

# Demand Based Equivalence Scale Estimates for Australia and the UK<sup>α</sup>

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## Abstract

This paper compares alternative approaches for estimating equivalence scales from household expenditure data. Discussion focuses upon the limitations of cross-sectional household survey data, and the implications for model estimation. Taking into consideration the biases that are associated with alternative models, a demand system with fixed price effects is identified as the preferred equivalence scale estimation methodology using pooled cross-sectional survey data. The relativities of household expenditure estimated for Australia and the UK are similar, and suggest that there are likely to exist larger economies of scale in the UK.

Key Words: Equivalence Scales, Household Expenditure

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## I Introduction

Equivalence scales relate the needs of heterogeneous households and are consequently an important analytical tool. This paper compares four alternative approaches for estimating equivalence scales from household expenditure data.

The approaches that are considered were selected on the basis of the attention

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that has been paid by the associated literature. Potential biases are discussed, and emphasis is placed on the importance of selecting a model that imposes sufficient structure to enable estimation subject to data limitations. A demand system model with fixed price effects is identified as the preferred methodology using pooled cross-sectional survey data for a 10 year period from Australia, and for a 6 year period from the UK. The relativities of household expenditure estimated for the two countries are similar, and suggest that there are likely to exist larger economies of scale (with respect to the number of household members) in the UK than in Australia.

There exists a vast literature that considers household equivalence scales. Much of this literature is concerned with associated welfare comparisons between households. However, Pollak and Wales (1979, p. 216) have notably criticised this focus on the basis that “the equivalence scales required for welfare comparisons are logically distinct from those which arise in demand analysis”.<sup>1</sup> The central difficulty is that demand analysis fails to provide a basis for making cardinal comparisons of welfare between households, and so equivalence scales that are estimated from expenditure data necessarily depend upon exogenously imposed value judgements. The current study is not concerned with welfare comparisons, and as such makes no attempt to address the criticism of Pollak and Wales. The equivalence scales considered here are simply taken to relate the expenditure costs that are associated with alternative household demographic specifications.

Section II presents some preliminary discussion regarding the equivalence scales that are considered by this paper, including a brief review of the associated literature. Section III considers scale estimates obtained using the Engel and Rothbarth methods. Demand system estimates are reported and compared with those derived using the Engel and Rothbarth methods in Section IV. Concluding comments are made in Section V.

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<sup>1</sup>Deaton and Muellbauer (1980, chapters 7, 8 and 9) provide a comprehensive discussion of the theoretical underpinnings of the demand based approach for estimating equivalence scales. Muellbauer (1975) pointed out the difficulties of making welfare comparisons, given taste differences of the type used in equivalence scale models.

## II Alternative Approaches

The Engel method of estimating equivalence scales assumes that the welfare of a household is accurately reflected by the expenditure share on food. Engel (1895) proposed this method after observing that there exists a negative relation between the expenditure share on food and total expenditure, and that the expenditure share on food increases with household size.<sup>2</sup> In 1943, Rothbarth departed from Engel's methodology by focusing on goods that are specific to adult well-being (rather than the well-being of all household members). Rothbarth (1943, p. 123) justified his framework by stating that "there will be a broad correspondence between the standard-of-living attained by the parents and the standard-of-living of the child." This assumption is similar to a related assumption underlying the Engel method, where "One would scarcely expect differences in age and sex to have an effect upon food expenses different from that which they had upon food needs in calories."<sup>3</sup> It is evident, however, that the Rothbarth assumption is more demanding. The principal advantage of the Rothbarth compared with the Engel method is that it avoids contamination associated with disparities between the consumption patterns of adults and children; the issue of bias is returned to later in the paper.<sup>4</sup>

The majority of the contemporary literature has interpreted family well-being in terms of utility following Barten (1964) and Muellbauer (1974, 1977), where associated equivalence scales are estimated using consumer demand theory.<sup>5</sup> This framework is useful for estimating equivalence scales because, unlike the Engel and Rothbarth methods, it can capture the effect that demographic change has on household preferences. In the Barten (1964) model, for example, altered household preferences are captured through demographic dependent

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<sup>2</sup> See also, Engel (1875).

<sup>3</sup> Sydenstricker and King (1921), quoted by Nelson (1993, p. 476).

<sup>4</sup> For a contemporary analysis of both the Engel and Rothbarth models, refer to Tsakloglou (1991)

<sup>5</sup> See also, Deaton and Muellbauer (1986).

prices in the cost (or expenditure) function:

$$c(p, z, u) = f(p^a, u) \quad (1)$$

where  $p$  is a vector of expenditure group prices,  $z$  is a vector of consumption unit characteristics,  $u$  denotes utility, and  $p^a$  is a vector comprised of elements  $p_i^a = p_i/\phi_i(z)$ . The function  $\phi_i(z)$  adjusts prices in response to the consumption of additional family members. In the words Gorman (1976), "a penny bun costs threepence when you've a wife and child."<sup>6</sup> In addition,  $\phi_i(z)$  reflects behavioural responses to demographic change, such as the melancholy description by Prais and Houthakker (1955, p. 132) of "the father who is driven to drink by his wife and children."

In the spirit of Rothbarth (1943), most of the contemporary literature focuses on adult conditional utility, where children have an effect on the utility of their parents only insofar as they affect adult consumption.<sup>7</sup> This framework may be compared with one that is specified in terms of adult unconditional utility, where the increase in utility derived by parents from their offspring is explicitly taken into account. Use of the unconditional form is based on the premise that failure to include children in the utility function results in an excessively high estimate for the cost of children. As noted by Atkinson (1983, p. 51), "This is a matter for social judgement. Indeed, it might be argued that the benefits of having children are such that in a country where birth control is widespread no allowance should be made for the cost of children."<sup>8</sup>

The use of unconditional utility does not, however, account for the fact that children have no say in their birth, and rarely whether they obtain an additional sibling. In this sense a focus on unconditional utility underestimates the equivalence scale, where concern is for the welfare of every family member.

