t,\tau}^{C,x}$ is indirect taxes net of subsidies collected on consumption goods. Thus $c_{t,\tau}^{C,x}$ is the total consumption by the cohort measured at basic prices rather than market prices.

The need to distinguish produced capital from land complicates our analysis. It arises, on the one hand because the latter does not accrue through saving and, on the other hand, because its stock cannot be expected to grow in line with labour input adjusted for growth in productivity. Table 6 in section 7 shows that in France this comprises 45% of national wealth, with a figure of 55% for the United Kingdom (end-2005 figures). There are no data from Italy and Spain. We assume, for these countries, that the value of non-produced capital takes the same multiple of consumption at current market prices as is observed in France.

The fact that the supply of land is fixed means that the price of land has to be expected to rise relative to the price of labour and, as a consequence, the overall rate of consumption growth has to be lower than the rate of growth of labour income. Balanced growth is not possible in an economy with a fixed factor of production.

We make the following assumptions in order to resolve this problem.

A1. Consumption is a composite good; we distinguish consumption of the services provided by land, consumption type *a* from the consumption of goods and services produced from labour and from capital which is itself produced, consumption type *b*.

The composite consumption good is the numeraire. The price of the former is denoted p_t^a and of the latter p_t^b.

A2. There is a unit elasticity of substitution between the two types of consumption.

A3. Labour productivity grows at a constant rate and that international mobility of capital means that the real rate of return is constant.

A4. The rate of return equals the rate of discount or alternatively the elasticity of intertemporal substitution is zero.

Thus

$$c_{t-\tau,\tau}^{C,x} = p_t^a a_{t-\tau,\tau}^x + p_t^b b_{t-\tau,\tau}^x \quad \text{with } p_t^a a_{t-\tau,\tau}^x = \rho c_{t-\tau,\tau}^{C,x} \text{ and } p_t^b b_{t-\tau,\tau}^x = (1-\rho) c_{t-\tau,\tau}^{C,x} \tag{2}$$

The assumption of unit within period elasticity substitution with aggregate consumption the numeraire implies that the two prices always satisfy the following constraint

$$(p_{t+\theta}^a)^\rho (p_{t+\theta}^b)^{1-\rho} = 1 \tag{3}$$

Labour income is measured in units of the consumption type *a,* the point being that overall consumption cannot be expected to rise in line with labour income if a part of that consumption comes from rental income on land and the stock of land is fixed. The volume of labour income in year *t* is denoted $y_{t-\tau,\tau}^{L,x}$ and the consumption value of that labour income is $p_t^a y_{t-\tau,\tau}^{L,x}$.

We assume that there is a given exogenous rate of growth of labour productivity, *g* and that consumption *per capita* measured in terms of labour income grows at the same rate[2]. We relate the *per capita* labour income and consumption of cohort *t** when it reaches the age of τ to those of the cohort aged τ observed in year *t* as

$$y_{t^*-\tau,\tau}^{L,x} = y_{t-\tau,\tau}^{L,x}(1+g)^{t^*-t} \tag{4}$$

$$c_{t^*-\tau,\tau}^{C,x} = \frac{p_{t^*}^a}{p_t^a} c_{t-\tau,\tau}^{C,x}(1+g)^{t^*-t}; \quad t^* \geq t; \; 95 \geq \tau \geq 20; x = m,w. \tag{5}$$

[2] With a unit elasticity of demand this is equivalent to assuming that the volume of goods and services generated by labour/produced capital grows at the rate of growth of labour productivity.

The behavioural model which underlies the consumption assumption is set out in Appendix 2. However, since a part of consumption is the outcome of collective rather than individual decision-making, excessive weight should not be placed on this framework.

Assumption A2 implies a unit elasticity with respect to total consumption spending for the services provided by land. To explore the impact of this we need to take account of population change. We assume that at time *v* the number of people of sex *x* in the cohort born in year *j*, and therefore of age $t-\tau$ is $n^{x}_{t-\tau,\tau}$. We denote the vector showing the number of people of each age in year *t* as $\mathbf{n}^{\mathbf{x}}{}_{t}$ and use other vector variables in a similar way.

With a fixed supply and a constant rate of return the price of land is proportional to the price of the consumption services provided by land. We define the quantum of land so that the price of land is also measured by p^{b}_{t}; this implies that

$$\frac{p^{b}_{t+1}}{p^{b}_{t}} = \frac{\sum_{x=m,w} \mathbf{n}^{x'}_{t+1}.\mathbf{c}^{x}_{t+1}}{\sum_{x=m,w} \mathbf{n}^{x'}_{t}.\mathbf{c}^{x}_{t}} = \frac{(1+g)\sum_{x=m,w} \mathbf{n}^{x'}_{t+1}.\mathbf{c}^{x}_{t}}{\sum_{x=m,w} \mathbf{n}^{x'}_{t}.\mathbf{c}^{x}_{t}} \qquad (6)$$

It should be noted that this implies that p^{b}_{t} and $\sum_{x=m,w} \mathbf{n}^{x'}_{t}.\mathbf{c}^{x}_{t}$ are simultaneously determined. A given path for the price of services from land affects the consumption value of labour income and thus the total growth of consumption spending. However, equations (6) can be solved jointly with equations (2) and (3) and (5) to determine the prices of land and labour. With a rate of return *r*, equation (6) implies that the aggregate rent on land excluding the capital gain is, with *Q* being the value of the stock of land in the year in which $p^{b}_{t}=1$,

$$D_{t} = \left\{(1+r)\frac{p^{b}_{t}}{p^{b}_{t+1}} - 1\right\} Q p^{b}_{t} \qquad (5)$$

This indicates the return to land which can be consumed while retaining the stock of land to be bequeathed rather than sold to subsequent generations. It is also the

measure of income which appears in the national accounts. We assume that the income from rent is credited to the adult population on a *per capita* basis, so that the amount received *per capita* is $d_t = \frac{D_t}{\mathbf{i}'(\mathbf{n}_t^m + \mathbf{n}_t^w)}$ where **i** is a vector of ones.

For given profiles of consumption, labour income and rent we can then calculate the value of the wealth holding $w^x_{t-\tau,t}$ needed to ensure that the budget constraint is met. The assumption that all produced wealth is annuitised means that survival rates also enter into this function. In this expression both *c* and *d* are already measured in consumption units, while effective labour supply needs to be multiplied by the price of labour in consumption terms,

$$w^x_{t-\tau,t} = \sum_{\theta=\tau}^{95} \frac{(\left[c^{C,x}_{t-\tau,\theta} - p^a_{t-\tau+\theta} y^{L,x}_{t-\tau,\theta}\right] - d_{t+\theta-\tau}) s^x_{t-\tau,\theta}}{s^x_{t-\tau,\theta}(1+r)^{(\theta-\tau)}}; \quad 95 \geq \tau \geq 20. \tag{6}$$

This analysis identifies the relationship between composite consumption and labour income which has to hold if each cohort does pay its own way. But, there is no reason why the current wealth holding of each cohort, its labour income expectation and its consumption plan should be consistent with the budget constraint; the constraint depends in any case on the assumptions made about future labour income and consumption plans. If we assume, as we do, that everyone plans ahead, then the budget constraint is represented by the restriction that the initial value of wealth, for the cohort currently aged 20, is zero, $w^x_{t-20,20}$=0. The observed consumption profile may not satisfy this.

We then scale the consumption profile derived from the data to impose the budget constraint, $w^x_{i,i+20}$=0. If consumption satisfying the budget constraint is denoted $\tilde{c}^{C,x}_{t-20,\theta} = \lambda^x_t c^{C,x}_{t-20,\theta}$ where

$$\lambda^x_t = \sum_{\theta=20}^{95} \frac{s^x_{t-20,\theta}\{ p^a_{t+\theta-20} y^{L,x}_{t-\theta,\theta}(1+g)^{(\theta-20)} + d_{t+\theta-20}\}}{(1+r)^{(\theta-20)}} \Big/ \sum_{\theta=20}^{95} \frac{s^x_{t-20,\theta} c^{C,x}_{t-\theta,\theta}(1+g)^{(\theta-20)}}{(1+r)^{(\theta-20)}} \tag{7}$$

can be calculated from the observed profile and assumptions about the growth rate and the interest rate. In these expressions in should be noted that consumption is, in effect, measured in terms of the numeraire of base-period labour income.

The affordability ratios for men and women inevitably differ. We focus on the average affordability ratio calculated as

$$\lambda_t = \frac{\sum_{x=m,w}\sum_{\theta=20}^{95} \frac{s^x_{t-20,\theta}\{\, p^a_{t+\theta-20} y^{L,x}_{t-\theta,\theta}(1+g)^{(\theta-20)} + d_{t+\theta-20}\,\}}{(1+r)^{(\theta-20)}}}{\sum_{x=m,w}\sum_{\theta=20}^{95} \frac{s^x_{t-20,\theta} c^{C,x}_{t-\theta,\theta}(1+g)^{(\theta-20)}}{(1+r)^{(\theta-20)}}} \tag{8}$$

This measure indicates the ratio of the discounted value of the affordable life-time consumption of a man and a woman added together to the value of the discounted life-time consumption as calculated from observed cross-section consumption. It defines what we subsequently refer to as affordability and is our first guide to savings adequacy.

