Taxation, Reranking and Equivalence Scales[*]

Justin van de Ven[†] and John Creedy[‡]

Abstract

This paper considers whether an equivalence scale implicit in transfer policy can be inferred from summary measures of reranking. It is conjectured that, if the government has a distributional objective and formulates tax policy with a view to equitable treatment of income units, then adopting the scale that is implicit in government transfer policy should only identify reranking that has no equity foundation. This motivates the question: Is the incidence of reranking associated with a transfer system minimised by the equivalence scale that is implicit in the transfer system? The analysis presented in this paper suggests that the equivalence scale which minimises reranking, while not necessarily equal to the closest approximation to the one that is implicit in transfer policy, is nevertheless in its vicinity.

Key Words: Reranking, Equivalence Scales, Taxation

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[†] National Institute of Economic and Social Research. Fax No. +44 207 654 1900. Email: jvandeven@niesr.ac.uk.

[‡] New Zealand Treasury. John.Creedy@treasury.govt.nz

1 Introduction

The use of adult equivalent scales, to allow for non-income-relevant differences between income units, is widespread in the empirical analysis of inequality. A wide variety of scales exists and it is known that they can have substantial effects on income distribution comparisons; see Coulter et al. (1992b). While it is recognised that the choice of scales involves value judgements, surprisingly little attention has been given to the question of the value judgements implicit in the tax and transfer system. The aim of this paper is to examine whether information about such judgements can be obtained from the analysis of changes in the rank order of pre-tax incomes compared with that of the post-tax distribution.

Survey data usually reveal that the rank order of income units is altered after taxes and benefits are added to gross income, an effect that is commonly referred to as reranking. One explanation for reranking is that the order based on income gross of taxes and benefits (hereafter referred to as pre-tax income) is 'unfair' because it fails to take into consideration the differing needs of a heterogeneous population. Most transfer systems adjust for household need by basing tax and benefits payments on the number, age and health status of household members, and these adjustments can affect the post-tax order of households. However, it is unlikely that all rank reversals are attributable to an unfair distribution of pre-tax income.[1] Reranking can also arise as the result of government policy that is tangential to equity objectives. For example, unemployment benefits may be designed to encourage labour market participation, or certain types of income may be treated differently on efficiency grounds. Any reranking that arises as a result of these non-equity based considerations is an issue of concern for studies that consider the redistributive effects of taxation. This is because such reranking opposes the extent to which a transfer system reduces inequality of pre-tax incomes, and thus contradicts a progressivity objective.

Adult equivalent incomes are commonly used to focus on reranking that is perceived as having no equity justification (hereafter referred to as inequitable reranking).[2] This methodology requires the selection of an equivalence scale

[1] Shoup (1969, p.23) made a similar point in the context of unequal treatment of pre-tax equals, or horizontal inequity. The relationship between horizontal inequity and reranking is discussed in Section 2.

[2] For recent literature that considers reranking, see Creedy and van de Ven (2001), Lambert

that is used to adjust incomes for differences in unit need. Despite a considerable research effort, however, no compelling criteria have been identified for selecting an equivalence scale. This problem is important for applied studies of reranking because observed reranking typically depends upon the equivalence scale adopted. Hence applied studies of reranking are commonly criticised for failing to differentiate between reranking that is attributable to assumed value judgements, and reranking that arises due to inequities of the tax and benefits system.[3]

The question considered here is whether it is possible to infer a set of equivalence scales that are implicit in tax and benefits policy, by identifying the equivalence scale specification that minimises observed reranking.[4] Unlike estimation methods that have been suggested elsewhere[5], this approach does not depend on an assumed tax function for reference units. It is based on the premise that, if the government has a distributional objective and formulates tax policy with a view to equitable treatment of income units, then adopting a scale that is implicit in government transfer policy should identify only inequitable reranking.

The conjecture that there exists a single equivalence scale implicit in transfer policy warrants some comment. Specifically, it is frequently assumed that the adjustments made by fiscal policy for household heterogeneity describe a range of equivalence scales, rather than a single set of relativities.[6] It is possible, however, to show that the equivalence scale framework is sufficiently flexible to describe any redistributive system (see, for example, Muellbauer and van de Ven, 2003).[7] In this paper we follow the methodology of Muellbauer and van de Ven (2003), and define the equivalence scale implicit in transfer policy as the scale that minimises horizontal inequity.[8]

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and Ramos (1997a, b), Lerman and Yitzhaki (1995), Aronson et al. (1994), and Jenkins (1988a).

[3] On the scepticism that is associated with measures of reranking due to the equivalence scales assumed, see Lambert (2003, footnote 2).

[4] This suggestion was also mentioned briefly in Creedy and van de Ven (2001). A related issue concerns the aversion to inequality. Attempts to impute a value of inequality aversion implicit in government tax decisions include Christiansen and Jansen (1978) and Stern (1977); see also Mera (1969), Moreh (1981) and Brent (1984).

[5] See, for example, Muellbauer and van de Ven (2003).

[6] See, for example, Coulter et al. (1992a).

[7] See also Seneca and Taussig (1971).

[8] Horizontal equity requires 'equal treatment of equals'. This is defined by Muellbauer and van de Ven (2003) as the requirement that households with the same pre-tax equivalent

The issue of how reranking should be interpreted is discussed at greater length in Section 2. This discussion is extended in Section 3, where a formal model is used to identify the factors that determine the incidence of reranking. Section 4 considers whether the reranking observed for a transfer system is indeed minimised by the equivalence scale that is implicit in the transfer system. It is argued, based on a hypothetical population comprised of two income unit types, that reranking need not be minimised by the equivalence scale that is implicit in tax and benefits policy. It is, however, conjectured that, in practice, the scale that minimises reranking is likely to be closely related to the scale that implicitly underlies transfer policy. To test this conjecture, survey data are considered in Section 5.

Previous work on the role of equivalence scales in distributional analyses has focused on inequality and poverty measurement.[9] A study that has examined the impact of equivalence scales on observed reranking is by Nolan (1987). He expressed surprise on finding that adult equivalent income showed more reranking than unadjusted incomes.[10] The observations based on survey data that are presented in Section 5 suggest that Nolan's findings (which appear to contradict the idea that an implicit equivalence scale can be identified by minimising observed reranking) are attributable to the focus of the equivalence scale that he used. Conclusions are in Section 6.

