Limiting mortgage debt: Aggregate demand externalities and housing market distortions
PRELIMINARY DRAFT
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Abstract

Most households must borrow to buy a home. Homeownership provides security of tenure, and it is the only way to insure against rent increases when housing derivatives markets are incomplete. However, highly indebted households may impose aggregate demand externalities when there are nominal rigidities and monetary policy is constrained. Optimal macroprudential limits on mortgage borrowing would trade off the costs of distorting the housing market against the benefits of reducing aggregate demand externalities. In a model calibrated to match features of UK data, we find that debt limits affect interest rates, house prices and rents. Depending on the size and incidence of these general equilibrium effects, macroprudential policy can have different distributional consequences.

1 Introduction

This paper presents a model in which a desire to own housing rather than rent it motivates poor households, who inherit little or no housing, to borrow from rich ones, who inherit a large amount. When borrowing and lending, households do not internalize the fact that the accumulation of debt may reduce future aggregate output. We show that, aside from mitigating this externality, a macroprudential debt limit can have distributional consquences through its general equilibrium effects on interest rates, house prices and rents. Rich and poor households' preferred settings of macroprudential policy depend on the relative strength of these effects, which in turn depends on the model parameters. We illustrate this point with alternative calibrations based on UK data.

*The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England or its committees.

Mortgage debt played a prominent role in the 2007–2008 financial crisis and the Great Recession. US subprime mortgage losses contributed to the downfall of many financial institutions. Although press and policymaker attention focused mainly on these financial failures at the start of the crisis, there is now a widespread view that household indebtedness played an independent role in triggering and exacerbating the recession (Zabai, 2017; Mian and Sufi, 2018; Aikman et al., 2018). Countries and regions that saw a greater build-up of household debt prior to the crisis tended to suffer the greatest falls in output and employment during the recession.

The build-up of household debt has been associated with financial crises and recessions (Schularick and Taylor, 2012; Zabai, 2017). Jordà, Schularick and Taylor (2016) find that recessions following mortgage lending booms are particularly severe. Mian, Sufi and Verner (2017) and Alter, Feng and Valckx (2018) find that growth in household debt relative to GDP ratio is associated in the medium run with lower GDP growth and higher unemployment. US postal code areas where households suffered greater leveraged housing losses saw greater declines in economic activity (Mian, Rao and Sufi, 2013). Central banks now monitor the proportion of highly indebted households as a potential source of economic and financial instability (Cateau, Roberts and Zhou, 2015; Reserve Bank of Australia, 2017; Bank of Canada, 2017; Bank of England, 2017).

An important reason why household debt could exacerbate recessions is that highly indebted households tend to deleverage sharply in response to shocks, either for precautionary reasons or because they are forced to by a credit crunch. If other households could be induced to borrow more, deleveraging by one group of households should not lead to a reduction in aggregate demand. However, in the presence of nominal rigidities and an effective lower bound on nominal interest rates, monetary policy may be unable to achieve the reduction in the real interest rate that would be necessary to keep output at potential (Eggertsson and Krugman, 2012; Guerrieri and Lorenzoni, 2017).

1.1 Aggregate demand externalities

A recent strand of literature has explored the impact of macroprudential policies to correct aggregate demand externalities originating from the accumulation of high levels of household debt. By and large, this literature models the reason for borrowing as a difference in impatience parameters across households. While this is a convenient modelling expedient, we believe that there are other intrinsic reasons why households choose to borrow. In particular, we introduce property ownership as a motive for borrowing.

Korinek and Simsek (2016) provide a model of aggregate demand externalities from household debt. When choosing how much to borrow, households know that they may find themselves credit constrained in future, unlike in related papers by Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017). Although households take this risk of being constrained into account, they do not internalize the fact that their deleveraging will reduce output if there are nominal rigidities and monetary policy is constrained by the zero lower bound. Macroprudential limits on debt can therefore be efficient.

Farhi and Werning (2016) propose a very general setting in which debt restrictions help improve on the unregulated equilibrium. In one of many applications, they explore the impact of household deleveraging that brings the economy to the zero lower bound on nominal interest rates. They introduce housing services in the utility function and analyse the interaction between aggregate demand and pecuniary externalities by introducing a collateral constraint that depends on the price of a house. However, their analysis features no meaningful distinction between renting and owning: the price of housing in their model is in fact analogous to what would be the rental price of housing services in our analysis. Furthermore, households' decision to borrow remains linked to a difference in discount factors.