Cross-section wealth holding *per capita* is given by

$$\tilde{w}^x_{t-\tau,\tau} = \sum_{\theta=\tau}^{95} \frac{\{(\lambda_t{}^x c^{C,x}_{t-\theta,\theta} - p^a_{t+\theta-\tau} y^{L,x}_{t-\theta,\theta})(1+g)^{(\theta-\tau)} - d_{t+\theta-\tau}\}s^x_{t-\tau,\theta}}{s^x_{t-\tau,\theta}(1+r)^{(t-\tau)}} \tag{9}$$

From these variables we can now calculate net national income, total wealth and total consumption. Total labour income, wealth and consumption are given as

$$\begin{aligned} Y^L_t &= \mathbf{n}^{m'}_t\ \mathbf{y}^{L,m}_t + \mathbf{n}^{w'}_t\ \mathbf{y}^{L,w}_t; \\ \tilde{W}_t &= \mathbf{n}^{m'}_t \tilde{\mathbf{w}}^m_t + \mathbf{n}^{w'}_t \tilde{\mathbf{w}}^w_t; \\ \tilde{C}^C_t &= \mathbf{n}^{m'}_t \mathbf{c}^{L,m}_t + \mathbf{n}^{w'}_t \mathbf{c}^{L,w}_t; \end{aligned} \tag{10}$$

From these variables we calculate the required national saving rate, our second measure of savings adequacy, as

$$\tilde{\sigma}_t = \frac{p^a_t Y^L_t + D_t + r\tilde{W}_t - \tilde{C}^C_t}{p^a_t Y^L_t + D_t + r\tilde{W}_t} \tag{11}$$

with the actual savings rate denoted σ_t

Even, of course if $\lambda_t = 1$, so that the intertemporal budget constraint can be balanced with consumption taking its unscaled value, there is no guarantee that past saving has

been adequate. Thus $\sigma_t = \tilde{\sigma}_t$ does not on its own guarantee that the consumption profile can be afforded. For our third measure of savings adequacy, we identify the required holding of wealth measured as the ratio of required produced wealth to labour income

$$\tilde{\gamma}_t = \tilde{W}_t / p_t^a Y_t^L \qquad (12)$$

and compare this with the actual ratio of produced wealth to labour income

$$\gamma_t = V_t / p_t^a Y_t^L \qquad (13)$$

This provides a means of assessing the adequacy of past saving rather than simply calculating current saving needs on the assumption that past saving has been adequate.

Finally there is a question how far current holdings of wealth can finance the consumption profiles represented by $\boldsymbol{c}^{\boldsymbol{x}}_t$ rather than $\tilde{\mathbf{c}}^x_t$; if saving in the past has been high, it might be possible to pay for the current consumption profile for those currently economically active even though the consumption of current young people is too high to be financed out of their life-time income.

Our fourth indicator of savings adequacy therefore looks at the sum of actual wealth and the present discounted value of the future labour and rent income of the current population relative to the present discounted value of their consumption plan. It is given by

$$\mu_t = \frac{\left\{ V_t + \sum_{x=m,w} \sum_{\tau=20}^{T_{max}} n^x_{t-\tau,\tau} \sum_{\theta=\tau}^{T_{max}} \frac{\{ p^a_{t+\theta-\tau} y^{L,x}_{t-\theta,\theta} (1+g)^{(\theta-\tau)} + d_{t+\theta-\tau} \} s^x_{t-\tau,\theta}}{s^x_{t-\tau,\tau} (1+r)^{(\theta-\tau)}} \right\}}{\sum_{x=m,w} \sum_{\tau=20}^{T_{max}} n^x_{t-\tau,\tau} \sum_{\theta=\tau}^{T_{max}} \frac{c^{c,x}_{t-\theta,\theta} (1+g)^{(\theta-\tau)} s^x_{t-\tau,\theta}}{s^x_{t-\tau,\tau} (1+r)^{(\theta-\tau)}}} \qquad (14)$$

Here the numerator is the present discounted value of the resources available to those currently aged twenty or more, as the sum of actual produced wealth and the present

discounted values of labour income and rent. The values of these *per capita* are aggregated taking account of the size of each cohort. The denominator is the present discounted value of unscaled consumption calculated on the same basis. Thus the ratio of the two, denominated by $_t$ indicates the extent to which consumption must be scaled in the light of the amount of accumulated capital accrued through past saving. This contrasts with $_t$ which shows how the consumption of the current twenty-year olds needs to change to that they can satisfy their life-time budget constraint.

3. Data Sources and Measurement Issues

Income and consumption profiles

It is clear from the previous sections that the profiles for labour income and consumption form the core data required for our analysis. Such profiles, in so far as they relate to household or individual activities, are derived from the results of household income and expenditure surveys. In our context this is relevant for the whole of labour income since it all accrues to individuals. It is also relevant to household consumption, but not to consumption activity carried out by the government, whether collective or allocable to individuals.

There are, in broad terms, two possible approaches to the construction of these household profiles. The simpler one is to use the cross-section provided by an annual survey. This in essence constructs a snapshot view of the labour income or consumption of people of different ages in any particular year and uses this cross-section to construct an estimate of the expected income/taxation or planned consumption of younger people. In a general environment in which incomes and consumption rise over time it would be unreasonable to assume that the consumption of a seventy year-old today is a good forecast of the consumption plan of a current thirty year old in forty years time. But if we make an assumption that underlying growth is superimposed on this cross-section profile, then the idea that expectations about future incomes or consumption plans can be derived using the cross-section profiles is much more reasonable. This is the approach of equation (3).

This approach has the great advantage that the income data and the individual consumption data can be drawn from a consistent data set. It is, nevertheless, open to the objection that it confuses time effects, age effects and cohort effects. Deaton

(1997) showed that, from a succession of cross-section surveys and with appropriate identifying restrictions, it was possible to separate these out by regressing the variables of interest on appropriate dummy variables. While, when the underlying framework is stable, the approach has obvious attractions, there is, nevertheless a major disadvantages. The profile showing consumption or labour income as a function of age is best interpreted as an average for the estimation period. This might not be very helpful if, for example there is substantial fluctuations in the labour market participation rate of old people or trends in the relationship between consumption and income during the period in question. In such circumstances the profiles might well provide a less satisfactory indicator of prospective consumption and labour income patterns than could be evaluated from current data. We therefore use recent household income and expenditure surveys as set out in Appendix 1.

Observed government consumption for the UK, $\tilde{c}_{it}^{G,x}$ and indirect taxes $\tilde{\tau}_{it}^{C,x}$ are taken from updated profiles[3] used in the construction of generational accounts (Cardarelli, Sefton and Kotlikoff, 2000). These profiles again provide cross-section values and are growth-adjusted before use. For the other countries the OECD provides information on public spending per person classified by age. We assume that indirect taxes are represented by net taxes on production and imports as shown in the national accounts and that the profile of these is proportional to that of personal consumption. We have scaled the age profiles so that, given the age structures of the population, the profiles are consistent with national accounts data for 2005 as shown in Table 6 of section 9.

4. Households and Individuals

An issue arises over the distinction between households and individuals. The household budget surveys used in this study that collect consumption information are constructed around the concept of the household. The reason for this is obvious. Some components of consumption (such as food) may not be consumed by the individual responsible for procurement; other types of consumption such as costs related to housing are collective rather than individual in nature. In either case an arbitrary allocation is needed to attribute consumption to individuals. The income information is, however set out in terms of individuals; in the case of benefits calculated in the

[3] We are grateful to James Sefton for providing these profiles.

light of family circumstances this means that the income is assumed to accrue to the person legally entitled to receive the benefit.

Some households comprise more than two adults and these are inherently difficult to unravel. We exclude these from the consumption data, so that all households in the sample used to calculate the profiles consist of one or two adults[4]. There are in fact three types of family unit[5] to consider; single men, single women and couples. All three types of family unit may have dependent children present.

We allocate reported consumption between men and women by making the assumption that, at each age, the consumption of women is proportional to that of men with the ratio of the two being given the ratios of the present discounted values of net non-property income for women and men evaluated at the age of 20[6]. This provides the required profiles for individual consumption by sex. The budget surveys analysed in this paper allow us to identify the proportion of men and women of each age who live as couples rather than singly; we denote these proportions ψ_t^M and ψ_t^W and make the simplifying assumption that in any couple both partners are of the same age. We extend our earlier notation so that the total number of family units is given as $n_{t-\tau,\tau}^F$ with family consumption $c_{t-\tau,\tau}^F$ for units born in year *t*- and observed in year *t*. With $n_{t-\tau,\tau}^M$ representing the number of men and $n_{t-\tau,\tau}^W$ the number of women (provided by the mid-year population estimates), then

$$n_{t-\tau,\tau}^F c_{t-\tau,\tau}^F = n_{t-\tau,\tau}^M c_{t-\tau,\tau}^M + n_{t-\tau,\tau}^W c_{t-\tau,\tau}^W \qquad (15)$$

as a matter of identity. We also calculate the number of family units as

$$n_{t-\tau,\tau}^w = \frac{1}{2}\left(\psi_t^m n_{t-\tau,\tau}^m + \psi_t^w n_{t-\tau,\tau}^w\right) + (1-\psi_t^m) n_{t-\tau,\tau}^m + (1-\psi_t^w) n_{t-\tau,\tau}^w \qquad (16)$$

With the ratio of men's to women's consumption given by

[4] The point is often made that in Italy and Spain most older and very old people live with their children. In fact we did not find that. In Italy the survey showed 74% of men and 86% of women aged 71-80 living on their own or with one other adult. There are, however problems with adults in their twenties, since only 20% of women and 26% of mean aged 20-29 live on their own or with one other adult. There are also problems of parents with adult children. Only 39% of mean and 36% of women aged 51-59 live on their own or with one other adult. There is a risk that selection bias may result in the consumption of people in their twenties being overstated since, presumably, it is a high income which makes it possible for people to live on their own or with one other adult.

[5] Family units were formerly referred to as benefit units in the United Kingdom.

[6] Using a real interest rate of 4% p.a. and a growth rate of 1.5% p.a. which represents our "core" projection.

$$\phi = c^{m}_{t-\tau,\tau} / c^{w}_{t-\tau,\tau} \qquad (17)$$

the three equations (15)-(17) can be used to calculate $c^{I,m}_{t-\tau,\tau}$ and $c^{I,w}_{t-\tau,\tau}$ from the observed consumption per household unit. The calculations provide us with profiles which take account of the fact that the ratio of single to married people changes with age, and also of the fact that the balance of single men to single women changes considerably with age; single old people are predominantly female. At ages where most people live in couples the consumption of men and women is well below that of the average family unit while in old age when there are many more single people and women predominate the consumption of men given by the above approach rises above that of the average family unit.

This approach carries with it the implication that consumption by adults supporting their children is treated on the same basis as consumption by adults. It offers a simple framework in which to work, albeit with the disadvantage that changes in family size over time are not reflected in changes to the consumption profile.