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<sup>6</sup>A limitation of the Barten model is that families of varying characteristics are restricted to consuming only those goods that are taken by the reference household. To relax this restriction, Gorman (1976) added a fixed cost term dependent on  $z$  to the cost function - a specification referred to as the Gorman-Barten model.

<sup>7</sup>See, for example, Barten (1964), Muellbauer (1974, 1977), Gorman (1976), Deaton and Muellbauer (1986), and Gronau (1988)

<sup>8</sup>See also, Pollak and Wales (1979). For a recent applied study, see Ferriera et al. (1998).

Given random changes in economic status, the assumption that observing children in a family implies that some net benefit is enjoyed by the parents is also questionable (Muellbauer, 1977, p. 461).<sup>9</sup>

Although the utility framework provides a fundamental foundation for equivalence scale analysis, it is also the subject of considerable criticism. A large volume of recent research has focused on the problems associated with the existence and specification of family utility functions (see Phipps and Burton, 1995, for a survey). In addition, defining equivalence scales in terms of utility implies the need for interpersonal utility comparisons, the difficulties associated with which have been explored prominently by Sen (1987). From a practical perspective, increasingly complex models require an attendant increase in the information necessary for estimation. Demand system models, for example, require data that include price variation to enable the associated coefficients to be identified. Even where the prices defined by the data do exhibit substantial variation, the information may not be sufficient to identify the model parameters. This is because prices generally vary between periods while the majority of family characteristics vary within periods for survey data (Dickens et al., 1993). The temporal nature of price variation also requires stable behavioural patterns to enable meaningful equivalence scales to be estimated when cross-sectional data spanning a significant number of years are pooled.

### III Single Equation Estimates - The Engel and Rothbarth Methods

Two of the oldest, and most frequently employed single equation methods of estimating equivalence scales are those suggested by Engel (1895) and Rothbarth (1943). The Engel method is usually calculated using a variant of the Working

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<sup>9</sup>See, Bojer and Nelson (1999), criticising Ferreira et al. (1998). Also, see, Sen (1973), Pollak and Wales (1979), and Pashardes (1991) on the limitations of using cross-sectional data. See Bradbury (1989) for a survey of scales based on subjective data that attempt to account for the inadequacies of standard cross-sectional surveys.

(1943) - Leser (1963) specification:

$$w_{fi} = \alpha + \beta (\ln(e_i) - h(z_i)) + \xi_i \quad (2)$$

where  $w_{fi}$  is a measure of household  $i$ 's expenditure share on food,  $e_i$  is total household expenditure, and  $\xi_i$  is a random error term. The equivalence scale associated with equation (2) is derived by equating  $w_{fi}$  to the expenditure share of a reference household,  $w_{fr}$ . Rearranging to obtain the ratio of expenditure for each household, the equivalence scale is defined by:

$$a_{Ei} = \frac{e_i}{e_r} = \exp \{ \beta [h(z_i) - h(z_r)] \} \quad (3)$$

where the  $r$  subscripts identify reference household values.

The Rothbarth method of estimating equivalence scales is identical to the Engel method, with the exception that per adult consumption of 'adult goods' is considered rather than expenditure share on food. Hence, the specification associated with equation (2) that is estimated under the Rothbarth method is given by:

$$\frac{q_i}{n_{di}} = \alpha + \beta (\ln(e_i) - h(z_i)) + \xi_i \quad (4)$$

where  $q_i$  is household expenditure on the adult good, and  $n_{di}$  is the number of adult household members.

For the analysis considered here, heterogeneity is limited to the number of children (aged 17 years and under) and the number of adults (aged 18 years and over) in each household. These characteristics are entered via three specifications for  $h(\cdot)$ :

$$h(n_{di}, n_{ci}) = \begin{cases} \gamma_{pc} \frac{n_{ci}}{(n_{ci} + n_{di})} + \gamma_n \ln(n_{di} + n_{ci}) & 1 \\ \gamma_c n_{ci} + \gamma_d n_{di} + \gamma_n \ln(n_{di} + n_{ci}) & 2 \\ \gamma_c n_{ci} + \gamma_d n_{di} + \gamma_{2c} n_{ci}^2 + \gamma_{2d} n_{di}^2 + \gamma_{cd} n_{di} n_{ci} & 3 \end{cases} \quad (5)$$

where  $n_{ci}$  denotes the number of children of household  $i$ . The data used are

from the 1993 Household Expenditure Survey (HES) for Australia, and the 1993 Family Expenditure Survey (FES) for the UK. The HES and FES provide detailed microdata that describe the demographic composition, income sources, and expenditure behaviour of, respectively, 8,389 and 6,979 households.<sup>10</sup> After editing the HES and FES survey populations, the estimations presented here are based on data for 8,205 Australian and 6,970 UK households.<sup>11</sup>

Estimates for the models described above are reported in Table 1.<sup>12</sup> The 'h(n<sub>c</sub>,n<sub>d</sub>) specification' listed in the table refers to the characterisations of equation (5). Comparing the three equation standard errors (and R-squares) obtained for each of the four series displayed in Table 1, indicates that the alternative specifications of h(.) reflect similar proportions of the variation observed in the data, with the first specification performing slightly better for all but the Engel model estimated from Australian data. The R-squares indicate that the UK data are modelled better than the Australian data for both the Engel and Rothbarth models, with the largest disparity observed for the Engel model.