2 Interpretations of Reranking

The appropriate interpretation of reranking depends on the importance that is assigned to the rank order of pre-tax incomes, which is reflected in alternative views regarding distributive justice.[11] In the absence of a widely accepted principle of redistributive justice, some guidance may be obtained by relating reranking to horizontal equity, which was described by Musgrave (1959, p.160) as 'perhaps the most widely accepted principle of equity in taxation'. The relation between reranking and horizontal equity is defined here with the use of a

incomes should have the same post-tax equivalent incomes.

[9] See, for example, Buhmann et al. (1988), Coulter et al. (1992a, b), and Banks and Johnson (1994).

[10] Nolan's use of scales based on Supplementary Benefit rates followed the Royal Commission on the Distribution of Income and Wealth.

[11] For example, with a Rawlsian view involving a maximin strategy, emphasis is on the minimum income level and no significance is attached to the rank order of incomes.

simple analytical framework.

Assume that the redistributive objectives of tax policy designers are framed in terms of income per equivalent adult. Values in adult equivalent terms, as measured by the government, are denoted by a * superscript. Let x_i^* and y_i^* define the pre-tax and post-tax equivalent income of unit i. Given the distribution of x_i^* for a population, the government is considered to impose an 'equivalised tax function', $T(x_i^*)$, so that:

$$y_i^* = x_i^* - T(x_i^*) + \varepsilon_i \tag{1}$$

where ε_i allows for the possibility of horizontal inequity, and assume that $T'(x^*) \leq 1$. This restriction prohibits an explicit reranking objective of government in terms of adult equivalent incomes: it was formulated by Feldstein (1976), and adopted by Kakwani and Lambert (1998) as one of their three axioms of equity in taxation.[12]

Add to this framework the following three assumptions. First, the population is sufficiently large such that there are several individuals with the same pre-tax income as any other individual. Second, that the government specifies taxation policy to minimise the individual-specific effect, ε_i. Third, suppose the government's objective is achieved on average such that $E(\varepsilon_i) = 0$ for all i.

Under these conditions, any reranking that arises must also be identified as horizontal inequity. Hence, if horizontal inequity is deemed to be undesirable, then it is reasonable to suppose that the same can be said for reranking, given the direct correspondence between the two concepts.[13] This conclusion, combined with the fact that precise pre-tax equals are seldom observed in survey data, explains why many applied studies of horizontal inequity focus upon reranking.[14] However, this conclusion applies only to reranking that has no explicit equity justification. Furthermore, in practice it is important to weigh the undesirable

[12] Fei (1981) called this restriction 'incentive preservation', referring to the fact that an individual who is subject to a marginal tax rate in excess of 100 per cent has an incentive to reduce pre-tax income.

[13] See Musgrave (1990) for a brief survey of the relationship between alternative concepts of redistributive justice and horizontal equity. The concept of horizontal equity is not universally accepted. Gordon (1972) suggested that the initial ordinal ranking of individuals merits no normative emphasis. Also, horizontal equity may not be satisfied by the solution that maximises a utilitarian social welfare function (see, for example, Atkinson and Stiglitz, 1980).

[14] See the definitions of horizontal equity suggested by Feldstein (1976, p.83), and Plotnick (1985, p.241). Jenkins (1988b, p.308) referred to the 'no-reranking' condition as 'strong horizontal equity', and the 'equal-treatment-of-equals' condition as 'weak horizontal equity'.

implications of observed reranking against alternative issues of concern, such as those mentioned in the introduction.

3 Equivalence Scales and Reranking

In terms of observed incomes, equation (1) translates to:[15]

$$y_i = x_i - a_i^* T\left(\frac{x_i}{a_i^*}\right) + a_i^* \varepsilon_i \tag{2}$$

where a^* defines the equivalence scale implicit in tax policy. To explore the sources of observed reranking, assume that the government's desired redistributive policy involves the linear function:

$$T(x_i^*) = t x_i^* - G \tag{3}$$

$$a_i^* T(x_i^*) = t x_i - a_i^* G \tag{4}$$

where the same marginal tax rate, $t < 1$, is applied to all income units irrespective of their characteristics, and there is a single transfer payment, G, which is adjusted according to the government's equivalence scale. Assuming that $\varepsilon_i = 0$ for all i, and substituting equation (4) into (2), gives:

$$y_i = (1 - t)x_i + a_i^* G \tag{5}$$

Thus, for any two income units, A and B, for which $x_A < x_B$, reranking of unequivalised incomes arises if $y_B < y_A$, that is if:

$$(x_B - x_A) < (a_A^* - a_B^*)\frac{G}{(1-t)} \tag{6}$$

This reranking is a deliberate objective of transfer policy in response to a value judgement regarding the 'needs' of the respective income units.

Suppose that an independent judge wishes to examine the redistributive effects of the transfer system defined by equation (3) using an equivalence scale $\tilde{a}_i$ instead of the government's implicit scale. Use of $\tilde{a}_i$ implies that pre-tax and post-tax equivalent incomes are related (assuming $\varepsilon_i = 0$) by:

$$\tilde{y}_i = (1 - t)\tilde{x}_i + \frac{a_i^*}{\tilde{a}_i} G \tag{7}$$

[15] The derivation of equation (2) assumes that the same equivalence scale is applicable for pre-tax and post-tax income.

Hence, reranking may be observed unless a_i is distributionally equivalent to a_i^*, such that a_i^*/a_i is constant. Consider, for example, two income units, for which $x_A < x_B$. From equation (7), reranking is observed ($y_B < y_A$) if:

$$(1 - t)(x_B - x_A) < \left(\frac{a_A^*}{a_A} - \frac{a_B^*}{a_B} \right) G \tag{8}$$

that is, if the difference between the post-tax benefit derived from pre-tax equivalent income by the two income units is more than offset by the difference between the equivalent transfer benefits that arises due to the equivalence scale used.