The profiles for government spending and also for indirect taxes net of subsidies provided to us were constructed *per capita* with resources as well as taxes being allocated to children as well as to adults; separate profiles exist for men and women. To put them on the same basis as the profiles for individual consumption, we construct matrices showing the total number of children of each sex classified by the age of the head of the family unit. For male children we then multiply by the estimate of the overall number of family units estimated above and divide by the number of men in each age. This gives an array which allocates the boys across the male adult population by age of each and is used to allocate the public consumption and net indirect taxes in the same way. Consumption and taxes attributed to girls in the generational accounting profiles are allocated to women in the same way.

5. Survival Rates

As equation (6) makes clear, the results depend on the survival rates of the different cohorts. The availability of forecasts of survival rates varies considerably from

country to country[7]. France provides projections of mortality rates by age. The French data[8] are provided for 2005 to 2050 and show death rates tabulated by age reached during that year for all age from 0 to 120. We extrapolate these death rates to 2080 by taking the average annual change in the log death rate at each age from 2041 to 2050 and applying this trend rate of change to the log of the death rates from 2050 onwards.

Italy and Spain publish interim life tables but do not publish projections of mortality rates (although they must make projections of some sort in order to publish population projections). We therefore construct our own forecasts of mortality rates using the method proposed by Lee and Carter (1992). Working with mortality rates by age for each year since 1970 taken from the Human Mortality Database[9], we calculate the first principal component of the correlation matrix of the log mortality rate by age for each year from 1970 to 2005. We project this component by from its 2005 value, using the average growth rate over the period 1970 to 2005 and use the correlations between the component and the log mortality rates to project the latter up to 2080. This method ensures that the age structure of log mortality rates is largely preserved. Again we carry out separate calculations for men and for women.

The United Kingdom Government Actuary publishes tables of forecast cohort life expectancy by age up to 2056 but does not provide the estimates of mortality which underlie these. We therefore work from these published data as follows. We calculate the ratios of adjacent survival rates as

$$\frac{s_{t-\tau,\tau+1}}{s_{t-\tau,\tau}} = \frac{e_{t-\tau,\tau}-1}{e_{t-\tau,\tau+1}} \qquad (18)$$

In order to provide life expectancies beyond 2056 we assume that the trend increase log mortality rates after 2056 is the same as that between 2046 and 2056 in order to derive the required survival rates.

[7] Scholz *et al.* (2006) do not consider the issue of rising survival rates, simply assuming that people can plan on the basis of 2002 period life tables.
[8] http://www.insee.fr/en/ppp/ir/accueil.asp?page=projpop0550/dd/projpop0550-hypotheses.htm
[9] http://www.mortality.org/

The combination of interpolation and the fact that the numbers from the Government Actuary[10] are rounded means that at young ages survival rate ratios very slightly above one can be generated. These ratios are set to one. Separate calculations are carried out for men and women.

The results of the projections are summarised in the table below which shows the forecast probability of a twenty-year old in 2005 surviving to the age of ninety-five. We compare the forecast probabilities with those derived from the interim life tables for the countries concerned. The table demonstrates the powerful impact of rising survival rates on the prospects for youth. Comparison of the two sets of survival probabilities allows us to demonstrate the impact of rising longevity on savings needs.

Table 1 The Probability of a Twenty-year old in 2005 Surviving to Age Ninety-five

	Man 20 Period	Woman 20 Period	Man 20 Cohort	Woman 20 Cohort
France	0.058	0.169	0.217*	0.525*
Italy	0.056	0.144	0.281**	0.548**
Spain	0.046	0.112	0.241**	0.432**
UK	0.063	0.189	0.316*	0.443*

* Calculated by extrapolation of mortality rates provided by national sources
** Calculated by use of Lee-Carter method applied to 1970-2004 for Italy and 1970-2006 for Spain

These tables show, for the youngest cohorts fairly high survival rates to the age of 95, which we have set as the upper age limit. Despite this the impact of the approximation for our measures of interest is small because of the degree of discounting involved.

6. Determinants of Savings Needs

It is clear from the above analysis that there are going to be five determinants of the savings needs of each country. These are

i) The profile of labour income. A country which tends to earn its labour income early in life will, other things being equal, need to hold more wealth and to save more for its retirement than one which earns its labour income later in life.

[10] http://www.gad.gov.uk/Life_Tables/eoltable.htm

ii) The profile of income from land. While is obviously not as important as that from labour income, our assumption that it accrues on a *per capita* basis may turn out to be of importance for savings behaviour.

iii) The profile of consumption. Countries in which the consumption of people post-retirement declines markedly with age do not need to save as much for their retirement as countries where consumption of older and very old people holds up.

iv) The population structure. A disproportionate number of very young people will probably lead to low saving because most studies suggest that labour incomes peak in middle age. With reasonably stable consumption by age, so too will saving. Similarly, since the life-cycle framework suggests that people dis-save after retirement, a disproportionate number of older and very old people will lead to low saving.

v) Mortality rates. A population with low mortality rates will need to plan for a long period of post-retirement consumption. This needs to be financed by saving during working life. If there is no economic growth and the population structure is stable, then, of course, there will be no net saving. A country with low mortality rates will nevertheless, need a high ratio of wealth to income.

vi) The rate of return. This obviously influences the current cost of future consumption with high returns reducing savings needs.

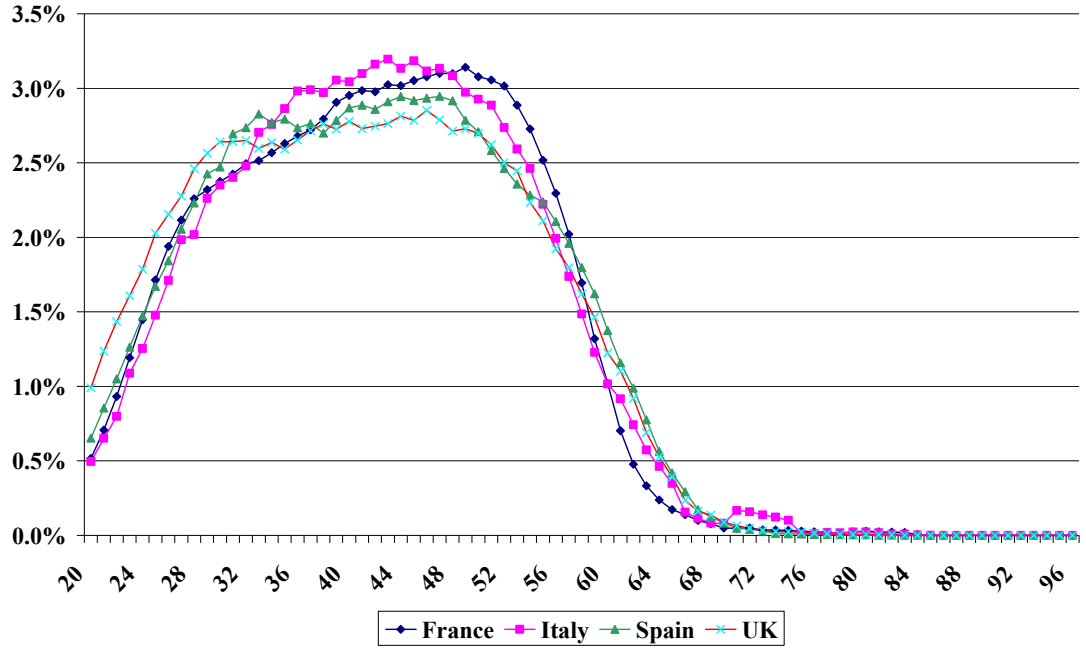
vii) The rate of growth. Conversely if consumption is planned to grow rapidly, savings needs are increased.

We discuss these factors in turn and the show how we can investigate the importance of demographic influences.

Labour Income Profiles

We show in Figure 1 labour income by age as a proportion of total life-time labour income. The figures are calculated by multiplying observed income at each age by the proportion of the twenty-year old population which survives to that age (based on the cohort survival probabilities for twenty-year olds in 2005), adding together the figures for men and women and dividing by the total of survival-adjusted labour income; thus the graph represents density functions for labour income.

Figure 1 Labour Income per capita as Proportion of Cross-section Total



The patterns in the four countries look very similar. There are, however, a number of differences of importance. First of all, we can observe that labour income falls off in the late fifties and early sixties faster in France and Italy than in Spain and the United Kingdom. This represents the different retirement policies and practices in the different countries. In particular, the late retirement in the United Kingdom, although not as late as in Spain, is often given as a reason for suggesting that the United Kingdom is less affected by problems of ageing than are its neighbours.

However, the graph also shows that young people in the United Kingdom accrue their labour income earlier than in the other countries. France and Italy, in particular, show a surge in earnings in the early fifties relative to the United Kingdom. Thus, despite working later, the United Kingdom might well need to start its saving early. British residents in their twenties also accrue a higher proportion of labour income than does this age group in the other countries, creating an additional need for early saving.

We can summarise this by calculating the mean labour income age. This is given as the sum over all ages of the proportion of labour income accruing at each age multiplied by the age in question.

$$P_t^L = \frac{\sum_{\tau=20}^{95} y_{t-\tau,\tau}^{L,m} s_{t-\tau,\tau}^{m} \tau + \sum_{\tau=20}^{95} y_{t-\tau,\tau}^{L,w} s_{t-\tau,\tau}^{w} \tau}{\sum_{\tau=20}^{95} y_{t-\tau,\tau}^{L,m} s_{t-\tau,\tau}^{m} + \sum_{\tau=20}^{95} y_{t-\tau,\tau}^{L,w} s_{t-\tau,\tau}^{w}}$$

As Table 2 shows, the mean labour income ages are very similar in France, Italy and Spain. In the United Kingdom, however, it is appreciably lower than in the other countries. Thus the profile of labour income, far from reducing savings needs in the United Kingdom because of late retirement, seems to increase them.