All of the estimates obtained for β are significant and have the expected signs. The equivalence scales associated with the first specification reported in Table 1 are given by:

$$a_i = \exp \left[ \frac{1}{2} \gamma_{pc} \frac{n_{ci}}{(n_{ci} + n_{di})} + \frac{3}{4} \gamma_n \ln(n_{ci} + n_{di}) \right] \quad (6)$$

Equation (6) implies that  $a_i = (n_{ci} + n_{di})^{\gamma_n}$  when  $\gamma_{pc} = 0$ . Hence the parameter  $\gamma_n$  reflects economies of scale within the household. When, for example,  $0 < \gamma_n < 1$ , economies of scale exist such that the costs of additional household members fall with household size. The parameter  $\gamma_{pc}$  indicates the impact that household composition has on the equivalence scale. Specifically,  $\gamma_{pc} < 0$

<sup>10</sup> Refer to the Australian Bureau of Statistics (ABS), and the Central Statistical Office (now the Office for National Statistics) for detailed information regarding the HES and FES respectively.

<sup>11</sup> Households were removed from the survey populations due to data inconsistencies, the observation of negative expenditure or net income, or the absence of any household members over the age of 17.

<sup>12</sup> Estimates of the equivalence scale parameters obtained for a quadratic specification of equation (2) in  $\ln(e_i)$  are not significantly different from those reported here.

Table 1: Engel Curve Estimates for Australia and the UK, 1993

h(nc,nd) specification	parameter	Australia		UK	
		Engel	Rothbarth	Engel	Rothbarth
1	$\beta$	-0.08808 (0.0022)	1668.56 (52.2295)	-0.10588 (0.0017)	5.83930 (0.1900)
	$\gamma_{pc}$	-0.60810 (0.0563)	-0.93955 (0.0566)	-0.64577 (0.0508)	-0.78147 (0.0695)
	$\gamma_n$	1.02571 (0.0275)	0.91798 (0.0291)	0.94181 (0.0234)	0.89806 (0.0364)
	s.e.	0.08196	1630.67	0.07703	6.27798
	R-square	0.25811	0.23547	0.41762	0.24699
2	$\beta$	-0.08721 (0.0022)	1671.53 (52.7559)	-0.10505 (0.0017)	5.82293 (0.1904)
	$\gamma_c$	0.00364* (0.0268)	-0.16114 (0.0269)	-0.0168* (0.0252)	-0.12347 (0.0314)
	$\gamma_d$	0.16105 (0.0328)	0.07577 (0.0318)	0.16497 (0.0322)	0.08069 (0.0392)
	$\gamma_n$	0.67597 (0.0747)	0.79189 (0.0782)	0.61142 (0.0679)	0.75384 (0.0893)
	s.e.	0.08210	1631	0.07720	6.28268
R-square	0.25572	0.23525	0.41502	0.24597	
3	$\beta$	-0.08766 (0.0022)	1669.45 (52.4540)	-0.10524 (0.0017)	5.86544 (0.1939)
	$\gamma_c$	0.34425 (0.0364)	0.24170 (0.0384)	0.32923 (0.0358)	0.27120 (0.0482)
	$\gamma_d$	0.93619 (0.0478)	0.88218 (0.0500)	0.84861 (0.0425)	0.91956 (0.0626)
	$\gamma_{2c}$	-0.00602* (0.0067)	-0.0137* (0.0073)	0.00316* (0.0055)	-0.0118* (0.0078)
	$\gamma_{2d}$	-0.09449 (0.0095)	-0.09060 (0.0089)	-0.08100 (0.0088)	-0.10362 (0.0133)
$\gamma_{cd}$	-0.05537 (0.0158)	-0.05409 (0.0131)	-0.08070 (0.0161)	-0.05643 (0.0210)	
s.e.	0.08195	1631.56	0.07711	6.28052	
R-square	0.25862	0.23492	0.41668	0.24671	

\* indicates insignificant coefficients at 95 per cent confidence level

Engel estimates from expenditure share of all food and non-alcoholic beverage

Rothbarth estimates from per adult expenditure on food taken outside the home

Coefficient standard errors provided in parentheses

indicates that the costs to a household of children are less than the costs of adults. Hence the coefficient estimates displayed in Table 1 indicate that the Rothbarth scales for both countries incorporate higher economies of scale than the Engel scales, and lower costs of children relative to adults. These results are consistent with the expected biases associated with each estimation method, which are discussed in detail below.

The second and third specifications considered in Table 1 indicate similar characteristics to the first. The marginally better performance of the first specification consequently supports its use for the subsequent analysis.

The Engel scale is expected to over-estimate the actual costs of children described by demand behaviour due to the assumption that households with the same expenditure share on food enjoy the same well-being, regardless of demographic composition. As Nicholson (1976, p. 2) recognised, "The variety of goods available to adults is greatly in excess of that competing for the allocation of income / expenditure on behalf of the child (whatever its age); and, at least for very young children, food expenditure is likely to bulk larger than it does for adults." Consequently, if the consumption share of food for adults is held constant after the addition of children, then the food share for the household taken as a whole would increase.

If the true equivalence scale,  $a_{Ti}$ , is defined by equating the food share of adults in any given household to the food share of adults in the reference household, then the above discussion implies that:

$$w_{fi} = w_{fr} + \phi_{fi} \quad (7)$$

When the reference household includes no children,  $\phi_{fi} \geq 0$  is an increasing function of the number of children in household  $i$ . Substituting the Working-Leser Engel curve specification into equation (7), and rearranging obtains the following for  $a_{Ti}$ :

$$a_{Ti} = \exp \left[ \frac{1}{2} \frac{\phi_{fi}}{\beta} + \ln a_{Ei} \right] \quad (8)$$

where  $a_{Ei}$  is the equivalence scale estimate derived using the Engel method. Since  $\phi_{fi} > 0$ , and  $\beta < 0$  for the food expenditure share specification, it is evident that  $a_{Ti} < a_{Ei}$ .