3.1 A Nonlinear Tax Function

Consider the implications for reranking of assuming an equivalence scale a_i, when the net effect of taxes and benefits is described by the generalised specification of equation (1):

$$y_i = x_i - \frac{a_i^*}{a_i} T \left(\frac{x_i}{a_i^*} \right) - \frac{a_i^*}{a_i} \varepsilon_i \tag{9}$$

Assume that the government's implicit scale and the scale adopted for analysis are related by:

$$a_i^* = a_i + \lambda_i \tag{10}$$

Substituting into equation (9) gives:

$$y_i = x_i - \frac{(a_i + \lambda_i)}{a_i} T \left(\frac{x_i}{a_i + \lambda_i} \right) - \frac{(a_i + \lambda_i)}{a_i} \varepsilon_i \tag{11}$$

Suppose that the equivalised tax function can be described by an Nth order polynomial (through a Taylor's series expansion), so that:

$$y_i = x_i - \frac{(a_i + \lambda_i)}{a_i} \sum_{j=0} \beta_j \left(\frac{x_i}{a_i + \lambda_i} \right)^j - \frac{(a_i + \lambda_i)}{a_i} \varepsilon_i \tag{12}$$

where β_j ($j = 0, ..., N$) are tax function coefficients. Equation (12) can be used to explore the effects of variations in an equivalence scale that approximates a_i^*.

For the jth term of the tax function:

$$
\begin{aligned}
\frac{(\alpha_i + \lambda_i)}{\alpha_i}\beta_j\left(\frac{x_i}{\alpha_i + \lambda_i}\right)^j &= \beta_j\frac{x_i^j}{\alpha_i(\alpha_i + \lambda_i)^{j-1}} \\
&\approx \beta_j\frac{x_i^j}{\alpha_i^j\left(1 + (j-1)\frac{\lambda_i}{\alpha_i}\right)} \\
&\approx \beta_j\frac{x_i^j}{\alpha_i^j}\left(1 - (j-1)\frac{\lambda_i}{\alpha_i}\right) \\
&= \beta_j\left(\frac{x_i}{\alpha_i}\right)^j - \beta_j(j-1)\frac{\lambda_i}{\alpha_i}\left(\frac{x_i}{\alpha_i}\right)^j \qquad (13)
\end{aligned}
$$

where the approximations assume small λ_i. Substituting for the tax function terms in equation (12):

$$
\begin{aligned}
y_i &= x_i - \sum_{j=0}\beta_j x_i^j + \sum_{k=0}\beta_k(k-1)\frac{\lambda_i}{\alpha_i}x_i^k \\
&\quad - \left(1 + \frac{\lambda_i}{\alpha_i}\right)\varepsilon_i + \psi_i \qquad (14)
\end{aligned}
$$

where ψ_i accounts for approximations made when deriving equation (13).

To explore the implications for reranking, consider the two income units referred to above, for which $x_A < x_B$. From (14), reranking ($y_B < y_A$) is observed if:

$$
\begin{aligned}
(x_B - x_A) - \sum_{j=0}\beta_j\left(x_B^j - x_A^j\right) &< \sum_{k=0}\beta_k(k-1)\left(\frac{\lambda_A}{\alpha_A}x_A^k - \frac{\lambda_B}{\alpha_B}x_B^k\right) \\
&\quad + \left[\left(1 + \frac{\lambda_B}{\alpha_B}\right)\varepsilon_B - \left(1 + \frac{\lambda_A}{\alpha_A}\right)\varepsilon_A\right] \\
&\quad + (\psi_A - \psi_B) \qquad (15)
\end{aligned}
$$

This result suggests the following conclusions regarding the sources of reranking. The disparity between the pre-tax equivalent incomes of income units A and B tends to oppose the inequality condition required for reranking: as the units move further apart in the pre-tax distribution cæteris paribus, it is less likely that reranking will be observed.

The deduction made by the equivalent tax function for increases in pre-tax equivalent income tends to support the inequality condition required for reranking: the more severely the equivalent tax function treats income unit B relative to A ceteris paribus, the more it is likely that reranking will be observed.

Assuming that $\lambda_i > -\alpha_i$ for all i, the larger is ε_B relative to ε_A ceteris paribus, the more it is likely that reranking will be observed. This observation, which relates to the relative severity of tax treatment for the two income units, is consistent with the preceding paragraph.

The influence on reranking of the relation between the equivalence scale adopted for analysis, α_i, and the equivalence scale implicit in the transfer system, a_i^*, depends upon elements of the equivalent tax function. When $\left(\sum_{j=0}^{N} \beta_j (j-1) x_i^{*j} - \varepsilon_i\right) = \Gamma_i > 0$ for $i = (A, B)$, λ_A tends to support, while λ_B tends to oppose, the inequality condition required for reranking: the smaller is α_A relative to a_A^*, and the larger is α_B relative to a_B^*, ceteris paribus, the more likely reranking is to be observed. When $\Gamma_i < 0$ for $i = (A, B)$, λ_A tends to oppose, and λ_B tends to support the inequality condition required for reranking. When $\Gamma_i = 0$ for $i = (A, B)$, λ_A and λ_B have no effect on the likelihood that reranking will be observed, ceteris paribus.

Observed reranking is therefore attributable to three principal factors, in addition to the distribution of pre-tax income. These are: the structural specification of the transfer system as described by $T(x^*)$ in equation (11); the heteroscedastic application of the transfer system as described by ε and; the disparity, λ, between the equivalence scale used for analysis and the scale implicit in transfer policy. Furthermore, if the equivalence scale adopted is distributionally equivalent to the scale implicit in the transfer system ($a_i^* = c\alpha_i$ implies $\lambda_i/\alpha_i = (c-1)$ for all i), (11) and (15) indicate that observed reranking can arise only from the structural specification of the transfer system and/or its heteroscedastic application, neither of which has an equity justification.

4 Reranking and Implicit Equivalence Scales

This section considers whether observed reranking is minimised by the equivalence scale implicit in transfer policy. The discussion is concerned with summary measures of reranking and while no reference is made to any particular index, discussion focuses on two properties of reranking, incidence and intensity. The first property refers to the frequency of observed reranking in a population, discussed in Section 3. The second property takes into consideration the fact that more concern may be associated with reranking when deviations between the pre-tax and post-tax distributions are large.

4.1 Minimising Observed Reranking

To consider whether reranking is minimised by the equivalence scale implicit in transfer policy, begin by focusing upon two income units, A and B, subject to scales α_A and α_B respectively. For any combination of strictly positive pre-tax and post-tax incomes, it is always possible to specify the equivalence scale, α_A/α_B, such that reranking will not be observed between the respective income units. Where, for example, $x_A > x_B$ and $y_A < y_B$, selecting $\alpha_A/\alpha_B > x_A/x_B$ or $\alpha_A/\alpha_B < y_A/y_B$ gives equivalised incomes:

$$\frac{x_A}{\alpha_A} = x_A^* \gtrless x_B^* = \frac{x_B}{\alpha_B}, \frac{y_A}{\alpha_A} = y_A^* \gtrless y_B^* = \frac{y_B}{\alpha_B} \tag{16}$$

which maintain pre-tax ranks in the post-tax distribution. This conclusion is independent of the relativities implicit in transfer policy, or the existence of horizontal inequity. Hence, for a population of two income units, it is possible that reranking is minimised by a range of equivalence scales that may bear little relation to the relativities implicit in transfer policy.