1.2 Housing tenure choice

A key feature of our model is that renting is perceived as a worse option than owning a house. Various arguments can be made to underpin this assumption. As pointed out in Sinai and Souleles (2005), owning a house insulates households against the risk of rent increases (at the cost of exposing them to house-price fluctuations). In the absence of markets for derivatives on local housing costs, taking on mortgage debt is the main way for households to insure themselves against this risk. Additionally, unlike homeowners, renters face the prospect of being required to leave at the end of their contracts, incurring considerable moving and search costs. Rental contracts typically place tight restrictions on modifications to the property. Renting also requires costly inspections to overcome moral hazard problems.

Iacoviello and Pavan (2013) propose a life-cycle model to analyse the mortgage debt and housing markets. Housing is lumpy in their setting and housing adjustment occurs infrequently, while house prices are not modelled. Similarly to our model, they assume households have a preference for owning over renting. However, they incorporate this feature in the utility function by assuming a larger utility coefficient in case of ownership, whereas we introduce a quadratic cost that is incurred by both landlord and tenant whenever housing is rented out.

Another life-cycle model of the housing market is developed by Kiyotaki, Michaelides and Nikolov (2011). They consider uninsurable idiosyncratic income shocks, focusing on the distributional impact of changes in fundamentals. However, they do not consider debt in their model, as housing is purely financed by equity. A preference for owning over renting is modelled in the form of a lower parameter in the utility function. They also show how this translates into a higher rental rate of housing services to be paid by the renter. This suggests that a version of the model we consider in which the renting friction is only paid in equilibrium by net tenants and not by net landlords would be equivalent to a setting in which a difference in preferences for owning versus renting is assumed.

1.3 Mortgages

Kaplan, Mitman and Violante (2017) propose a very rich model of the housing market to analyse the boom and bust episode during the Great Recession. They allow for mortgage default and foreclosure. Interestingly, they find that credit conditions do not play a role in house price dynamics, while they have an impact on home ownership. The rental market is featured as an alternative to buying, featuring both advantages as well as disadvantages. House purchases imply a minimum down-payment and a transaction cost in case of sale. On the other hand, it has a tax advantage insofar as mortgage interests are tax deductible. Furthermore, owning a house gives access to additional opportunities for borrowing in the form of Home Equity Line of Credit (HELOC) as the housing stock can be used as collateral.

2 Two-period, two-agent model with housing and debt

The economy lasts for two periods: $t \in \{1, 2\}$. All households are born at the beginning of the first period and die at the end of the second. In each period there are two goods: a non-durable consumption good produced by households, who all supply one unit of labour inelastically in each period, and durable housing, which is in fixed supply. Consumption goods and housing are both infinitely divisible. There are two types of households: rich and poor, who differ only in their initial endowment of housing: $h_0^R > h_0^P$. In each period there are spot markets for non-durable consumption goods c_t and housing occupancy s_t. In the first period only, there is a market for housing ownership h_1 and a market in riskless one-period debt d_2, where $d_2 > 0$ denotes net borrowing.[1] Housing transactions take place at the start of the period, so it is the purchaser of a unit of housing who has the right to live in it or rent it out.

2.1 Preferences and budget constraints

A household's lifetime utility is

$$u(c_1) + \eta v(s_1) + \beta(u(c_2) + \eta v(s_2)),$$

where $\eta > 0$ captures the intensity of the preference for housing and $\beta > 0$ is the intertemporal discount factor. The utility functions $u(c_t)$ and $v(s_t)$ have the usual properties: they are strictly increasing, concave, and continuously differentiable. For simplicity we focus on the special case with $u(\cdot) = v(\cdot) = \log(\cdot)$.

The period 1 and period 2 budget constraints are

$$c_1 + P_1(h_1 - h_0) + \rho_1(s_1 - h_1) + \mu m(s_1 - h_1) \leq y_1 + \frac{d_2}{1 + r_1} \text{ and}$$
$$c_2 + P_2(h_2 - h_1) + \rho_2(s_2 - h_2) + \mu m(s_2 - h_2) \leq y_2 - d_2,$$

where P_t and ρ_t are the purchase price of housing and the housing rental rate in period t.