This is precisely the type of bias that the Rothbarth scale is structured to avoid. However, the Rothbarth scale is likely to understate the true cost of children because there exist very few expenditure groups that relate solely to the consumption of adult household members. Where the adult good used for analysis is consumed partly by children, holding the value consumed by adults constant after the addition of children implies that total household consumption of adult goods must increase. Even if the good adopted is consumed only by adults, the total household expenditure on the adult good may be an increasing function of the number of children. As Deaton and Muellbauer (1986, p. 732) state, "the presence of children makes goods that are shared with children relatively more expensive than pure adult goods...so that there may be substitution toward adult goods in households with children." Both of these influences imply that the true equivalence scale may be defined by the condition:

$$\frac{q_i}{n_{di}} = \frac{q_r}{n_{dr}} + \phi_{AGi} \quad (9)$$

Again, when the reference household includes no children,  $\phi_{AGi} > 0$  is an increasing function of the number of children in the household. From this expression, the following can be obtained for  $a_{Ti}$ :

$$a_{Ti} = \exp \left[ \frac{1}{2} \frac{\phi_{AGi}}{\beta} \right] + \ln(a_{Ri}) \quad (10)$$

where  $a_{Ri}$  is the estimate of the equivalence scale obtained using the Rothbarth method. Since  $\phi_{AGi} > 0$ , and  $\beta > 0$  for the per adult expenditure on adult goods specification, it is evident that  $a_{Ti} > a_{Ri}$ . Consequently, the gap between the Rothbarth and Engel scales is usually interpreted as a range within which the actual household costs of children are defined.

## IV Demand System Estimates

Given the biases that are associated with the Engel and Rothbarth methods, it is useful to obtain equivalence scale estimates using a demand system. A non-linear extension of the Almost Ideal Demand System (EAIDS, Blundell et al., 1993) is used, which is defined using the Quadratic Logarithmic family of cost functions (Lewbel, 1990):

$$\ln c(p_t, u_{it}) = \ln g_i(p_t) + \ln f_i(u_{it}, p_t) \quad (11)$$

where  $\ln f_i(u_{it}, p_t)$  takes the form:

$$\ln f_i(u_{it}, p_t) = \ln \left[ \frac{\prod_{j=1}^J p_{jt}^{\beta_{ji}}}{(\beta_{0i} u_{it})^{\lambda_i} \prod_{j=1}^J p_{jt}^{\beta_{ji}}} \right] \quad (12)$$

The subscript  $i = 1, \dots, I$  refers to the household,  $j = 1, \dots, J$  to the consumption good, and  $t$  to time; all other variables are as defined previously. The (log) equivalence scale associated with this cost function is defined by:

$$\ln a_i(p_t, u_{0t}) = \ln g_i(p_t) + \ln f_i(u_{0t}, p_t) - \ln g_0(p_t) - \ln f_0(u_{0t}, p_t) \quad (13)$$

It is usual to assume that an equivalence scale is independent of base (Lewbel, 1989, and Blundell and Lewbel, 1991), such that it is unaffected by the level of utility (or income).<sup>13</sup> Although the empirical evidence to support the assumption of base independence is scarce, a number of papers have indicated that the associated effect (for demand based analyses) is slight.<sup>14</sup> Base independence with regard to the specification of equation (13) requires  $\ln f_i(u_{0t}, p_t) = \ln f_0(u_{0t}, p_t)$ , and hence  $\beta_{ji} = \beta_j$  and  $\lambda_i = \lambda$ .

The budget shares associated with the cost function defined by equations

<sup>13</sup> Base independence is referred to as Equivalence Scale Exactness by Blackorby and Donaldson (1993).

<sup>14</sup> Pendakur (1999) is a rare demand based analysis that has provided empirical support for base independence. Papers indicating that the effect of imposing base independence is slight include Blackorby and Donaldson (1993), Blundell and Lewbel (1991), Dickens et al. (1993), and Lancaster and Ray (1998).

(11) and (12), assuming a base independent form are:

$$\begin{aligned}
 w_{jit} &= \frac{\partial \ln c(p_t, u_{it})}{\partial \ln p_{jt}} \\
 &= \frac{\partial \ln g_i(p_t)}{\partial \ln p_{jt}} \\
 &\quad + \beta_j [\ln e_{it} - \ln g_i(p_t)] + \lambda [\ln e_{it} - \ln g_i(p_t)]^2 \quad \text{a} \quad (14)
 \end{aligned}$$

In equation (14),  $w_{jit}$  denotes the expenditure share of household  $i$  on consumption good  $j$  at time  $t$ , and  $e$  is total expenditure. Comparing equation (14) with equations (2) and (4) indicates that all three demand based estimation methods use similar specifications to identify the associated equivalence scales. Whereas the Engel and Rothbarth methods are based on consumption behaviour for specific goods, however, the demand system is based on behaviour for total expenditure, disaggregated into a range of consumption baskets.

It is evident from equation (14) that the data used to estimate the demand system must exhibit some price variation. Consequently, the 1993 HES and FES data described previously are pooled with data derived from the 1988 and 1984 HESs, and the 1992 and 1988 FESs. The 1988 and 1984 HESs (1992 and 1988 FESs) provide microdata for, respectively, 7,225 and 4,493 (7,419 and 7,265) households, from which 6,935 and 4,282 (7,411 and 7,258) are used to create a pooled Australian (UK) data set of 19,422 (21,639) households. Total household expenditure is disaggregated into eight consumption groups for the estimations presented here; food and non-alcoholic beverages, fuel and power, alcohol and tobacco, clothing and footwear, transport, entertainment, personal care, and miscellaneous (which is principally comprised of housing expenditure). The price data used are derived from quarterly reports of the Consumer Price Index published by the ABS for Australia, and the Retail Price Index publication issued by the CSO for the UK.<sup>15</sup> Prices are normalised to one for 1993, and both prices and expenditure are divided by the general price index for each year to adjust for inflation.<sup>16</sup>

<sup>15</sup> ABS Cat. No. 6401, Table 6A; CSO 1994 edition of the Retail Prices Index, Table 5.