It may be thought that this conclusion is relevant only for small populations. However, it can be shown to apply more generally. Consider a large population comprised of two subgroups, 1 and 2, within each of which income units are considered to be homogenous, and hence subject to the same equivalence scale. Altering the equivalence scale thus has no affect on the incidence of reranking within subgroups, but affects reranking between groups.

The impact of equivalence scales on reranking for this population can be explored using Figure 1. In Panel A, the pre-tax incomes of subgroup 1 are distributed between x_1^- and x_1^+, while those of subgroup 2 are distributed between x_2^- and x_2^+.[16] Similarly, the right hand axis of Panel A indicates the distributions of post-tax income, which lie between y_1^- and y_1^+ for subgroup 1, and y_2^- and y_2^+ for subgroup 2. There is an overlap between the pre-tax and post-tax incomes of the two subgroups, indicated by the ranges $\left(x_1^-, x_2^+\right)$ and $\left(y_1^-, y_2^+\right)$.

[16] In practice, it is likely that the equivalence scale used differentiates between many more than two population subgroups. Furthermore, incomes within any given subgroup are unlikely to be bound within the type of range displayed in Figure 1. However, it is reasonable to interpret the figures as differentiating between one population subgroup and all others. Also, the income bounds defined in Figure 1 may be thought of as defining the range where the density of the respective population subgroups is highest.

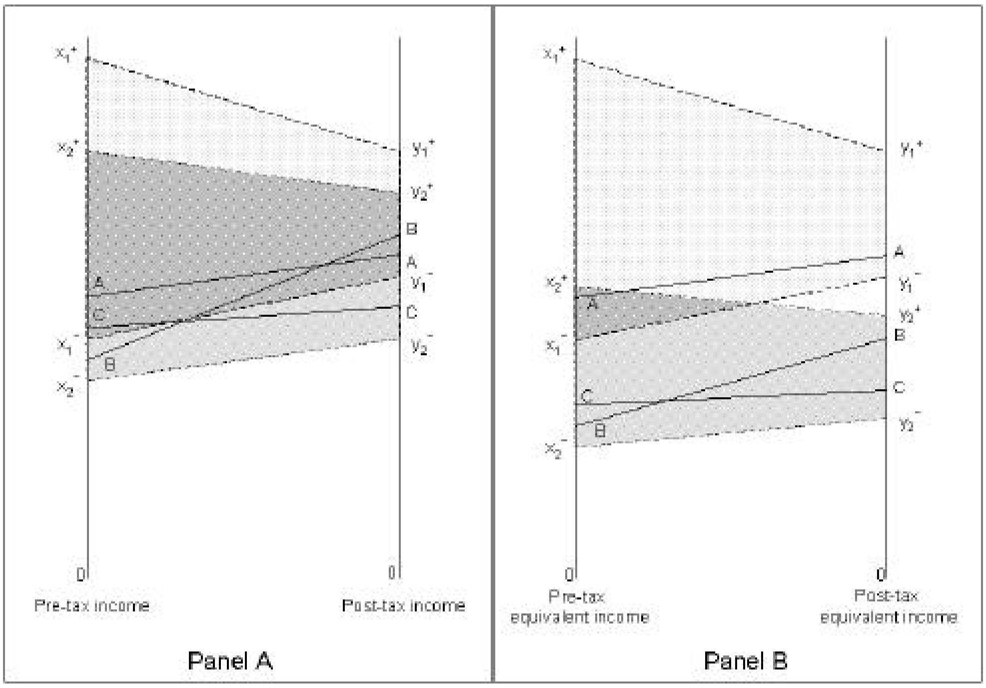


Figure 1: Reranking, Equivalence Scales and Population Subgroups

Any reranking between subgroups 1 and 2 must occur within the dark shaded region defined within these overlapping bounds. The trajectories from the pre-tax to the post-tax income distribution of three income units are also displayed. Unit A is a member of subgroup 1 (and hence lies between x_1^- and x_1^+, and y_1^- and y_1^+), and units B and C are members of subgroup 2. Panel A indicates that reranking is observed between income units A and B, and between units B and C.

Panel B illustrates the effect on the respective distributions of equivalent income if the equivalence scale reduces all incomes of subgroup 2 by 70 per cent relative to the incomes of subgroup 1 (the distributions of pre-tax and post-tax income depicted in Panels A and B hold subgroup 1 constant). Comparing Panels B and A reveals that the region within which reranking can be observed between the two subgroups is significantly diminished after the incomes of units in subgroup 2 are deflated.[17] This is likely to affect reranking in two respects.

[17] Unlike Panel A, any income units in subgroup 2 with pre-tax income in the range $\left(x_1^-, x_2^+\right)$ must exhibit some reranking in Panel B, assuming that there is at least one income unit in subgroup 1 with pre-tax income x_1^-.

First, between groups reranking may be affected, as is the case for the comparison between A and B when moving from Panel A to B. In the extreme case, where the ranges of pre-tax and post-tax income of each subgroup do not overlap, no reranking is observed between income units of the different subgroups. This effect bears obvious similarities with the two-income-unit case examined earlier.

Secondly, the intensity of reranking relative to mean population income is diminished in subgroups that are subject to larger equivalence scales, as is observed for the reranking between units B and C when moving from Panel A to B. Depending on the populations considered, these effects may evidently reduce reranking even if the scale considered is quite different from the relativities implicit in transfer policy. This statement can be made clear by the following example. Suppose the income units of subgroup 1 are subject to horizontally equitable tax burdens (so that no reranking is observed within the subgroup), but those of subgroup 2 involve some horizontal inequity. In this case, reranking of equivalised incomes can be minimised by applying an infinite equivalence scale to subgroup 2, which would omit reranking between the subgroups (since $x_1^i > 0$) and minimise the intensity of any reranking observed within subgroup 2 (relative to average population income).