2.2 Deadweight cost of rental housing

The function $m(s_t - h_t)$ denotes a deadweight cost of rental housing. This cost is incurred by both tenants and landlords, so $m(s_t - h_t) > 0$ whenever $s_t \neq h_t$. It is increasing in the amount of housing rented or rented out, so $m'(s_t - h_t) > 0$ for net tenants, for whom $s_t > h_t$, and $m'(s_t - h_t) < 0$ for net landlords, for

[1] As the second period is the final period, there is no scope for any new borrowing to be repaid, and no meaningful distinction between housing occupancy and ownership.

whom $s_t < h_t$. The marginal deadweight cost of rental housing is increasing, so $m''(s_t - h_t) > 0$. Owner-occupancy imposes no deadweight costs, so $m(0) = 0$. The parameter $\mu > 0$ captures the intensity of the rental market friction.

Each household supplies a fixed amount of labour and can produce y_t units of the non-durable consumption good in period t. For concreteness, we can interpret the deadweight cost of rental housing as landlords and tenants having to divert some of their effort towards monitoring and maintenance activities, or having to employ property management agents to do so at the prevailing wage. The assumptions on the curvature of the cost function $m(s_t - h_t)$ imply that these activities are subject to increasing marginal costs.

2.3 Debt and the aggregate demand externality

Households can borrow or lend between the two periods at the risk-free interest rate r_1, subject to the borrowing constraint

$$d_2 \leq \bar{d}_2.$$

The debt limit $\bar{d}_2$ is an exogenous parameter of the model, which we think of as being set in advance by a macroprudential authority. The focus of this paper is on comparative statics exercises in which we solve the model for different values of the debt limit and examine the implications for equilibrium outcomes and the welfare of the two types of households.

In the second period only, output can be affected by a reduced-form aggregate demand externality from debt as in Mian and Sufi (2017):

$$y_2 = \begin{cases} \bar{y}_2 & \text{if } D_2 \leq \bar{D} \\ \bar{y}_2 - f\left(\frac{D_2}{\bar{D}}, \phi\right) & \text{otherwise,} \end{cases}$$

where $\bar{y}_2$ is production capacity, D_2 is aggregate gross debt, $\bar{D}$ is the threshold debt level above which output is constrained, and ϕ captures the severity of macroeconomic frictions. The penalty function $f(\cdot)$ has the properties $f(1, \cdot) = 0$, $f_1 > 0$ and $f_{1,2} > 0$. As Mian and Sufi (2017) point out, this functional form can be motivated by a model with nominal rigidities and an effective lower bound on the nominal interest rate, as in Farhi and Werning (2016) and Korinek and Simsek (2016).

2.4 Market clearing and equilibrium

Markets are competitive in the sense that households take prices and aggregate quantities as given. Using the superscripts R and P to distinguish between rich and poor households, normalizing the total number of households to one, and using π to denote the fraction of poor households, the market-clearing conditions for goods, housing ownership and housing occupancy are:

$$\begin{aligned} (1-\pi)\left(c_t^R + \mu m(s_t^R - h_t^R)\right) + \pi\left(c_t^P + \mu(s_t^P - h_t^P)\right) &= y_t, \\ (1-\pi)h_t^R + \pi h_t^P &= H, \\ (1-\pi)s_t^R + \pi s_t^P &= (1-\pi)h_t^R + \pi h_t^P, \qquad t \in \{1, 2\} \end{aligned}$$

An equilibrium in this economy is a set of prices and a set of choices for each type of household such that all choices are individually optimal and all markets clear. As there are no shocks, the rational expectations equilibrium will feature perfect foresight.

3 Household optimization

Households of type $i \in \{P, R\}$ solve the following problem:

$$\max_{c_1^i, c_2^i, s_1^i, s_2^i, d_2^i, h_1^i, h_2^i} \quad u(c_1^i) + \eta v(s_1^i) + \beta \left((u(c_2^i) + \eta v(s_2^i) \right)$$

subject to the budget constraints

$$c_1^i + P_1(h_1^i - h_0^i) + \rho_1(s_1^i - h_1^i) + \mu m(s_1^i - h_1^i) \leq y_1 + \frac{d_2^i}{1 + r_1}, \tag{1}$$

$$c_2^i + P_2(h_2^i - h_1^i) + \rho_2(s_2^i - h_2^i) + \mu m(s_2^i - h_2^i) \leq y_2 - d_2, \tag{2}$$