<sup>16</sup> Homogeneity of the first degree between prices and expenditure implies that dividing both series by a constant (the general price level of the respective year) has no distributional effect.

It is common to assume a flexible translog specification for  $\ln g_i(p_t)$ :

$$\ln g_i(p_t) = \gamma_{0i} + \sum_j \gamma_{ji} \ln(p_{jt}) + \sum_j \sum_k \gamma_{jki} \ln(p_{jt}) \ln(p_{kt}) \quad (15)$$

Given the limited price variation of the data used, however, it was necessary to suppress the price interaction terms ( $\gamma_{jki} = 0$  for all  $j, k$ ).<sup>17</sup> Hence, the following CES form is assumed:

$$\ln g_i(p_t) = \alpha_0 + \sum_{m=1}^M \alpha_{0m} z_{mi} + \frac{1}{\rho} \ln \left[ \sum_{j=1}^J \alpha_j + \sum_{m=1}^M \alpha_{jm} z_{mi} \right] p_{jt}^\rho \quad (16)$$

where  $z_{mi}$  defines the income unit characteristics that affect the cost function. The equivalence scale associated with (16) is defined by:

$$\ln a_i(p_t) = \sum_{m=1}^M \alpha_{0m} (z_{mit} + z_{mrt}) + \frac{1}{\rho} \ln \left[ \sum_{j=1}^J \alpha_j + \sum_{m=1}^M \alpha_{jm} z_{mit} \right] \frac{p_{jt}^\rho}{\sum_{j=1}^J \alpha_j + \sum_{m=1}^M \alpha_{jm} z_{mrt} p_{jt}^\rho} \quad (17)$$

Normalising the reference characteristics so that  $z_{mrt} = 0$  for all  $m$ , and the reference prices  $p_{jr} = 1$  for all  $j$ , the equivalence scale (17) reduces to:

$$\ln a_i(p_r) = \sum_{m=1}^M \alpha_{0m} z_{mit} + \frac{1}{\rho} \ln \left[ 1 + \frac{\sum_{j=1}^J \alpha_j + \sum_{m=1}^M \alpha_{jm} z_{mit}}{\sum_{j=1}^J \alpha_j} \right] \quad (18)$$

It is consequently useful to normalise  $\sum_{j=1}^J \alpha_j = 1$ , and  $\sum_{j=1}^J \alpha_{jm} = 0$ , so that:

$$\ln a_i(p_r) = \sum_{m=1}^M \alpha_{0m} z_{mit} \quad (19)$$

The following specification is assumed for  $\sum_{m=1}^M \alpha_{0m} z_{mi}$ , in light of the results

Additive dummy variables by year are also included in the regression model to account for variation of  $\ln(e_{it})$ .

<sup>17</sup> This restriction is often imposed when the price data used exhibit limited variation; see, for example, Lancaster and Ray (1998).

reported in Section III:

$$\alpha_0 + \sum_{m=1}^M \alpha_{0m} z_{mit} = \alpha_0 + \gamma_{pc} \frac{n_{cit}}{(n_{ci} + n_{dit})} + \gamma_n \ln(n_{cit} + n_{dit}) \quad (20)$$

Given the limited variation of the price data used, the impact of household characteristics on price effects is explored by assuming:

$$\alpha_j + \sum_{m=1}^M \alpha_{jm} z_{mi} = \alpha_j + \alpha_{jc} n_{cit} + \alpha_{jd} n_{dit} \quad (21)$$

Regression results are reported in Table 2 for two models, the first of which restricts the parameters  $\alpha_{jc}$  and  $\alpha_{jd}$  to zero for all  $j$ .

Consider, first, the estimates obtained for the demand system that omits demographic effects on household prices, which are reported at the top of Table 2. Each country's estimates for the equivalence scale parameters,  $\gamma_{pc}$  and  $\gamma_n$ , from this model indicate higher economies of scale and less disparity between the costs of adults and children than the estimates derived using the Engel and Rothbarth methods. These two effects tend to offset one another with regard to the magnitude of the respective equivalence scales, an issue that is discussed in detail below. The significantly larger estimate obtained for  $\gamma_n$  from Australian data indicates fewer economies of scale than in the UK, and the larger magnitude of  $\gamma_{pc}$  indicates a more prominent disparity between the costs of adults and children. The estimates obtained for  $\lambda$  and  $\rho$  are significant though small, indicating that the use of a Cobb-Douglas specification for  $\ln g_i(p_t)$  and a linear specification for  $w_{jit}$  would not be excessively restrictive.<sup>18</sup>

Comparing the log-likelihood statistics of the two models reported in Table 2 for each country indicates that a significantly improved fit is obtained by the specification that allows demographic interactions with price effects. The estimates obtained for  $\gamma_{pc}$  and  $\gamma_n$  from Australian data are not significantly affected by allowing for demographic price interactions. However, the values

<sup>18</sup> Various starting values were used to verify that the small estimate obtained for  $\rho$  was not the result of a local maximum. Imposing a larger  $\rho$  on the model had no significant effect on the equivalence scale parameters.