In Section 1 it was suggested that identifying the equivalence scale implicit in transfer policy by minimising observed reranking could be useful because it does not rely on the assumption of a particular tax function for reference households. The analysis presented here reveals that omitting comparisons with the reference unit tax function can lead to a significant difference between the equivalence scale implicit in transfer policy and the scale that minimises observed reranking. It is possible, for example, for reranking to be minimised by a scale that shifts a subgroup of the population to a point in the equivalised income distribution that is otherwise scarcely populated, and also distorts the equivalised tax burden of the subgroup relative to the remaining population. In short, the equivalence scale that minimises reranking can trade decreased reranking for increased horizontal inequity (as horizontal inequity is defined by Muellbauer and van de Ven, 2003).

The equivalence scale that minimises the incidence of reranking does not therefore necessarily coincide with the scale that is implicit in transfer policy. However, it may be suggested that in practice - when incomes are dense and there is sufficient overlapping between groups - the two scales will be related. Indeed, it may be conjectured that the equivalence scale which minimises the incidence of reranking, while not necessarily equal to the closest approximation to the one that is implicit in transfer policy, will nevertheless be its vicinity. The above analysis suggests that this depends on the joint distributions of income and subgroup characteristics. This is examined in the following section.

5 Analysis of Survey Data

This section uses survey data to consider the relationship between equivalence scales and the reranking effects of taxation. A brief description of the data is given before presenting the empirical results.

5.1 The Data

The data were derived from the Confidentialised Unit Record Files (CURFs) of the 1997-1998 Survey of Income and Housing Costs (SIHC) for Australia, and the Family Expenditure Survey (FES) for the UK. Both surveys provide income and demographic data for individuals and households. The SIHC records annual household income measured in 1997 Australian dollars, whereas the FES provides 'normal' measures of weekly income denominated in 1997 British pounds. Both surveys attempt to account for all direct pecuniary flows. For the analysis undertaken here, no attempt is made to impute indirect taxes. The income unit adopted for analysis is the nuclear family, which comprises a single adult or married (registered or de facto) couple and any dependant children under the age of 17 years. After deleting income units with inconsistent data, negative income gross of taxes and benefits, or non-positive income net of taxes and benefits, the FES and SIHC were reduced, respectively, to 6,803 and 8,451 households.

5.2 Empirical Results

Measures of household income were adjusted for family size and composition using:

$$a_i = (\alpha c_i + w_i)^{\theta} \qquad (17)$$

where c_i and w_i refer respectively to the number of children and adults in income unit i, and $0 \leq (\theta, ©) \leq 1$. The coefficient θ determines the economies of scale implied by the equivalence scale, and © indicates the effect of children relative to adults. The equivalence scale described by (17) is defined with reference to single adults with no dependant children, for whom $a_i = 1$ regardless of the parameter values adopted. The specification assumes that the parameters θ and © are independent of income.[18] When calculating summary measures of reranking, measures of equivalent income were weighted by the number of individuals in each household.

The analysis presented here focuses upon the Atkinson (1979)-Plotnick (1981) summary measure of reranking, R.[19] This measure is affected by both the incidence and intensity of reranking and captures the area between the Lorenz curve and the concentration curve of the post-tax income distribution. The Lorenz curve is obtained by ranking individuals according to post-tax income, while the concentration curve orders individuals by pre-tax income. Since the Gini coefficient, G_y, measures the area between the Lorenz curve and the 45^0 line of equality, and the concentration index C_y measures the area between the concentration curve and the 45^0 line of equality, R is calculated by:

$$R = G_y - C_y \tag{18}$$

where;

$$G_y = \frac{2}{\bar{y}} cov\left(y, F(y)\right) \tag{19a}$$

$$C_y = \frac{2}{\bar{y}} cov\left(y, F(x)\right) \tag{19b}$$

The variables x and y denote pre-tax and post-tax income respectively, $F(.)$ is the respective cumulative distribution function, $cov(.)$ is the covariance operator, and $\bar{y}$ is the arithmetic mean of y. When the ordering of individuals by post-tax income is the same as the ordering by pre-tax income, $F(y) = F(x)$, $G_y = C_y$, and $R = 0$.

[18] This formulation was used by, for example, Cutler and Katz (1992), Banks and Johnson (1994), Jenkins and Cowell (1994), and Citro and Michael (1995). Furthermore, Buhmann et al. (1988) show, using a simplification where children are given the same weight as adults, that adjusting θ provides an approximation to a wide range of equivalence scales.

[19] For alternative summary indices of reranking see, for example, King (1983), Cowell (1985), and Jenkins (1988b).

The reranking measure, R, incorporates the same aggregation that is associated with the Gini coe¢cient. To explore di¤erent value judgements associated with the aggregation, the analysis has been repeated using the extended Gini coe¢cient. Since similar results are obtained for all of the measures considered, the discussion presented below focuses upon the Atkinson-Plotnick statistic, and results for alternative statistics are presented in Appendix A.

To obtain a detailed picture of the e¤ect on reranking of using di¤erent values of θ and ©, the unit square including all combinations of θ and © was divided into a grid of 31 intervals per coe¢cient. Varying the equivalence scales a¤ects redistribution, V, measured as the reduction of the Gini coe¢cient from the pre-tax to the post-tax equivalent income distribution.[20] This e¤ect is not the focus of interest here, so the measures of reranking are reported as percentages of redistribution. These percentages are plotted as surface projections for the Australian and UK survey data in Figures 2 and 4 respectively. Figures 3 and 5 provide the associated topographic maps, which divide the observed variation into …fty equally spaced iso-reranking lines.

The surface plot and topographic map of Figures 2 and 3 indicate that reranking based on the Australian data is highly dependent upon the parameter values selected for the equivalence scale. The Atkinson-Plotnick measure of reranking ranges between 5.677 and 10.184 per cent of associated redistribution. A similar range is observed for UK data, where R varies between 7.304 and 13.722 per cent of V.

Comparing Figures 2 and 3 with 4 and 5, indicates that data from the two countries produce a similar pro…le for the relation between reranking and the equivalence scale parameters. In each case, the corners of the unit square form local maxima, with an elongated basin around the minimum incidence of reranking.[21] Taking cross-sections with respect to the © axis of the pro…les of reranking indicates a U-shaped relationship with θ that is similar to the …ndings for inequality and poverty statistics obtained by Banks and Johnson (1994) and

[20] See, for example, Banks and Johnson (1994), for a consideration of the e¤ect of equivalence scale parameters on inequality.

[21] The pro…les are driven by variation in R rather than V: similar conclusions are obtained when pro…les of R rather than R/V are considered.