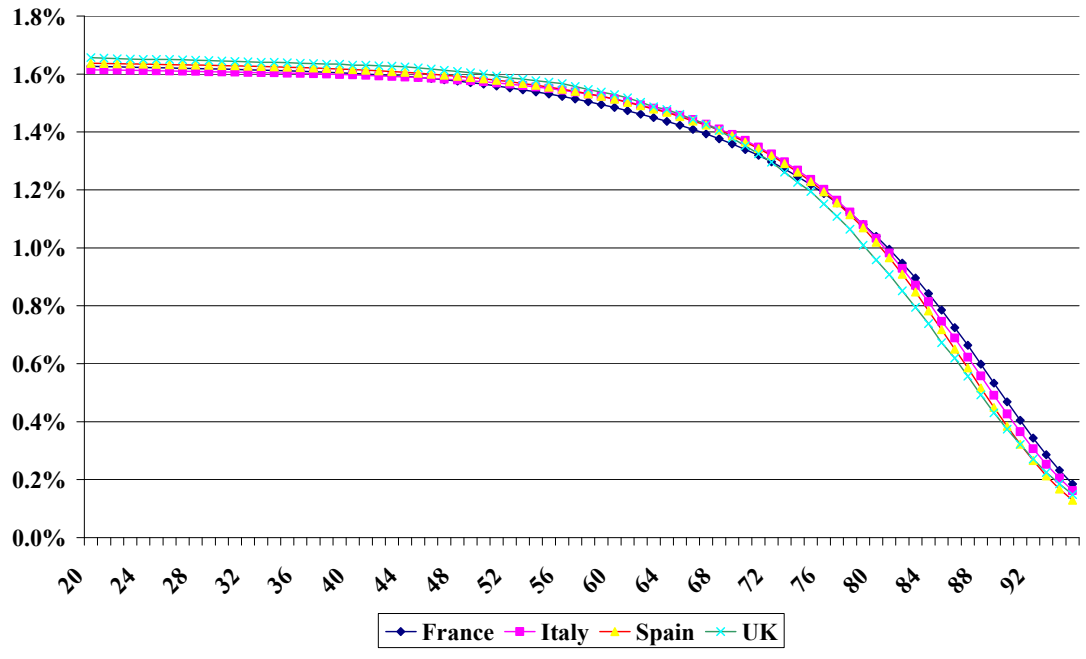
Table 2 The Mean Labour Income Age in 2005

Mean Labour Income Age in 2005			
France	Italy	Spain	UK
42.1	42.3	42.1	41.4

Land Income Profiles

The assumption that income from land accrues on a *per capita* basis defines the age profile of income from land. The analogy of the cross-section labour income profile is computed from the life tables which show the proportion of an initial population surviving to a given age. The profiles shown here are calculated from the interim rather than the cohort life tables by analogy with the labour income cross-section profiles.

Figure 2 Income from Land by Age per capital as Proportion of Total: Cross-section Data



Not surprisingly the pattern is very different from that of labour income. Computation of the mean age at which income from land accrues points to a figure appreciably higher than that found for labour income as Table 3 shows

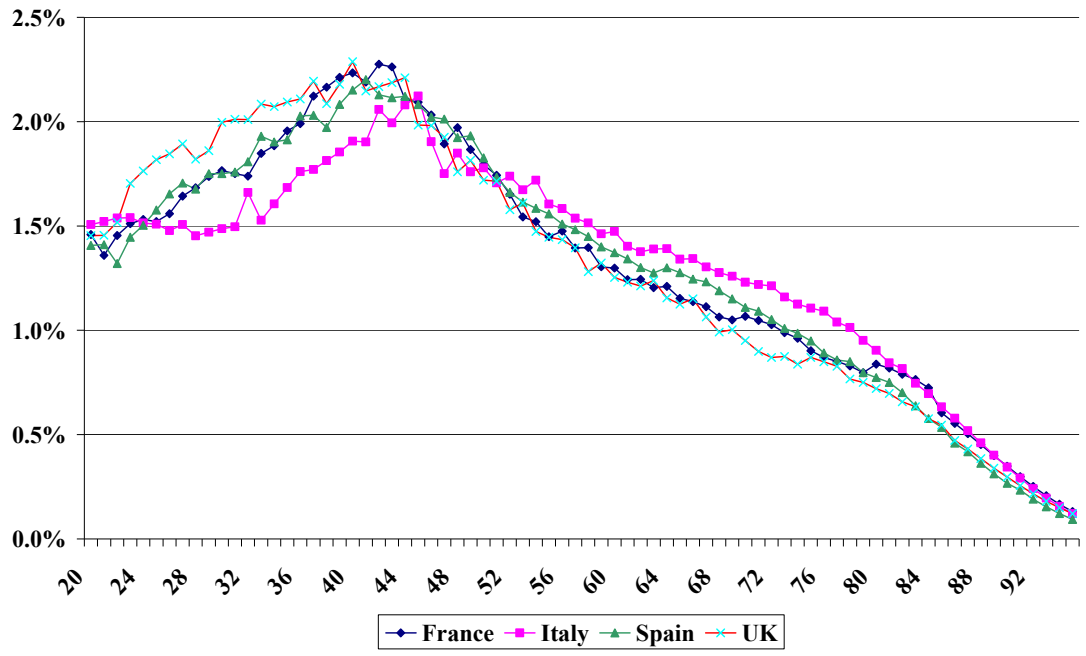
Table 3 The Mean Income for Land in 2005

Mean Income from Land in 2005			
France	Italy	Spain	UK
51.7	51.7	51.4	51.1

Consumption Profiles

These are calculated in the same way as the labour income and land profiles. We show them after the constraint of life-time budget balance has been imposed. This has a small impact because, although life-time budget balance is achieved by scaling the profile[11] - which has no effect on the proportions by age for men and women separately, it does mean that the balance between mean and women is varied slightly as a result of differential scaling.

Figure 3 Consumption Profiles as Proportion of Cross-section Total



It should be remembered that these profiles are survival-adjusted, which explains the decline in old age. Here it is striking that Italy has high consumption in old age relative to youth, while the opposite is true in the United Kingdom. As with labour income we can calculate a mean consumption age for each country as a way of summarising the profiles. This is shown in Table 4.

[11] Assuming our core interest rate of 4% p.a. and growth rate *per capita* of 1.5% p.a.

Table 4 The Mean Consumption Age

Mean Consumption Age			
France	Italy	Spain	UK
49.3	50.7	49.1	48.0

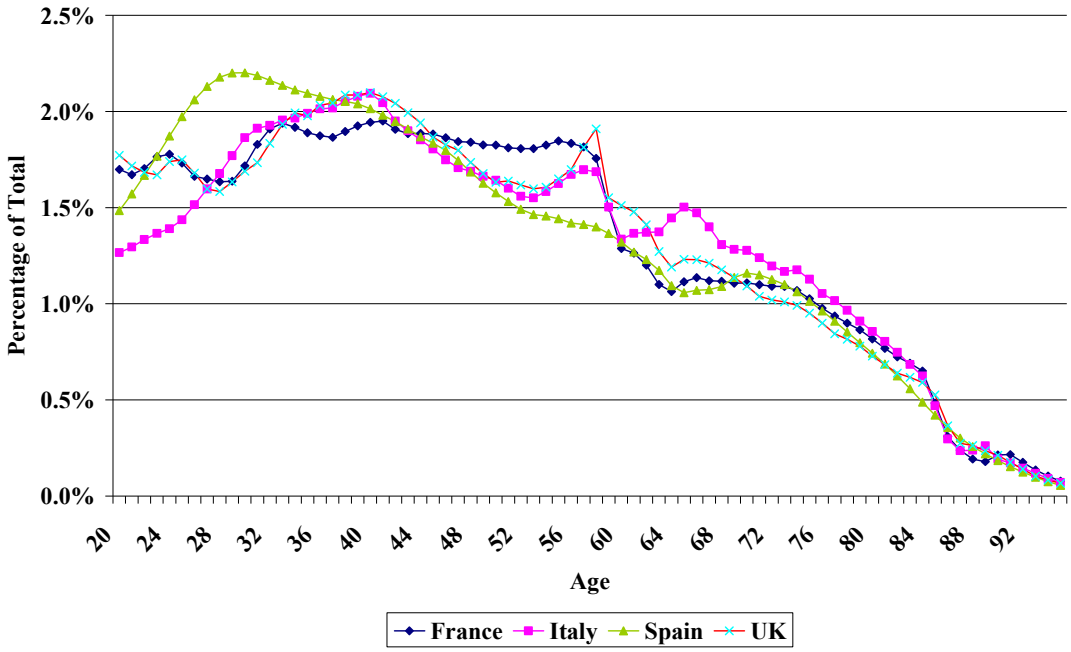
As the graph suggests, Italy is the latest consumer and the United Kingdom the earliest.

The land, labour income and consumption figures taken together suggest, in broad terms that people will need to save out of their labour income because labour income accrues before consumption. On the other hand income from land depresses saving because it accrues later in life. Thus the treatment of land could turn out to be of importance for the savings balance even though life-time income from land is smaller than life-time income from labour.

Population Structure

We summarise the actual population figures by showing the fraction of the population at each age in the four countries

Figure 4 Cross-section Population Structure



France, Italy and the United Kingdom all experienced a baby bulge at the end of the Second World War; this shows in the sharp rise in the proportion of the population aged fifty-seven in 2004 compared with that aged fifty-six. In France births continued

at a high rate while in Italy and the United Kingdom they dropped off, with the baby bulge then building up new momentum in the late fifties, reaching a peak in the mid sixties. Spain experienced a quite different pattern of births, with a much later baby bulge which reached its peak in the late seventies. Thus if substantial saving is carried out by people in their fifties, which is likely in countries where labour income peaks then, population structure boosts France's savings needs and depresses those of Italy. Savings needs in Italy are further depressed by the fact that it has a higher proportion of people aged sixty to seventy than do the other countries. These are almost certain to be dissavers.

Mortality Rates

The differences in mortality in the different countries are summarised in
Table 1. It is clear that there is a very substantial difference between a projection which ignores the question of rising mortality and one which takes it into account. Having done this, the differences between the countries are probably rather less important. Italy does, nevertheless have high projected survival rates for both men and women, raising savings needs.