Table 2: Demand System Regression Statistics

Parameter	Australia		UK	
specification without demographic price interactions				
$\gamma_{pc}$	-0.32349		-0.25754	
	(0.0253)		(0.0208)	
$\gamma_n$	0.67985		0.58032	
	(0.0130)		(0.0099)	
$\rho$	0.04566		-0.10901	
	(0.0168)		(0.0458)	
$\lambda$	-0.03241		-0.05841	
	(0.0059)		(0.0095)	
log(L)	191193		180922	
specification with demographic price interactions				
$\gamma_{pc}$	-0.44421		-0.20980	
	(0.0664)		(0.0507)	
$\gamma_n$	0.64827		0.50256	
	(0.0353)		(0.0250)	
$\rho$	0.05081		-0.06801	
	(0.0208)		(0.0353)	
$\lambda$	-0.03578		-0.05133	
	(0.0071)		(0.0099)	
	nc	nd	nc	nd
food	8.521E-03 (1.19E-03)	1.597E-02 (1.48E-03)	9.456E-03 (9.66E-04)	2.478E-02 (1.40E-03)
power	-2.62E-04* (3.38E-04)	-1.984E-03 (4.06E-04)	-2.126E-03 (5.64E-04)	-7.466E-03 (8.17E-04)
a&t	-5.930E-03 (3.34E-04)	3.082E-03 (6.01E-04)	-4.246E-03 (5.50E-04)	1.591E-02 (8.25E-04)
c&f	2.004E-03 (4.37E-04)	6.098E-03 (7.12E-04)	6.990E-03 (5.06E-04)	3.075E-03 (7.55E-04)
transport	-3.449E-03 (9.96E-04)	1.628E-02 (1.46E-03)	-9.89E-04* (9.35E-04)	1.600E-02 (1.38E-03)
entertainment	-6.482E-03 (7.96E-04)	1.15E-03* (1.14E-03)	-3.586E-03 (7.98E-04)	-6.866E-03 (1.18E-03)
personal care	-9.771E-04 (1.45E-04)	1.305E-03 (2.50E-04)	-1.087E-03 (3.10E-04)	1.688E-03 (4.64E-04)
log(L)	192040		182424	

\* indicates insignificant coefficients at 95 per cent confidence level  
standard errors in parentheses

of the estimates obtained from the model that includes demographic effects on prices imply a smaller equivalence scale than estimates obtained from the restricted demand system. Estimates obtained from UK data also imply that a smaller scale is associated with the demand system that includes demographic-price interactions.

Extensive sensitivity analysis revealed that the estimates obtained from the model that includes demographic price interactions should be treated with caution. Specifically, significant heterogeneity with regard to the non-price related parameters was identified between the three cross-sections included in the pooled data sets. Given a small number of pooled cross-sections, heterogeneity between the cross-sections can bias the estimates associated with price variation, and hence the model as a whole. This effect is somewhat constrained for the estimates reported in Table 2 by the specification adopted for  $\alpha_{0m} z_{imt}$ . However, substituting the third specification identified in equation (5) into (20), obtains estimates that imply equivalence scales which decrease with the addition of household members for the demand system that allows demographic characteristics to affect prices. This is a common problem faced when using data that are drawn from a small number of cross-sections.

The effects of demographic characteristics on prices are reported in the lower section of the Table 2. Relative to otherwise similar households without children, the coefficient estimates suggest that Australian households with children tend to spend more on food, clothing, and miscellaneous goods (mainly housing), and less on alcohol and tobacco, transport, entertainment, and personal care. Similarly, otherwise similar households with an additional adult tend to have a reduced preference for fuel and power, and miscellaneous expenditure (housing). These effects compare approximately with the results obtained for the UK.

The equivalence scales obtained from the Engel, Rothbarth, and demand system estimates are displayed in Table 3.<sup>19</sup> The equivalence scale estimates derived using the Engel method are all greater than the estimates obtained from

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<sup>19</sup> The equivalence scales derived by the Engel and Rothbarth methods are calculated from estimates of the first specification of equation (5).

Table 3: Equivalence Scales by Estimation Method

nd	nc	Australia				UK			
		Engel	Rothbarth	DS(a)	DS(b)	Engel	Rothbarth	DS(a)	DS(b)
1	0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1	1	1.50218	1.18119	1.36273	1.25513	1.39087	1.26081	1.31453	1.27564
1	2	2.05744	1.46542	1.70102	1.51597	1.82974	1.59303	1.59337	1.51022
1	3	2.62706	1.76461	2.01345	1.76039	2.27345	1.93261	1.84289	1.71489
1	4	3.20381	2.06638	2.30569	1.98969	2.71603	2.27094	2.07084	1.89838
2	0	2.03596	1.88947	1.60197	1.56729	1.92094	1.86355	1.49518	1.41673
2	1	2.51975	2.00436	1.89470	1.75791	2.26920	2.06706	1.73620	1.61961
2	2	3.05839	2.23182	2.18305	1.96715	2.67177	2.34958	1.96545	1.80724
2	3	3.61814	2.49355	2.45979	2.17455	3.09047	2.65511	2.18030	1.97973
2	4	4.18886	2.76887	2.72498	2.37596	3.51481	2.96870	2.38237	2.13957
3	0	3.08594	2.74150	2.11043	2.03846	2.81421	2.68214	1.89183	1.73693
3	1	3.56054	2.82273	2.36694	2.19820	3.13988	2.85652	2.09616	1.90456
3	2	4.08605	3.00902	2.62420	2.37658	3.51654	3.10427	2.29554	2.06457
3	3	4.63566	3.23824	2.87593	2.55854	3.91420	3.38166	2.48686	2.21571
3	4	5.19895	3.48834	3.12074	2.73912	4.32178	3.67291	2.67001	2.35857

(a) Equivalence scale estimates from demand system with fixed price effects

(b) Equivalence scale estimates from demand system with demographic dependent price effects

the Rothbarth method for both countries. Comparing the scales for the two countries indicates that the Engel estimates for Australia are all greater than, and the Rothbarth estimates predominantly less than, the associated scales derived for the UK. For the Australian estimates, the Engel and Rothbarth methods imply that a single parent with two dependent children has costs that are between 1.47 and 2.06 times those of a single adult. These are contrasted with the associated estimates derived for the UK, which indicate that a single parent with two dependent children has costs between 1.59 and 1.83 times a single adult. The fact that the Australian estimate for  $\gamma_n$  derived using the Engel method is (insignificantly) greater than one results in scales that suggest a two adult household without children has costs that are higher per household member than a single adult household, consistent with dis-economies of scale.