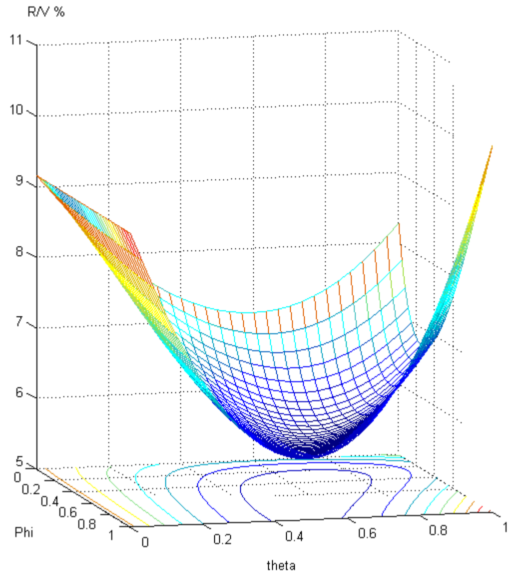


Figure 2: Reranking by Equivalence Scale Parameters - Surface Profile: Australia

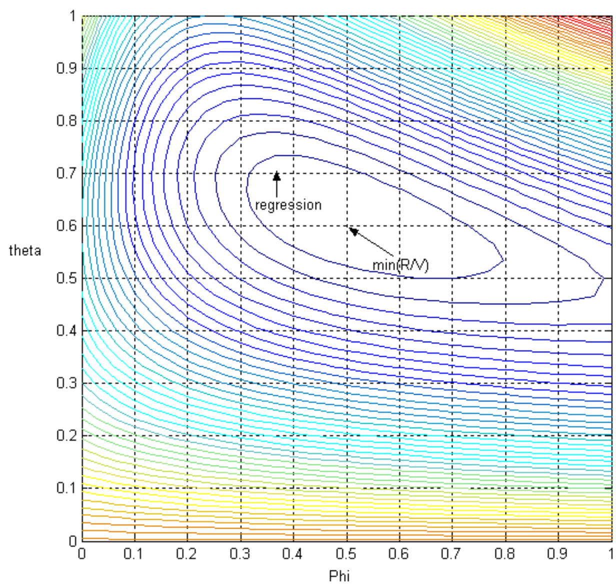


Figure 3: Reranking by Equivalence Scale Parameters - Topographic Map: Australia

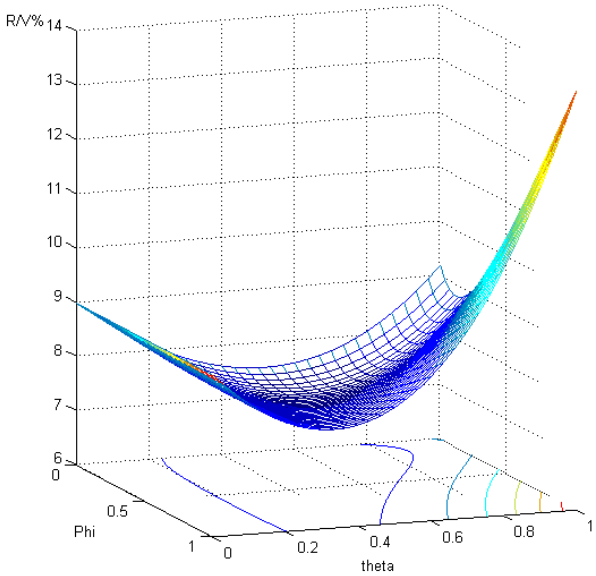


Figure 4: Reranking by Equivalence Scale Parameters - Surface Pro…le: UK

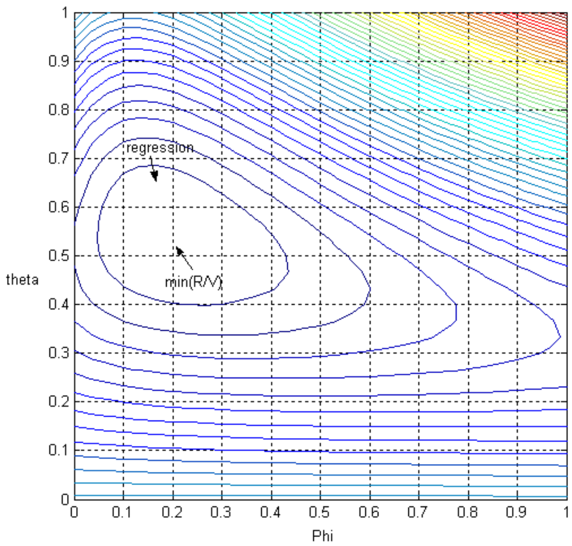


Figure 5: Reranking by Equivalence Scale Parameters - Topographic Map: UK

Coulter et al. (1992b).[22]

The reranking minimising scale parameters may be compared with econometric estimates of the scale implicit in transfer policy, based on the use of an explicit tax function following Muellbauer and van de Ven (2003). For Australia, a quadratic polynomial equivalent tax function and a logarithmic specification for tax function heteroscedasticity were used as follows:

$$T^{*}(x_i^{*}) = \beta_0 + \beta_1 x_i^{*} + \beta_2 x_i^{*2} + \varepsilon_{Ti} \quad (20)$$

$$\varepsilon_{Ti} \sim N\left(0, \sigma^2_{Ti}\right), \sigma^2_{Ti} = \exp\left(e_1 + e_2 x_i\right) \quad (21)$$

For the UK, a third-order polynomial with homoscedastic errors was found to provide a better fit for the tax function. The estimates are displayed in Table 1.[23]

Table 1: Econometric Estimates of Tax Policy Implicit Scale

	Australia	UK
Φ	0.36536	0.15512
	(0.0189)	(0.0141)
θ	0.70962	0.71991
	(8.5E-03)	(1.1E-02)
β0	-7283.08	-86.1408
	(70.896)	(1.1093)
β1	0.43645	0.52920
	(2.6E-03)	(1.5E-02)
β2	3.71E-08	-2.63E-04
	(1.5E-08)	(3.4E-05)
β3		6.76E-08
		(1.1E-08)
e1 / std error (tax fn)	14.836	31.778
	(0.1417)	(0.5733)
e2	4.26E-06	
	(1.4E-06)	
std error (eqiv scale)	0.30261	0.09304
	(0.0262)	(0.0244)
R-Squared	0.94992	0.94422

Standard Errors in Parentheses

It can be seen that the equivalence scale parameters that minimise R/V displayed in Figures 2 to 5 are in the vicinity of the estimates in Table 1, although

[22] However, the results considered here correct for the effect of the equivalence scale relativities on inequality per se. Furthermore, unlike the profiles of inequality and poverty reported by Banks and Johnson (1994), the profiles of reranking displayed in Figures 2 to 5 exhibit a global minimum with © > 0.