and the borrowing constraint

$$d_2^i \leq \bar{d}_2.$$

Letting λ_t^i denote the Lagrange multiplier on the period t budget constraint for household type i, and letting $\tilde{\lambda}_2^i$ denote the Lagrange multiplier on the borrowing constraint for household type i, the Lagrangian becomes:

$$\begin{gathered} u(c_1^i) + \eta v(s_1^i) + \beta \left(u(c_2^i) + \eta v(s_2^i) \right) \\ + \lambda_1^i \left(y_1 + \frac{d_2^i}{1 + r_1} - c_1^i - P_1(h_1^i - h_0^i) - \rho_1(s_1^i - h_1^i) - \mu m(s_1^i - h_1^i) \right) \\ + \lambda_2^i \left(y_2 - d_2 - c_2^i - P_2(h_2^i - h_1^i) - \rho_2(s_2^i - h_2^i) - \mu m(s_2^i - h_2^i) \right) \\ + \tilde{\lambda}_2^i \left(\bar{d}_2 - d_2^i \right). \end{gathered}$$

Differentiating with respect to the household's choice variables, we have

$$c_1^i: \qquad u'(c_1^i) = \lambda_1^i \tag{3}$$

$$c_2^i: \qquad \beta u'(c_2^i) = \lambda_2^i \tag{4}$$

$$s_1^i: \qquad \frac{\eta v'(s_1^i)}{\rho_1 + \mu m'(s_1^i - h_1^i)} = \lambda_1^i \tag{5}$$

$$s_2^i: \qquad \frac{\beta \eta v'(s_2^i)}{\rho_2 + \mu m'(s_2^i - h_2^i)} = \lambda_2^i \tag{6}$$

$$d_2^i: \qquad (1 + r_1) \left(1 + \frac{\tilde{\lambda}_2^i}{\lambda_2^i} \right) = \frac{\lambda_1^i}{\lambda_2^i} \tag{7}$$

$$h_1^i: \qquad \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)} = \frac{\lambda_1^i}{\lambda_2^i} \tag{8}$$

$$h_2^i: \qquad \mu m'(s_2^i - h_2^i) = \rho_2 - P_2 \tag{9}$$

Combining (3) with (5) and (4) with (6) yields an intratemporal optimality condition for each period:

$$\frac{\eta v'(s_1^i)}{u'(c_1^i)} = \rho_1 + \mu m'(s_1^i - h_1^i), \tag{10}$$

$$\frac{\eta v'(s_2^i)}{u'(c_2^i)} = \rho_2 + \mu m'(s_2^i - h_2^i). \tag{11}$$

Within each period the ratio of a household's marginal utility of housing occupancy to the marginal utility of its non-durables consumption must equal the relative price it faces between these two goods. The price of non-durables is normalized to one, so for a given level of housing ownership the relative price of housing occupancy is given by the housing rental rate ρ_t plus the marginal rental friction associated with an increase in net renting. Note that for landlords, with $s_t^i < h_t^i$, this marginal friction is negative because occupying more of the housing they own and renting less of it out reduces the deadweight rental cost they must pay.

Combining first-order conditions (3), (4), and (8) gives us an intertemporal optimality condition:

$$\frac{u'(c_1^i)}{\beta u'(c_2^i)} = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)}. \tag{12}$$

The ratio of the discounted marginal utilities of a household's consumption across the two periods must equal the effective rate of return it faces on housing ownership. The rate of return on owning a unit of housing is its future sale price P_2 divided by the net resources that must be forgone today to aquire it. As well as the purchase price P_1, we must take into account the rental rate ρ_1 that a landlord earns or a tenant saves, and the marginal deadweight rental cost of their housing purchase. For a given level of housing occupancy, purchasing an additional unit of housing worsens the rental friction for a landlord but alleviates it for a tenant.

By combining first-order conditions (7) and (8), we obtain an equation relating the effective rates of return on bonds and housing :

$$(1 + r_1)\left(1 + \frac{\tilde{\lambda}_2^i}{\lambda_2^i}\right) = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)}. \tag{13}$$

Households for whom the borrowing constraint is binding, and for whom the Lagrange multiplier on the borrowing constraint, $\tilde{\lambda}_2^i$, is therefore positive, will face a higher effective rate of return on housing than on bonds. For households who are unconstrained by the debt limit and for whom the Lagrange multiplier on the borrowing constraint is therefore zero, condition (13) simplifies to the following no-arbitrage condition:

$$1 + r_1 = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^i - h_1^i)}. \tag{14}$$

Households who can borrow or save as much as they like must be indifferent between using bonds and housing as financial assets to transfer resources between the two periods.