Rates of Return

We work both with illustrative rates of return, so as to facilitate comparison between countries and with country-specific rates of return. The latter are, for France and the United Kingdom, calculated from national accounts data. The fact that the capital stock is measured at the start of the year while prices may be rising has the effect of reducing the value of the initial capital relative to the income it generates and thus biasing upwards the rate of return. We generate end-year consumption price increases as the means of the reported consumption deflators for adjacent years. We use these to deflate the capital stock figures and conventional consumption deflators to deflate the flow of property income so as to remove this inflation bias. This provides a flow estimate of the rate of return before taking into account the capital gains on land identified by equation (5). We evaluate these using the actual growth in total consumption in each year, taking the view that cyclical effects are likely to balance out over the period of our data.

However, we need to take account also of changes in the price of capital goods relative to consumption goods. In the United Kingdom there were substantial falls in the replacement cost of capital goods in the 1990-1992 recession, and these have not been made up since. Such relative price movements plainly affect the ability of the produced capital to finance the retirement of the people who paid for it. Our model does not project these continuing in future but due account should be taken of them in calculating the rate of return adjusted for capital gains[12].

For France the data start in 1979. For the United Kingdom they start in 1987. However the UK figures show a three-fold increase in the value of non-produced capital between 1987 and 1988. We regard this as suspect, and work from the 1988 data. Since the balance sheets relate to end-year data, this means that our series for France cover the period 1980-2006 and for the United Kingdom 1989-2006.

In both cases, we identify the flow of property income as operating surplus plus mixed income/3[13] plus net property income from abroad less depreciation.

In Figure 5 we show the rate of return before and after taking account of capital gains in France.

[12] There is a clear difference between this approach and that which includes all capital gains in the estimate of the rate of return. The one-off capital gains generated by a fall in required returns are included in the latter but not in our measure.
[13] Assuming that 2/3 of mixed income is return to labour.

Figure 5 The Rate of Return in France

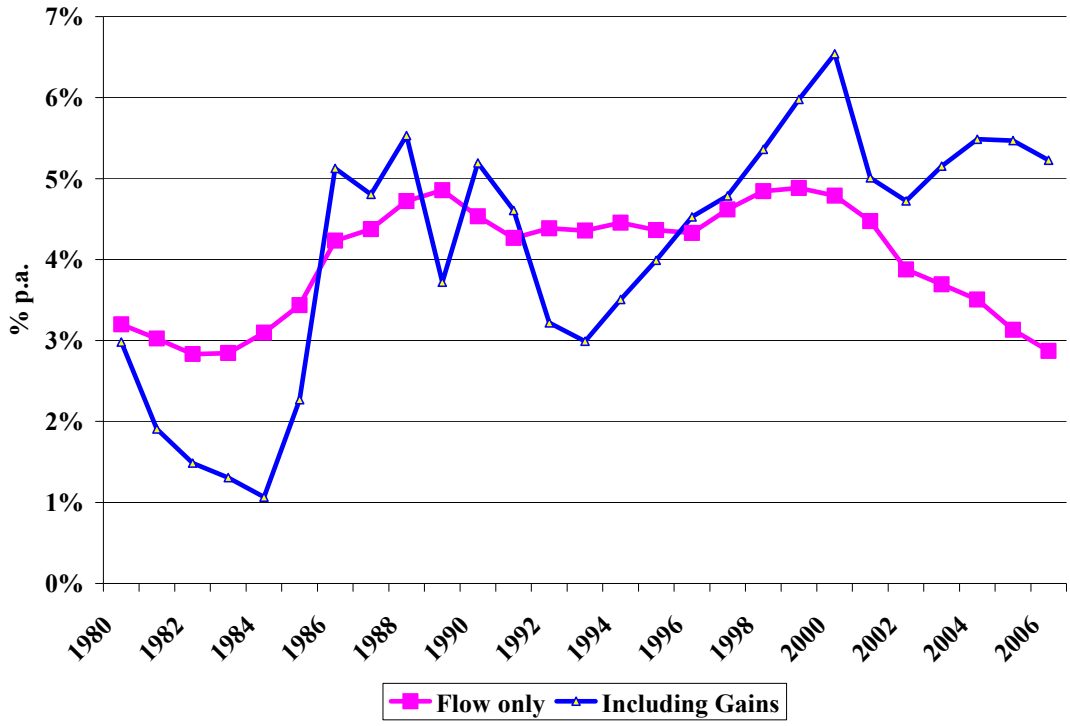


Figure 6 The Rate of Return in the United Kingdom

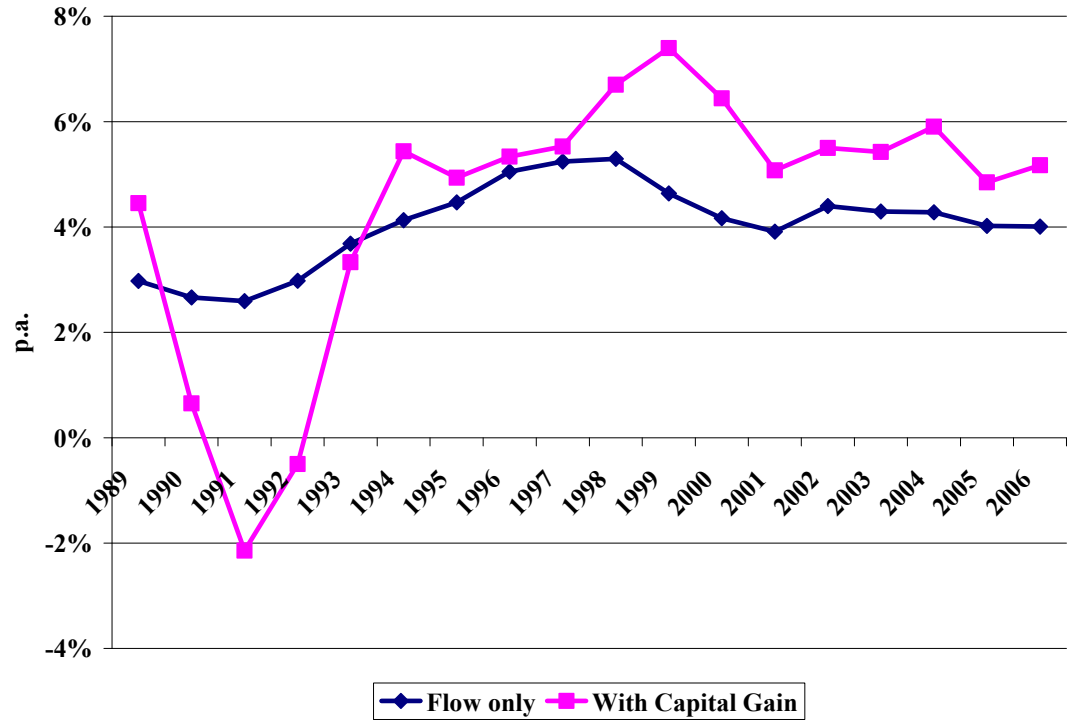


Figure 6 shows the same figures for the United Kingdom. The period averages of the two returns in each country in Table 5, suggest a reassuring degree of coincidence

Table 5 Average Returns in France and the United Kingdom

	France	United Kingdom
	1980-2006	1989-2006
Flow only	4.00% p.a.	4.04% p.a.
Including Capital Gains	4.14% p.a.	4.39% p.a.

For Italy and Spain with no balance sheets there is little we can do except produce figures for 2005 only. These are derived using the assumption that the ratio of non-produced capital to consumption in these countries is the same as in France. We then make the assumption that the rate of return in each country is the same on land as one produced assets. However the calculations take into account the capital gain on land holdings described by equation (4) and that the income distributed on holdings of land is reduced to reflect this as shown in equation (5). The relevant rates of return are shown in Table 6

Spain has a problem with a very low rate of return. This poor rate of return may be connected with its poor productivity performance (Mas and Quesada, 2007). Otherwise the figures are very similar. The rate of return in the United Kingdom is boosted by the fact that it received positive net property income on its overseas net assets despite the fact that the net value of these assets is negative[14]

Rates of Growth

We work with illustrative rates of growth from 0.5% p.a. to 2% p.a. regarding a growth rate of 1.5% p.a. as our core assumption.

7. National Income and National Wealth

We now turn to provide a macro-economic picture of our four economies. This is shown in Table 6.

[14] The data for 2006 and 2007 show a sharp decline in net income from abroad for the United Kingdom, falling from 2% of GDP in 2005 to an estimated -0.4% in 2007.

Table 6 The Macro-Economic Picture in 2005

	France €bn	Italy €bn	Spain €bn	UK £bn
Derivation of Net Saving				
Income from Employment	896.3	578.2	428.5	686.2
+Mixed Income	113.1	216.6	153.9	75.7
+Gross Operating Surplus	472.4	435.4	228.0	320.8
+Net Indirect Taxes	236.2	187.0	99.1	150.6
=GDP at Market Prices	1717.9	1417.2	908.5	1234.0
+Net Income from Abroad	1.3	-5.8	-15.2	26.3
+ Other Transactions	12.3	1.2	2.5	-1.6
=Gross National Income	1731.5	1412.6	895.7	1258.7
-Capital Consumption	223.9	207.8	138.7	131.1
+Net Foreign Transfers	-26.8	-9.8	-6.9	-11.1
=Net National Disposable Income	1480.8	1195.0	750.1	1116.5
-Net Indirect Taxes				
= Net Disposable Income at Factor Cost	1244.6	1008.0	650.9	965.9
-Household Consumption	718.8	641.8	417.7	610.2
-Public and NPISH Consumption	431.0	292.9	171.8	300.5
=Saving	94.9	73.4	61.5	55.3
Net Savings as Proportion of Net Income	**7.6%**	**7.3%**	**9.4%**	**5.7%**
Net Savings as Proportion of GDP	**5.5%**	**5.2%**	**6.8%**	**4.5%**
Wealth and Labour Income				
Labour Income=Employment Income				
+ (2/3) Mixed Income	971.7	722.6	531.1	736.6
As per cent of net income at factor cost	78.1%	71.7%	81.6%	76.3%
Start of Year Wealth				
Produced	5452.1	4528.9	3297.8	2658.1
Total	9387.8	7755.9	5241.0	5915.0
End of Year Wealth				
Produced	5916.3	4778.1	3975.6	2755.0
Total	10761.3	8716.6	6159.2	6198.7
Inflation-adjusted start of year produced wealth[15]	5504	4578	3353	2685
Produced capital/labour income[16]	5.9	6.4	6.6	3.7
Rates of Return				
Net Operating Surplus	287.5	294.0	125.4	241.2
Rate of Return[16] % p.a.	4.14%	4.26%	3.43%	4.39%

[15] The initial stock of produced wealth augmented for the effects of six months inflation, based on annual HICP rates for 2005 of 1.9%, 2.2%, 3.4% and 2.1% in France, Italy, Spain and the United Kingdom.