[23] The estimation method involves searching for parameter values that minimise deviations from horizontal equity. Given a close association between reranking and horizontal inequity, it is likely that the values also minimise observed reranking.

the differences are statistically significant. However, the measures of R/V associated with the regression estimates are within the 95 per cent confidence interval about the global minimum observed for R/V using both Australian and UK data. For Australia, the R/V measure calculated at the parameter estimates in Table 1 is equal to 5.728, and for the UK the respective R/V measure is equal to 7.520. These compare with a minimum R/V of 5.677 with a 95 per cent confidence interval of 5.340-6.039 for Australia, and a minimum of 7.304 with a 95 per cent confidence interval of 6.945-7.719 for the UK.[24] These results suggest that the reranking-minimising equivalence scales may provide a conservative basis for scales implicitly used by policy makers.

Figures 4 and 5 for the UK also help to explain the observations reported by Nolan (1987), who found more reranking for equivalised than unequivalised incomes. Consideration of Figure 5 reveals that combinations of θ and © on the upper-diagonal spanning between ($\theta = 1$, © ' 0.2) to (θ ' 0.65, © = 1) are associated with a higher measure of R/V than the value obtained when $\theta = 0$ (for which the equivalence scale is unresponsive to household size). The equivalence scale parameter combinations defined within this upper-diagonal consequently imply the same ordinal relation observed by Nolan for equivalised and unequivalised incomes. Hence, it is possible to conclude that the results reported by Nolan are attributable to the specific relativities of the scale that he imposed, rather than a conclusion that holds more generally.

The parameters that minimise R/V imply larger economies of scale (a smaller value for θ) and a larger adjustment for children relative to adults (a larger value of ©) compared with the associated regression estimates for both countries. This finding can be explained with reference to the analysis in Sections 3 and 4. The survey populations considered in Figures 2 to 5 can be divided into three groups; income units for which a larger equivalence scale is associated with the parameter estimates derived using the two-stage regression displayed in Table 1 (households comprised predominantly of adults), income units for which a larger equivalence scale is associated with the parameter values that minimise the incidence of reranking (households with children), and income units for which the equivalence scale is unaffected by the parameters adopted

[24] Standard errors were calculated using the Bias Corrected and Accelerated Bootstrap following Efron and Tibshirani (1993).

(reference households).

Australian and UK data reveal that the average pre-tax equivalent income (adjusted by the scales associated with the regression estimates) of income units in the first of these three groups is greater than the average pre-tax equivalent income in the second group ($24,648 compared with $10,261 for Australia, and £ 269.04 compared with £78.36 for the UK). Thus, raising equivalence scales above those from regression estimates for a subgroup of income units that have, on average, low pre-tax equivalent incomes tends to reduce the R/V observed, and vice versa for a subgroup of income units with high pre-tax equivalent incomes.

Consider two representative individuals drawn from the first and second population subgroups referred to above, and denote them A and B respectively. Let a^* denote the equivalence scale defined by the parameter estimates in Table 1, and assume that the respective pre-tax incomes of A and B are such that $0 < x_A/a_A^* = x_A^* < x_B^* = x_B/a_B^*$. Suppose that the equivalence scale used for analysis is $\tilde{a}$, where $\tilde{a}_A > a_A^*$ and $\tilde{a}_B < a_B^*$. Hence:

$$0 < x_A/\tilde{a}_A = \tilde{x}_A < x_A^* < x_B^* < \tilde{x}_B = x_B/\tilde{a}_B \tag{22}$$

The pre-tax equivalent incomes of A and B exhibit greater disparity based on the equivalence scale $\tilde{a}$ than on a^*. This suggests it is less likely that reranking is observed between A and B. Indeed, assuming that the marginal rate of the equivalent tax function is strictly less than one, the left hand side of equation (15) opposes the inequality condition required for reranking. This is offset by uncertainty regarding the effect on the right hand side of equation (15), which depends upon the equivalent tax function parameters and the heteroscedastic application of taxation in addition to the alteration considered for the equivalence scale. Assuming the same equivalence scale is applicable for pre-tax and post-tax income, $\tilde{y}_A < y_A^*$ and $\tilde{y}_B > y_B^*$, which supports the conclusion that it is less likely that reranking between A and B is observed using equivalence scale $\tilde{a}$ than a^*. These effects are consistent with the discussion in Section 4 regarding Figure 1, where the relative increase considered for the equivalence scale of subgroup 2 resulted in the omission of observed reranking between income units A and B and a reduced area in which reranking between subgroups could occur.

6 Conclusions

Reranking can have a signi…cant e¤ect on the progressivity achieved by a tax-transfer system, and is closely related to concepts of equity and redistributive justice. However, empirical analyses of reranking are typically subject to criticism on the basis of the value judgements that they exogenously impose. This paper examined the implications for reranking of di¤erent value judgements regarding the needs of heterogeneous income units. It was conjectured that an appropriate estimation strategy for the equivalence scale implicit in transfer policy is to seek the relativities that minimise reranking. This strategy does not require an assumed speci…cation for the tax function of reference units. Analytical results showed that the equivalence scale that minimises observed reranking can di¤er from the scale that is implicit in transfer policy. Nevertheless, analysis of survey data from Australia and the UK revealed no signi…cant di¤erence between the minimum measure of reranking, and the reranking observed for regression-based scales that minimise horizontal inequity using an explicit tax function. Hence, the reranking-minimising equivalence scales may provide a conservative basis for scales implicitly used by policy makers.