4 Analytical results

Using the household optimality conditions and market-clearing conditions above, we can show the following about the equilibrium of our model.

First, there will be no renting in the second period. Intuitively, since the second period is the final period, housing ceases to be a financial asset and so the distinction between owning and renting breaks down. Due to the deadweight cost of renting, the rental rate would need to be lower than the purchase price of housing in order to induce households to become tenants. Since landlords must also pay the deadweight cost, households would need to earn a rental rate in excess of the purchase price of housing in order to induce them to become landlords. These two requirements are contradictory, so the only equilibrium in the second period is one in which all housing is owner-occupied and the purchase price of housing is equal to its rental rate.

Proposition 1. *In the second period the purchase price of housing is equal to its rental rate and all housing is owner-occupied:* $P_2 = \rho_2$, $s_2^R = h_2^R$ *and* $s_2^P = h_2^P$.

Proof. The first-order condition (9) must be satisfied for both rich and poor households. All households face the same rental rate ρ_2 and the same purchase price of housing P_2, so the right-hand side of this equation is identical for all households. For equation (9) to be satisfied for all households, the left-hand side must also be the same for all households. The second derivative of the deadwdeight rental cost function is positive for all levels of net renting, so in order for the first derivative $m'(s_2^i - h_2^i)$ to be equal across households we must have $s_2^R - h_2^R = s_2^P - h_2^P$. For the housing ownership and occupancy markets to clear in the second period, we must have $(1-\pi)(s_2^R - h_2^R) + \pi(s_2^P - h_2^P) = 0$. For these two conditions to both be satisfied we must have $s_2^R = h_2^R$ and $s_2^P = h_2^P$. This in turn implies that the left-hand side of the first-order condition (9) is equal to zero, which means we must have $P_2 = \rho_2$ in order for the right-hand side to equal zero. □

Second, if neither rich nor poor households are constrained by the debt limit, then there will be no renting in the first period, either. Similar to the previous result, the intuition is that the deadweight cost of renting drives a wedge between the rates of return on housing for landlords and tenants. If households face no constraints on their asset positions, they must all be indifferent between holding bonds and housing in equilibrium. They all face the same rate of return on bonds, but due to the rental friction they will only face the same effective return on housing if their net rental positions are the same. This can only be the case when all housing is owner-occupied.

Proposition 2. *If the borrowing constraint is slack for both rich and poor households,* $\tilde{\lambda}_2^R = \tilde{\lambda}_2^P = 0$*, then in the first period all housing is owner-occupied:* $s_1^R = h_1^R$ *and* $s_1^P = h_1^P$.

Proof. If the borrowing constraint is slack for both rich and poor, then the no-arbitrage condition (14) must be satisfied for both types of households. All households face the same interest rate r_1, house prices P_1 and P_2 and rental rate ρ_1. For equation (14) to be satisfied for both types of households, the marginal deadweight rental friction $m'(s_1^i - h_1^i)$ must therefore be equal across households. The second derivative of the deadweight rental cost function is positive for all levels of net renting, so equality of marginal frictions can only be achieved if net renting is equal across households: $s_1^R - h_1^R = s_1^P - h_1^P$. Market clearing in the housing ownership and occupancy markets requires $(1-\pi)(s_1^R - h_1^R) + \pi(s_1^P - h_1^P) = 0$. For these two conditions to both be satisfied we must have $s_1^R = h_1^R$ and $s_1^P = h_1^P$. □

We have shown that in order for the rental market to be active, one of the household types must find itself constrained by the debt limit. If this is the case, we can also show that the constrained households will be the tenants and the unconstrained households will be the landlords. Intuitively, households constrained by the debt limit are unable to purchase as much housing as they would like to.