[16] Capital/labour ratios are calculated using inflation-adjusted wealth at the start of the year. For Italy and Spain rate of return is calculated from this and from the mean of the opening and closing values of end-year wealth net of six months expected appreciation. The expected appreciation is calculated using an assumed rate of growth of per capita consumption measured in terms of labour income of 1.5% p.a. See equations (4) and (5). For France and the UK the rates of return are time series averages from Table 5.

The table shows substantial differences between the saving behaviour and wealth holdings in the different countries. Given the analysis of section 5, we focus on produced wealth holdings rather than total wealth holdings. Produced wealth includes produced capital (stocks as well as fixed capital) and also net foreign assets, since these are the outcome of saving in excess of domestic investment.

The United Kingdom continues to be a low-saving country and, as a consequence of its history of low saving, it has a low ratio of produced wealth to labour income. At the other extreme Spain, with a high savings ratio shows a high ratio of produced wealth to labour income.

For the purpose of scaling our profiles and calculating rates of return we need estimates of total labour income. We have assumed that 2/3 of gross mixed income is a return to labour rather than to capital and added this to income from employment to as to provide the value of labour income which we use to align the profiles calculated using the same definition of income from the micro-economic sources.

We align separately household and public consumption, with the consumption of non-profit institutions serving households included with the latter. This provided the total used for a final scaling of publicly-provided consumption.

8. Savings Needs Compared

Using the framework set out above we explore the implications of the consumption and spending patterns identified above. We calculate affordability, the required savings rate, the required ratio of wealth to labour income and the wealth gap conditional on different assumptions about the growth rate and the real rate of interest. We do this first for illustrative growth rates from 0.5% pa. to 2% p.a. and rates of return of 4% p.a. and 5% p.a..

Table 7 Affordability Ratios of Consumption Profiles for Young People ($_t$

		Labour Income Growth Rate			
Real interest rate		0.5% p.a.	1% p.a.	1.5% p.a.	2% p.a.
France					
4% p.a.	Without Land	0.863	0.852	0.838	0.821
	With Land	0.983	0.957	0.929	0.898
5% p.a.	Without Land	0.876	0.871	0.863	0.852
	With Land	1.030	1.009	0.987	0.962
Italy					
4% p.a.	Without Land	0.812	0.799	0.783	0.763
	With Land	0.952	0.921	0.887	0.849
5% p.a.	Without Land	0.827	0.821	0.811	0.798
	With Land	1.005	0.980	0.953	0.921
Spain					
4% p.a.	Without Land	0.926	0.912	0.895	0.874
	With Land	1.027	0.999	0.967	0.930
5% p.a.	Without Land	0.944	0.936	0.926	0.912
	With Land	1.078	1.056	1.031	1.002
United Kingdom					
4% p.a.	Without Land	0.837	0.826	0.813	0.796
	With Land	0.948	0.922	0.893	0.860
5% p.a.	Without Land	0.851	0.845	0.836	0.826
	With Land	0.996	0.976	0.953	0.927

The results show the importance of land as a means of financing consumption. If it is ignored affordability ratios are substantially below those computed after the income it generates is taken into account.

It can be seen that Italy has very severe affordability problems with young people needing to reduce consumption sharply unless the rate of growth is low or the return on capital is high. Spain by contrast has no great difficulties unless growth is rapid and the rate of return is 4 per cent per annum; indeed with a real rate of return of 5 per cent per annum it could afford to increase its consumption. The United Kingdom's position is only slightly more favourable than Italy's while France's situation is appreciably more favourable. But a general conclusion from these results is that, unless rates of return are close to, or above 5 per cent per annum, young people need

to consume less than our consumption profiles imply. They might also, of course, work longer to raise their labour incomes, an issue which we discuss in section 11.

Table 8 The Required National Savings Rate (net saving/net national income) using Actual Population and Rising Survival Rates ($\tilde{\sigma}_t$)

		Labour Income Growth Rate				
Real interest rate		0.5% p.a.	1% p.a.	1.5% p.a.	2% p.a.	Actual
France						
4% p.a.	Without Land	10.4	12.6	15.1	17.8	
	With Land	8.0	9.8	12.0	14.5	
5% p.a.	Without Land	9.1	10.9	13.0	15.3	
	With Land	6.7	8.1	9.8	11.8	7.6
Italy						
4% p.a.	Without Land	11.1	13.6	16.5	19.6	
	With Land	8.1	10.5	13.2	16.4	
5% p.a.	Without Land	9.7	11.9	14.3	17.0	
	With Land	6.7	8.6	10.9	13.5	7.3
Spain						
4% p.a.	Without Land	9.1	11.4	14.0	16.9	
	With Land	5.7	7.7	10.3	13.2	
5% p.a.	Without Land	7.6	9.4	11.6	14.0	
	With Land	4.3	5.9	7.8	10.1	9.4
United Kingdom						
4% p.a.	Without Land	9.4	11.6	14.0	16.7	
	With Land	7.2	9.1	11.3	14.0	
5% p.a.	Without Land	8.2	10.0	12.0	14.3	
	With Land	5.9	7.3	8.9	11.0	5.7

Using the framework set out above we calculate in Table 8 the required national savings rate. Except with the assumption of a very low growth rate and a high rate of return the actual savings rate for all the countries (though less so for Spain) is well below what is required. This implies that people either have to accept lower consumption paths than those assumed or work longer to deliver more labour income if one assumes that each cohort is assumed to pay its own way. This is obviously a consequence of the fact that the affordability ratios shown in Table 7 are below one. If consumption needs to be reduced, then saving must be expected to rise. However the

differences between the required and actual savings rates are smaller than the differences implied by the affordability ratios, because the required savings rates reflect the actual age structures of the population.

We now turn, in Table 9 to look at the required ratio of produced wealth to labour income required by the scaled consumption profiles, on the assumption that people older than twenty have, in the past, done the saving required to pay for the consumption indicated by the scaled profile of current twenty-year olds suitably adjusted for the effects of growth.

Table 9 The Ratio of Produced Wealth to Labour Income using Actual Population and Rising Survival Rates ($\tilde{\gamma}_t$)

		Labour Income Growth Rate				
Real interest rate		0.5% p.a.	1% p.a.	1.5% p.a.	2% p.a.	Actual
France						
4% p.a.	Without Land	3.4	3.7	4.1	4.4	
	With Land	2.8	3.1	3.5	3.9	
5% p.a.	Without Land	2.7	3.0	3.4	3.7	
	With Land	2.1	2.4	2.7	3.0	5.9
Italy						
4% p.a.	Without Land	4.5	4.9	5.3	5.6	
	With Land	3.9	4.3	4.7	5.2	
5% p.a.	Without Land	3.7	4.1	4.4	4.8	
	With Land	3.0	3.4	3.8	4.2	6.4
Spain						
4% p.a.	Without Land	3.3	3.6	4.0	4.3	
	With Land	3.0	3.4	3.8	4.2	
5% p.a.	Without Land	2.7	3.0	3.3	3.6	
	With Land	2.4	2.6	3.0	3.3	6.6
United Kingdom						
4% p.a.	Without Land	3.3	3.6	4.0	4.3	
	With Land	3.0	3.3	3.7	4.1	
5% p.a.	Without Land	2.7	3.0	3.3	3.6	
	With Land	2.3	2.6	2.9	3.2	3.7

The table shows that, in France, Italy and Spain, holdings of wealth are higher than those required by the scaled profiles. They also show, once again, the importance of land as a means of financing life-cycle consumption. Given the excess of wealth holdings there is a question whether wealth levels are high enough to finance consumption paths without any scaling to impose solvency on the youngest cohorts.

In Table 10 we therefore show the scaling adjustment to current consumption needed so that the existing stock of produced wealth, plus current and future labour income and rent on land are adequate to finance current consumption plans. As with the earlier findings, the results obviously depend on the assumed rates of growth and rates of return, but they give a general impression that Spain has plenty of produced wealth unless returns are low and or growth rates high. In the other countries there will be a deficiency unless the rate of growth is very low or the rate of return high. The adjustment needed in the UK is the most marked and reflects its historically low savings rates.

Table 10 Affordability of Current Consumption to the Current Adult Population ($_t$)

		Labour Income Growth Rate			
Real interest rate		0.5% p.a.	1% p.a.	1.5% p.a.	2% p.a.
France					
4% p.a.	Without Land	0.969	0.934	0.899	0.863
	With Land	1.100	1.052	1.003	0.953
5% p.a.	Without Land	1.035	1.003	0.969	0.935
	With Land	1.204	1.158	1.111	1.063
Italy					
4% p.a.	Without Land	0.896	0.861	0.826	0.790
	With Land	1.036	0.984	0.930	0.876
5% p.a.	Without Land	0.929	0.895	0.860	0.824
	With Land	1.111	1.058	1.005	0.951
Spain					
4% p.a.	Without Land	1.071	1.033	0.993	0.951
	With Land	1.178	1.124	1.067	1.007
5% p.a.	Without Land	1.148	1.113	1.075	1.036
	With Land	1.289	1.239	1.187	1.131
United Kingdom					
4% p.a.	Without Land	0.848	0.822	0.795	0.766
	With Land	0.977	0.934	0.888	0.839
5% p.a.	Without Land	0.898	0.873	0.848	0.822
	With Land	1.069	1.029	0.988	0.943

9. Actual Rates of Return

While the figures above illustrate the effects of particular rates of return and rates of growth on the four countries, these are purely illustrative. Accordingly we also present results for the variables above evaluated at the returns shown in Table 6 for 2005.