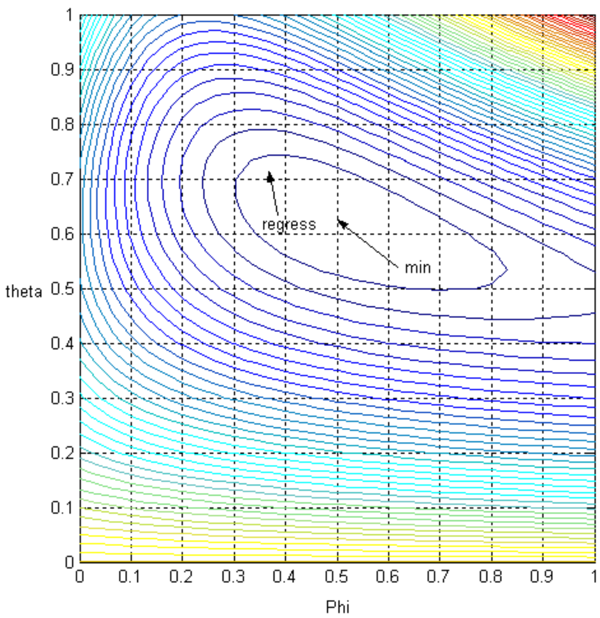
The analysis presented here consequently provides the analyst with a framework for selecting the parameters of a given equivalence scale speci…cation. It does not, however, provide an adequate response to criticism that observed reranking may be due to horizontal inequity 'imposed from the outside'; selection of an equivalence scale speci…cation can have an important e¤ect on the measures of reranking obtained. Clearly, the practitioner must continue to exercise care when selecting an equivalence scale, and when interpreting associated distributional observations.

Appendix A: Alternative Reranking Measures

The reranking measure, R, incorporates the same aggregation that is associated with the Gini coefficient. To explore different value judgements associated with the aggregation, the analysis that is presented in Section 5 has been repeated using the extended Gini coefficient $G(v)$ where:[25]

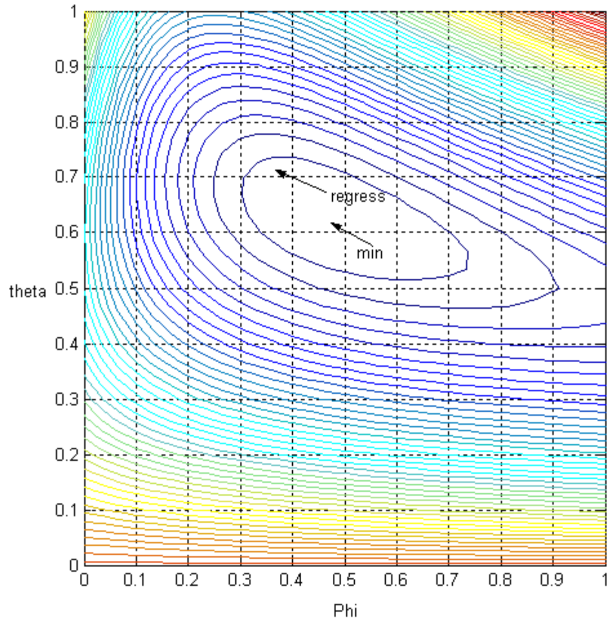
$$G_y(v) = -\frac{v}{\bar{y}} cov\left(y, (1-F(y))^{(v-1)}\right) \tag{23}$$

where $v > 1$ is a parameter that reflects aversion to inequality, and all other terms retain their definitions from above. As v increases from 1 toward infinity, $G(v)$ places a greater weight on the lower end of the income distribution. A value of $v = 2$ is equivalent to the standard Gini coefficient (which allocates the same weight to each income unit). The resulting topographical plots, for alternative values of v for both Australia and the UK, are shown below. Table 2 reports associated confidence intervals.

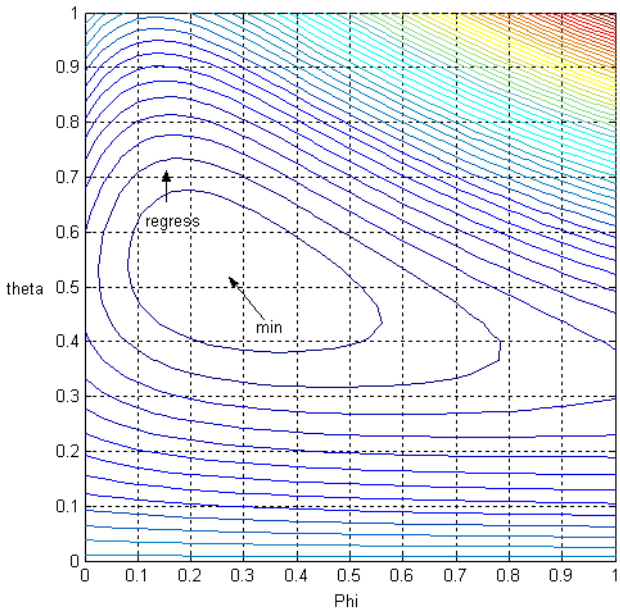


Australia: v=4

[25] See Yitzhaki (1983) on the extended Gini coefficient. See Lerman and Yitzhaki (1995) for an analysis of reranking based on the extended Gini coefficient.



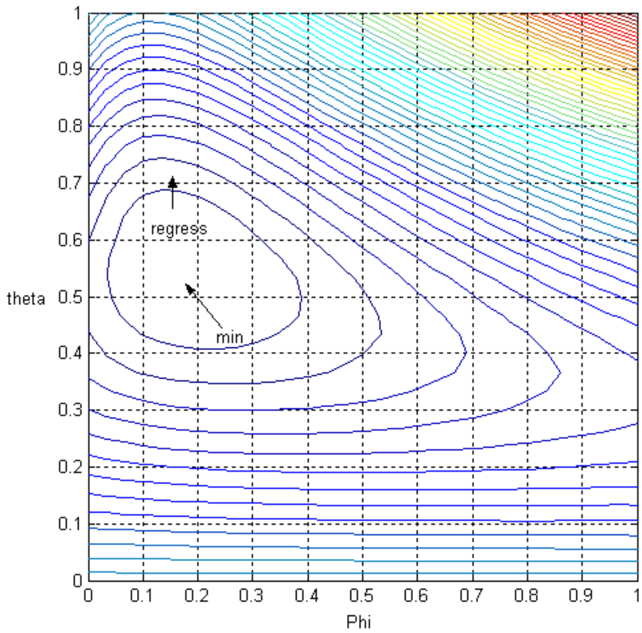
Australia: v=1.5



UK: v=4

Table 2: Reranking and Alternative Inequality Aversion Parameters

Equiv scale	Regress R/V	Min R/V	95% Con...dence Lower	Upper
Australian Data				
v = 1.5	4.006	3.975	3.737	4.224
v = 4.0	12.916	12.825	11.955	13.785
UK Data				
v = 1.5	6.319	6.209	5.895	6.548
v = 4.0	11.487	11.173	10.501	11.940



UK: v=1.5

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