The equivalence scales derived from the demand systems estimates are less than the scales derived by either the Engel or Rothbarth methods for all of the household specifications comprised of two or more adults. This reflects the larger economies of scale that are associated with the demand system estimates. Focusing on single adult households, the estimates obtained for the restricted demand system using Australian data imply equivalence scales that lie between the Engel and Rothbarth scales, consistent with the biases that are associated

with the Engel and Rothbarth methods.<sup>20</sup> The restricted demand system estimates derived using Australian data, however, imply scales that fall below the Rothbarth scales for three or more children, which is also the case for estimates of the restricted demand system obtained using UK data. Similarly, the UK equivalence scale estimates obtained for the unrestricted demand system fall below the Rothbarth scales for single adult households with two or more children. These observations suggest that the equivalence scales implied by the demand system estimates may under-represent the household costs of constituent members, a result that is attributable to the small number of cross-sections used for estimation.

The fact that the equivalence scales derived from the demand system estimates exhibit larger economies of scale for adult household members than either the Engel or Rothbarth scales can be attributed, at least in part, to biases associated with the Engel and Rothbarth methods. With regard to the Rothbarth method, for example, Rothbarth (1943, pp. 123-124) states that, "The method in its simplest form applies only to families with the same number of adults. If the number of adults varies, we might take excess income per adult as our index." There are a number of adult goods for which it is reasonable to suppose that expenditure per adult will fall with the number of adults when household welfare is held constant due to shared consumption or reduced preference. When such goods are used for the dependent variable of the Rothbarth model, the true scale may be obtained by:

$$\frac{q_i}{n_{di}} = \frac{q_r}{n_{dr}} + \delta_{AGi} \quad (22)$$

where  $\delta_{AGi}$  reflects the altered per adult consumption of the adult good associated with the addition of adult household members. When the reference household is comprised of a single adult,  $\delta_{AGi} \geq 0$  is an increasing function in the number of adults. From this specification, it can be seen that the relation

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<sup>20</sup> The equivalence scales obtained from the estimates of the demand system with fixed price effects for Australia are also similar to those reported by Lancaster and Ray (2002, Table II).

between the true scale and the scale estimate obtained by the Rothbarth method will be characterised by:

$$a_{Ti} = \exp \left[ a_{Ri} + \frac{\delta_{AGi}}{\beta} \right]^{\frac{1}{2}} \quad (23)$$

Given that  $\beta > 0$  for the Rothbarth specification,  $a_{Ti}$  defined by equation (23) is less than the Rothbarth scale,  $a_{Ri}$ .

A similar argument can be used for the Engel method. It is evident that, in order to enjoy the same level (and quality) of nutrition, otherwise similar households will need to spend similar amounts on food per adult, regardless of the number of adults in a household; that is there are small (if any) economies of scale with respect to food expenditure. The same, however, cannot be said for non-food expenditure, where for example, consumption of housing can be shared amongst all members of a household. Hence it is reasonable to expect that, as the number of adults in a household increases, the share of non-food expenditure necessary to maintain a given level of welfare will be a decreasing function of the number of adults in the household; and conversely that the food expenditure share will be an increasing function of the number of adults in a household. With regard to identifying the true equivalence scale, these observations imply that:

$$w_{fi} = w_{fr} + \delta_{Ei} \quad (24)$$

where  $\delta_{Ei}$  reflects the impact on the food expenditure share of the number of adults in household  $i$ . When the reference household is comprised of a single adult,  $\delta_{Ei} \rightarrow 0$  is increasing in the number of adults in household  $i$ . This specification implies that the true equivalence scale will be characterised by:

$$a_{Ti} = \exp \left[ a_{Ei} + \frac{\delta_{Ei}}{\beta} \right]^{\frac{1}{2}} \quad (25)$$

and since  $\beta < 0$  for the Engel specification, it is evident that  $a_{Ti}$  will be less than the equivalence scale obtained using the Engel method.

## V Conclusions

This paper contrasts equivalence scales derived for Australia and the UK using four demand based models; the Engel model, the Rothbarth model, a demand system that includes demographic price interactions, and a demand system with restricted price effects. Particular care is taken with regard to model selection given the limited data used for estimation.

Equivalence scale estimates derived using the Engel method suggest that the proportional increase of household costs associated with the addition of children are higher in Australia than they are in the UK, whereas the reverse is true of estimates obtained using the Rothbarth method. Given the biases associated with these estimation methods, the results consequently suggest that child costs in Australia may be equivalent to those in the UK. Three specifications of equivalence scale are estimated, all of which indicate significant economies of scale with regard to household size in both countries.

The equivalence scales associated with estimates derived for the demand systems (with or without demographic price interactions) of both Australia and the UK exhibit more significant economies of scale and less disparity between the costs of adults and children than do estimates obtained by either the Engel or Rothbarth methods. Australian estimates of child costs obtained from a demand system that omits demographic price interactions fall between the Engel and Rothbarth estimates consistent with expected biases. However, estimates obtained from a demand system that allows demographic characteristics to affect household prices suggest child costs below the Rothbarth estimates for common household specifications based on data from both countries. This unexpected result is attributed to biased estimates obtained as a result of including an insufficient number of cross-sections in the pooled data sets used.

The discussion undertaken suggests that the estimates derived by the Engel method provide an upper bound for the equivalence scale that most accurately reflects the proportional costs of heterogeneous households. Similarly, the esti-

mates obtained for the demand system that excludes demographic dependent price effects can be referenced as a lower bound. Each of these equivalence scales imply that higher economies of scale exist for households in the UK than Australia. However, the range defined between the Engel and demand system scales estimated for Australia substantially overlaps the range of estimates obtained for the UK - compromising any definitive comparison between the two countries.