Table 11 Savings and Wealth Needs with Observed Rates of Return

	France	Italy	Spain	United Kingdom
Rate of Return (% p.a.)	4.14	4.26	3.43	4.39
Affordability for twenty-year olds (t	0.938	0.905	0.924	0.918
Required Savings Rate (%)($\tilde{\sigma}_t$)	11.7	12.6	11.9	10.3
Actual Savings Rate (%)(σ_t)	7.6	7.3	9.4	5.7
Required Produced Wealth/Labour Income ($\tilde{\gamma}_t$)	3.4	4.5	4.3	3.3
Actual Produced Wealth/Labour Income Ratio (γ_t)	5.9	6.4	6.6	3.7
Affordability for current population ($_t$	1.018	0.950	0.996	0.928

There are a number of similarities between the countries. The consumption profiles of twenty-year olds is not affordable and needs to be reduced by an amount ranging from just over 6% in France to nearly 10% in Italy. The three continental countries have similar required savings ratios although France requires less produced wealth than Italy and Spain. UK savings needs and required wealth holdings are appreciably lower than on the continent. However, there are much greater disparities in actual ratios of wealth to labour income. The high levels of actual wealth, resulting from historically high saving means that the population currently economically active comes close to affording its consumption plan in France and Spain. Although past saving in Italy has been high it has not been high enough given the country's population structure and spending patterns. A reduction of 5% in consumption is needed. Low past saving in the United Kingdom means that existing wealth is not nearly high enough to meet the budget constraint, and that consumption needs to be reduced by 7.2% if those currently alive are to pay their own way.

10. Implications of Declining Mortality

One might reasonably ask how far these results are affected by the declining mortality shown in Table 1. We therefore show in Table 12 estimates of savings needs with mortality rates remaining at those observed in 2005.

Table 12 Savings and Wealth Needs with Constant Mortality Rates

	France	Italy	Spain	United Kingdom
Rate of Return (% p.a.)	4.14	4.2	3.2	4.4
Affordability for twenty-year olds (t	0.958	0.914	0.957	0.935
Required Savings Rate (%)($\tilde{\sigma}_t$)	9.3	11.5	8.0	7.4
Actual Savings Rate (%)(σ_t)	7.6	7.3	9.4	5.7
Required Produced Wealth/Labour Income ($\tilde{\gamma}_t$)	3.2	4.4	4.0	3.0
Actual Produced Wealth/Labour Income Ratio (γ_t)	5.9	6.4	6.6	3.7
Affordability for current population ($_t$	1.048	0.965	1.045	0.963

With no increase in mortality the consumption problems of young people are mitigated, but it remains the case that the current consumption profile is not affordable. Savings and wealth needs also fall. With no increase in mortality, France and Spain would no longer have any problem in financing current consumption plans. On the other hand Italy and the United Kingdom would remain with a substantial shortfall. For all countries, however, the results demonstrate that falling mortality has an important influence on savings needs, suggesting that the results presented by Scholz *et al;*. (2006) might have been rather different if they had used projections of cohort mortality rates instead of 2002 period mortality rates in their calculations (see footnote 7).

11. Working Longer

It is often suggested that extending working life is a means of resolving the problems arising from population ageing and falling mortality rates. We explore the implications of this by assuming that between the ages of fifty-five and sixty people earn the labour incomes shown by our profiles at age fifty-five. From age sixty-one and onwards they earn the labour incomes currently accruing at age fifty-six and onwards. Thus we look at extending working life by five years with the wage reflecting what people earn late in their working lives.

Table 13 Savings and Wealth Needs with Working Life Extended Five Years

	France	Italy	Spain	United Kingdom
Rate of Return (% p.a.)	4.14	4.2	3.2	4.4
Affordability for twenty-year olds (t	0.996	0.905	0.985	0.959
Required Savings Rate (%)($\tilde{\sigma}_t$)	8.2	12.6	8.3	7.6
Actual Savings Rate (%)(σ_t)	7.6	7.3	9.4	5.7
Required Produced Wealth/Labour Income ($\tilde{\gamma}_t$)	1.8	4.5	2.8	1.8
Actual Produced Wealth/Labour Income Ratio (γ_t)	5.9	6.4	6.6	3.7
Affordability for current population ($_t$	1.142	0.950	1.108	1.029

These results show that working five years longer is inadequate to make the current consumption pattern affordable to today's young adults although it is close to affordable in France. Working for five years extra does not quite bring the required savings rate down quite to observed rates. On the other hand it much reduces the need for wealth to finance consumption. Thus we find that current wealth holdings are well above those needed to finance consumption with the consequence that, even in the United Kingdom, working five years longer solves the budgetary problems of the population currently active; their consumption on average could rise by nearly 3%. Working longer allows more substantial increases in consumption in France and Spain but in Italy is is not enough to avoid the need for a 5% cut to consumption.

12. Conclusions

The findings of this study are fairly clear. In none of the four countries studied is the current pattern of consumption by old people affordable for young people. Thus they need to choose some combination of reducing consumption and raising income probably by working longer. On the other hand except in the United Kingdom, existing levels of produced wealth are high. In France and Spain this means that the

average member of the currently active population does not need to make a large adjustment to their behaviour, while in Italy wealth holdings are still not adequate. There and in the United Kingdom consumption levels of both the current young and the average member of the population are substantially above what can be afforded if each cohort pays its own way.

Falling mortality is an obvious influence on savings needs. But the consumption profiles we identify would not be affordable to young people even if we assumed no change to mortality rates. On the other hand existing wealth holdings would be well surplus to requirements in France and Spain. In Italy and the United Kingdom a sizeable gap would remain.

Extending working life by five years comes close to making the consumption profile affordable for young people in France and Spain although the results are obviously sensitive to the assumptions made about the wage rates people command in their extended working lives as well as the growth rates and rates of return which we focus on here. The extension of working life considered here may not be practical given that healthy life expectancy seems to be rising less rapidly than overall life expectancy. But the analysis here highlights the quantitative choice between saving more and working longer.

Of course it can be objected that the consumption profiles may not reflect the consumption plans that current cohorts have in mind for the future. Equally the labour market participation plans of current cohorts may be different from those implied by the profiles shown here.

But this objection misses the point. If current cohorts have in mind lower consumption in the future they do not need to save so much. The results here show that, relative to the consumption profiles identified, balance can be achieved by a reduction to consumption spread over the remaining life-course But balance could also be achieved by means of a small adjustment in the short-term and a rather larger adjustment later on. The framework can be adapted to investigate the implications of any consumption plan. Similarly, if they plan to raise their incomes, relative to the observed profile, then the budget can be balanced. The case of raising incomes by working longer is illustrated.

The fact is that none of the four countries studied can afford to carry on as they are; in France and Spain it is only the youngest people who need to adjust while in Italy and the United Kingdom the whole population needs to. The adjustment may be planned or unplanned but it needs to happen. Alternatively current consumption can be supported at the expense of future generations.

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Appendix 1. Data

In view of the fact that national saving rates differ widely across the EU, it is essential to take into full account all sources of income and expenditure. We work with the latest available data for each country. We use representative cross-sectional surveys for each country to construct age profiles of all resources and consumption, and we then benchmark these profiles by age and gender against national aggregates.

a) France

Aggregate data are taken from the *Tableau Economique d'Ensemble, 2004*[17]. Population figures for 2004 are taken from the Human Mortality Database. Projected mortality rates are computed from the official projections for population by age and for deaths[18].

The income and expenditure profiles are drawn from the *Enquete Budget des Familles* (EBF) 2000, which is a representative repeated cross sectional survey of French households.

The following income variables were added together to provide labour income

TOTAL SALAIRES ET AUTRES REM. REDRESSE	rev20
TOTAL REVENUS D'ACTIVITES INDEPENDANTES	rev10
TOTAL SALAIRES AUTO VERSES	rev11
TOTAL REVENUS ACT. SECONDAIRES REDRESSE	rev21

The measure of consumption used was

CONSOMMATION TOTALE SUR 12 POSTES	ctotale

b) Italy

Our main source for the aggregate data are taken from. *Conti economici nazionali - Anni 1970-2005*[19]. Information on mixed incomes taken from *Conti economici nazionali per settore istituzionale Anni 1999-2005*[20]. Estimates of net foreign assets

[17] http://www.insee.fr/fr/indicateur/cnat_annu/base_2000/tableaux/tab_eco_ensemble.htm

[18] http://www.insee.fr/fr/ppp/ir/accueil.asp?page=projpop0550/dd/projpop0550-scenarios_sd.htm. These provide population figures for 2005 but not for 2004.

[19] http://www.istat.it/dati/catalogo/20070626_00/testointegrale.pdf

[20] http://www.istat.it/salastampa/comunicati/non_calendario/20070226_00/testointegrale.pdf

are taken from the National Institute's database. No data exist for non-produced capital and we have assumed that the value of these bears the same ratio to consumption as in France. Population data are provided by ISTAT [21] with mortality data taken from the Human Mortality Database.

The source for the micro data is the Bank of Italy's Survey of Household Income and Wealth (SHIW) 2004, which provides data on income, consumption, and wealth from a sample that is representative of the population (see Jappelli and Pagano (1994); Jappelli and Pagano (1999) for a comprehensive discussion).

The variables from the Survey of Household Income and Wealth (SHIW) 2004 that we used are:

From File ricfam04.dta

c	Consumption

From File carcom04.dta

pesofl	Unit sampling weight (defined at household level)
cfred	Head of household, defined as the major income earner
eta	Age (years)
sex	Sex
staciv	Marital Status
ncomp	Number of household members

From rfam04.dta

y	Net disposable income
yl	Compensation of employees
yl1	Net wages and salaries
yl2	Fringe benefits
ym	Net income from self-employment
yma1	Income from self-employment
yma3	Entrepreneurial income
yt	Pensions and net transfers
ytp	Pensions and arrears
ytp1	Pensions
ytp2	Arrears
yc	Property income
yca	Income from buildings
yca1	Actual rents
yca2	Imputed rents

Net disposable income is given as yl+ym+yt+yc.