## VI References

- Atkinson, A.B. (1983) *The Economics of Inequality*. Clarendon Press: Oxford.
- Barten, A.P. (1964) 'Family composition, prices and expenditure patterns', in Hart, P.E., Mills, G. and Whitaker, J.K. (eds) *Econometric Analysis for National Economic Planning*. Butterworths: London.
- Blackorby, C. and Donaldson, D. (1993) 'Adult-equivalence scales and the economic implementation of interpersonal comparisons of well-being'. *Social Choice and Welfare*, 10, 335-61.
- Blundell, R. and Lewbel, A. (1991) 'The information content of equivalence scales'. *Journal of Econometrics*, 50, 49-68.
- Blundell, R., Pashardes, P. and Weber, G. (1993) 'What do we learn about consumer demand patterns from micro data'. *American Economic Review*, 83, 570-97.
- Bojer, H. and Nelson, J.A. (1999) 'Equivalence scales and the welfare of children: A comment on "Is there bias in the economic literature on equivalence scales?"'. *Review of Income and Wealth*, 45, 531-4.
- Bradbury, B. (1989) 'Family size equivalence scales and survey evaluations of income and well-being'. *Journal of Social Policy*, 18, 383-408.

- Deaton, A.S. and Muellbauer, J. (1980) *Economics and Consumer Behaviour*. Cambridge University Press: Cambridge.
- Deaton, A.S. and Muellbauer, J. (1986) 'On measuring child costs: With applications to poor countries'. *Journal of Political Economy*, 94, 720-44.
- Dickens, R., Fry, V. and Pashardes, P. (1993) 'Nonlinearities and equivalence scales'. *Economic Journal*, 103, 359-68.
- Engel, E. (1875) 'Die productions und consumtionsverhaltnisse des konigsreichs sachsen'. *Zeitschrift des Statistischen Bureaus des Koniglich Sachischen Ministeriums des Innern*, 3
- Engel, E. (1895) 'Die Lebenskosten belgischer arbeiter-familien fruher und jetzt'. *International Statistics Institute Bulletin*, 9, 1-74.
- Ferreira, M.L., Buse, R.C. and Chavas, J.P. (1998) 'Is there bias in computing household equivalence scales?'. *Review of Income and Wealth*, 44, 183-98.
- Gorman, W.M. (1976) 'Tricks with utility functions', in Artis, J.H. and Nobay, A.R. (eds) *Essays in Economic Analysis*. Cambridge University Press: Cambridge.
- Gronau, R. (1988) 'Consumption technology and the intrafamily distribution of resources: Adult equivalence scales reexamined'. *Journal of Political Economy*, 96, 1183-205.
- Lancaster, G. and Ray, R. (1998) 'Comparison of alternative models of household equivalence scales: The Australian evidence on unit record data'. *Economic Record*, 74, 1-14.
- Lancaster, G. and Ray, R. (2002) 'International poverty comparisons on unit record data of developing and developed countries'. *Australian Economic Papers*, 41, 129-39.
- Leser, C.E.V. (1963) 'Forms of Engel functions'. *Econometrica*, 31, 694-703.

- Lewbel, A. (1989) 'Household equivalence scales and welfare comparisons'. *Journal of Public Economics*, 39, 377-91.
- Lewbel, A. (1990) 'Full rank demand systems'. *International Economic Review*, 31, 289-300.
- Muellbauer, J. (1974) 'Household composition, Engel curves and welfare comparisons between households'. *European Economic Review*, 5, 103-22.
- Muellbauer, J. (1975) 'The cost of living and taste and quality change'. *Journal of Economic Theory*, 10, 269-83.
- Muellbauer, J. (1977) 'Testing the Barten model of household composition effects and the cost of children'. *Economic Journal*, 87, 450-87.
- Nelson, J.A. (1993) 'Household equivalence scales: Theory versus policy?'. *Journal of Labor Economics*, 11, 471-93.
- Nicholson, J.L. (1976) 'Appraisal of different methods of estimating equivalence scales and their results'. *Review of Income and Wealth*, 22, 1-11.
- Pashardes, P. (1991) 'Contemporaneous and intertemporal child costs: Equivalent expenditure vs. equivalent income scales'. *Journal of Public Economics*, 45, 191-213.
- Pendakur, K. (1999) 'Semiparametric estimates and tests of base-independent equivalence scales'. *Journal of Econometrics*, 88, 1-40.
- Phipps, S. and Burton, P. (1995) 'Social / institutional variables and behaviour within households: An empirical test using the Luxembourg income study'. *Feminist Economics*, 1, 151-74.
- Pollak, R.A. and Wales, T.J. (1979) 'Welfare comparisons and equivalence scales'. *American Economic Review*, 69, 216-21.
- Prais, S.J. and Houthakker, H.S. (1955) *The Analysis of Family Budgets*. Cambridge University Press: Cambridge.

- Rothbarth, E. (1943) 'Note on a method of determining equivalent income for families of different composition', in Madge, C. (eds) *War-time Pattern of Saving and Spending*. Cambridge University Press: Cambridge.
- Sen, A.K. (1973) 'Behaviour and the concept of preference'. *Economica*, 40, 241-59.
- Sen, A.K. (1987) *The Standard of Living*. Cambridge University Press: Cambridge.
- Sydenstricker, E. and King, W.I. (1921) 'The measurement of the relative economic status of families'. *Quarterly Publications of the American Statistical Association*, 17, 842-57.
- Tsakloglou, P. (1991) 'Estimation and comparison of two simple models of equivalence scales for the cost of children'. *Economic Journal*, 101, 343-57.
- Working, H. (1943) 'Statistical laws of family expenditure'. *Journal of the American Statistical Association*, 38, 43-56.