[21] http://demo.istat.it/pop2004/index.html

Gross labour income[22] is given as yl+ym*2/3.

14. All the variables are scaled so that the survey totals are consistent with the national accounts aggregates.

c) Spain

Aggregate data provided by the Instituto Nacional de Estadistica[23]. Capital stock data are provided by Fundacion BBV[24]. There are no estimates of the value of inventories, of non-produced assets or of net external assets. Population data are provided by the Instituto Nacional. [25] Estimates of net foreign assets are taken from the National Institute's database. No data exist for non-produced capital and we have assumed that the value of these bears the same ratio to consumption as in France.

The profiles are derived from two sources, the *Living Conditions Survey* (LCS) 2004, which provides information on labour incomes, and the *Household Budget Continuous Survey* (HBCS) 2004, which offers consumption expenditures of a representative panel survey of Spanish households. The panel keys are not made available with the rest of the data and we have treated the 2004 figures as cross-section data.

The variables from the Household Continuous Budget Survey (HCBS) 2004 that we used are:

From hogart1a04.dta; hogart2a04.dta; hogart3a04.dta; hogart4a04.dta

Gasto	Total expenditure
Edad	Age (years)
Sexo	Sex
Ecivil	Marital Status
nummiem	Number of household members

The variables from the Living Conditions Survey (LCS) 2004 that we used are:

From esudb04p.dta

Edad	Age (years)
Sexo	Sex

[22] We have included 2/3 of self-employment income, assuming that the remaining 1/3 accounts for deprecation and the net return to capital.
[23] http://www.ine.es/daco/daco42/cne00/dacocne_b00.htm
[24] http://w3.grupobbva.com/TLFB/stock07/tlfb_stock07_i5.html
[25] http://www.ine.es/inebase2/leer.jsp?L=0&divi=EPOB&his=0

Ecivil Marital Status
py010n Gross wage, salary
py050n Income from self-employment

Gross labour income[26] is given as py010n+py050n*2/3.

15. All the variables are scaled so that the survey totals are consistent with the national accounts aggregates.

d) United Kingdom

We use the *United Kingdom National Accounts: The Blue Book* (2006) as our main source for the aggregate data with information on produced capital drawn from *Capital Stocks, Capital Consumption and Non-Financial Balance Sheets 2007*. Population data are taken from the population projections for 2004 produced by the Government Actuary. Information on incomes[27] and consumption is take from the *Expenditure and Food Survey* (EFS) 2004-5, which is a repeated cross-section representative survey of British households.

The variables from the Expenditure and Food Survey (EFS) 2004-5 that we used are:

From File 2004-05_dvhh_ukanon.sav
p560tp Total Expenditure (National Accounts)
b038p Council Tax- Last Payment weekly amount (Great Britain only)
b030 Domestic Rates- last payment (N. Ireland only)
p396 Age of Household Reference Person
a062 Composition of Household (< 18 indicates 1 or 2 adults)
weighta Annual weight

From File 2004-05_dvper_ukanon.sav and 2004-05_rawper_ukanon.sav
a004 Sex
dvage Age
p008 Normal gross wage, salary (13 week rule)
p011 Gross wage - last time paid - (13 week rule sub) subsidiary employment
p029 NI contributions paid by non – employees
p031 Social security benefits included in income calcs
p037 Income from subsidiary self-employment
p047 Income from self-employment (main)
p049 Income from pensions, annuities
p050 Income from other sources
p051 Total personal gross income (normal)
p075 NI employees contribution – current

[26] We have included 2/3 of self-employment income, assuming that the remaining 1/3 accounts for deprecation and the net return to capital.
[27] The Family Resources Survey has a larger sample and also provides income data. However we took the view that it was better to use a common source for income and expenditure data.

p079 Income tax payments less refunds

The weights used are those from the household file- with each person being identified by the case number to which it belongs. The variable dvage is available on

2004-05_rawper_ukanon.sav but can be matched to the records in

2004-05_dvper_ukanon.sav in order to identify actual ages: the age variable in

2004-05_dvper_ukanon.sav has all ages over 80 set to 80.

Income from pensions and annuities comes in two forms. Civil Service pensions are unfunded and therefore a burden on current tax payers of the same type as social security payments. The survey does not distinguish funded from unfunded pensions; we therefore split them in the same proportion as is shown in the national accounts. This results in about half of pensions being included as non-property income.

Gross labour income[28] is defined as

p008+p011+(p031+p037)*2/3+employers' national insurance and pension contributions with the ratio of employers contributions to wages and salaries derived from the national accounts for 2004.

Net non-property income is p008+p011+(p031+p037)*2/3+employers' pension contributions+p031+γp049-p075-p079-p029 with γ being the share of unfunded pensions in the total and employers' contributions again being imputed from the data in the national accounts.

In the assessment of national spending and saving we use p560tp as the reference variable and deduct consumption taxes as allocated in the generational accounts. For looking at household spending and saving we add on to consumption at market prices council tax and domestic rates (b038p+b030).

[28] We have included 2/3 of self-employment income, assuming that the remaining 1/3 accounts for deprecation and the net return to capital.

All the variables are scaled so that the survey totals are consistent with the national accounts aggregates.

Appendix 2 Utility Maximisation

We set out here the optimising decisions which underlie our consumption profiles. Comprehensive consumption, $c_{t,\tau}^{C,m}$ has two components to it, consumption of goods produced from labour and produced capital, $a_{t,\tau}^{x}$ and consumption of the services provided by rent on land, $b_{t,\tau}^{x}$. There is a unit elasticity of substitution between these two types of consumption. The elasticity of intertemporal substitution is, however,

Remembering that $s_{t,\tau}^{x}$ is the number of people from cohort *t* surviving to age we write the optimisation problem for a representative individual of gender *x* aged from cohort *t*

$$\underset{a_{t,\theta}^{x},b_{t,\theta}^{x}}{Max}\sum_{\theta=\tau}^{95}\frac{s_{t,\theta}^{x}}{s_{t,\tau}^{x}}\frac{\pi_{\theta}\delta^{\theta-\tau}\{(a_{t,\theta}^{x})^{\rho}(b_{t,\theta}^{x})^{1-\rho}\}^{1-\varepsilon}}{1-\varepsilon}+\lambda\left(w_{t,\tau}^{x}+\sum_{\theta=\tau}^{95}\frac{p_{t+\theta}^{a}(y_{t,\theta}^{l,x}-a_{t,\theta}^{x})-p_{t+\theta}^{r}b_{t,\theta}^{x}}{(1+r)^{\theta-\tau}}\right)$$

Here is a discount rate and is a scaling factor which delivers the hump-shaped profiles for consumption of Figure 3.

The assumption of unit within period elasticity substitution implies that the two prices always satisfy the following constraint

$$(p_{t+\theta}^{a})^{\rho}(p_{t+\theta}^{b})^{1-\rho}=1$$

Labour income is measured in units of the consumption good *a,* the idea being that overall consumption cannot be expected to rise in line with labour income if a part of that consumption comes from rental income on land and the stock of land is fixed.

If we set

$$c_{t,\theta}^{C,x}=p_{t+\theta}^{a}a_{t,\theta}^{x}+p_{t+\theta}^{b}b_{t,\theta}^{x}$$

then the first order conditions yield

$$\varepsilon\log\frac{\pi_{\theta}}{\pi_{\theta-1}}=(\varepsilon-1)\log\frac{(a_{t,\theta}^{x})^{\rho}(b_{t,\theta}^{x})^{1-\rho}}{(a_{t,\theta-1}^{x})^{\rho}(b_{t,\theta-1}^{x})^{1-\rho}}+\log\frac{c_{t,\theta}^{C,x}}{c_{t,\theta-1}^{C,x}}+\log\frac{1}{\delta(1+r)}$$

Now the unit elasticity assumption, together with the normalisation of the price index, implies

$$(a^x_{t,\theta})^{\rho}(b^x_{t,\theta})^{1-\rho} = c^{C,x}_{t,\theta}\left(1-\rho\right)^{1-\rho}\rho^{\rho}$$

so that

$$\varepsilon \log \frac{\pi^x_{\theta}}{\pi^x_{\theta-1}} = \varepsilon \log \frac{c^{C,x}_{t,\theta}}{c^{C,x}_{t,\theta-1}} + \log \frac{1}{\delta(1+R)}$$

This then delivers the standard result that if *R)*

$$c^{C,x}_{t,\theta} = K_t \pi^x_{\theta}$$

so that the pattern of expenditure is driven by the π^x_{θ} terms in the utility function. K_t is a cohort-specific constant which reflects the life-time budget constraint. It is also worth noting that if the intertemporal elasticity of substitution 1/(*1*-) falls to zero then $c^{C,x}_{t,\theta} = K_t \pi^x_{\theta}$ for all discount and interest rates. The underlying framework can therefore be interpreted either as one where the interest rate equals the discount rate or where the intertemporal elasticity of substitution is zero.

The link between the observed consumption patterns in the cross-sectional data and the consumption profile of any cohort is derived as follows. c^x_{t-} is the observed consumption in year *t* of an individual from cohort *t*- Now, given the assumption of unit elasticity of substitution between the two types of consumption

$$\rho c^x_{t-\tau,\tau} = \rho K_{t-\tau}\pi^x_{\tau} = p^a_t a^x_{t-\tau,\tau}$$

It is now assumed that produced consumption grows in line with labour productivity $a^x_{t^*-\tau,\tau} = (1+g)^{t^*-t} a^x_{t-\tau,\tau}$.

These combine to give equation (5) of section 2,

$$c^{C,x}_{t^*-\tau,\tau} = \frac{p^a_{t^*}}{p^a_t}(1+g)^{t^*-t} c^{C,x}_{t-\tau,\tau}$$

so that , if $p^a_{t^*} < p^a_t$ overall consumption *per capita* at each age grows less fast than the rate of growth of labour productivity.