Proposition 3. *Suppose that the debt limit binds for one of the household types but not the other:* $d_2^U < d_2^C = \bar{d}_2$*. Then in the first period the constrained household type are tenants and the unconstrained type are landlords:*

$$s_1^C - h_1^C > 0 > s_1^U - h_1^U.$$

Proof. Let the superscripts C and U denote the constrained and unconstrained types of households. By assumption the borrowing constraint is binding for the constrained and slack for the unconstrained: $\tilde{\lambda}_2^C > \tilde{\lambda}_2^U = 0$. Non-satiation ensures $\lambda_2^C > 0$. From the household optimality condition (13) we have

$$\frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^C - h_1^C)} = (1{+}r_1)\left(1 + \frac{\tilde{\lambda}_2^C}{\lambda_2^C}\right) > 1{+}r_1 = \frac{P_2}{P_1 - \rho_1 - \mu m'(s_1^U - h_1^U)}.$$

Rearranging and simplifying yields $m'(s_1^C - h_1^C) > m'(s_1^U - h_1^U)$, and because $m''(s_t^i - h_t^i) > 0$ we must have $s_1^C - h_1^C > s_1^U - h_1^U$. Letting $\kappa > 0$ denote the share of the constrained type of household in the population, market clearing requires that

$$\kappa(s_1^C - h_1^C) = -(1-\kappa)(s_1^U - h_1^U).$$

Combining these gives us

$$s_1^C - h_1^C = -\frac{1-\kappa}{\kappa}(s_1^U - h_1^U) > 0 > s_1^U - h_1^U.$$

□

Proposition 4. *Suppose that the poor households start the period with a lower housing stock than the rich,* $h_0^P < h_0^R$*. Then the poor households will be borrowers and the rich households will be lenders.*

Proof. The allocation for consumption and housing services will be the same as in the previous section:

$$\begin{aligned}
c_1^i =& \frac{1}{(1+\beta)(1+\eta)}\left[y_1^i + \beta\frac{y_1}{y_2}y_2^i + \eta(1+\beta)\frac{h_0^i}{H}y_1\right] \\
c_2^i =& \frac{y_2}{y_1}c_1^i \\
h_t^i =& s_t^i = \frac{H}{y_t}c_t \\
d_2^i =& y_2^i - c_2^i - P_2(h_2^i - h_1^i)
\end{aligned}$$

We now show that if $y_t^R = y_t^P = y_t \ \forall t, \ \ h_0^P < h_0^R$, then $d_2^P > d_2^R$.

$$c_t^i = y_t \left(\frac{1}{1+\eta} + \frac{\eta}{1+\eta} \frac{h_0^i}{H} \right)$$
$$s_t^i = h_t^i = \frac{H + \eta h_0^i}{1+\eta}$$
$$d_2^i = \frac{\eta}{1+\eta} \left(1 - \frac{h_0^i}{H} \right) y_2$$

This shows that

$$d_2^P = \frac{\eta}{1+\eta} \left(1 - \frac{h_0^P}{H} \right) y_2 > \frac{\eta}{1+\eta} \left(1 - \frac{h_0^R}{H} \right) y_2 = d_2^R$$

as long as $h_0^P < h_0^R$. □

4.1 Closed-form solution with a non-binding debt limit and log utility

As shown in Propositions 1 and 2 above, if the borrowing constraint is slack for both rich and poor households then all housing will be owner-occupied in both periods. With $s_t^i = h_t^i$ for $t \in \{1, 2\}$ and $i \in \{R, P\}$, the household optimality conditions simplify to

$$\frac{\eta v'(h_t^i)}{u'(c_t^i)} = \rho_t, \tag{15}$$
$$\frac{u'(c_1^i)}{\beta u'(c_2^i)} = 1 + r_1 = \frac{P_2}{P_1 - \rho_1}. \tag{16}$$

With log utility, $u(\cdot) = v(\cdot) = \log(\cdot)$, these become

$$\eta c_t^i = \rho_t h_t^i, \tag{17}$$
$$\frac{c_2^i}{\beta c_1^i} = 1 + r_1 = \frac{P_2}{P_1 - \rho_1}. \tag{18}$$

By combining these with the household budget constraints and market clearing conditions, we can derive the following closed-form solution of our model when the debt limit is not binding for either household type:

$$\begin{aligned}
c_1^i &= \left(1+\eta\frac{h_0^i}{H}\right)\frac{y_1}{1+\eta},\\
c_2^i &= (1+r_1)\beta c_1^i = \left(1+\eta\frac{h_0^i}{H}\right)\frac{y_2}{1+\eta},\\
s_1^i &= s_2^i = h_1^i = \frac{\eta c_1^i}{\rho_1} = \frac{H+\eta h_0^i}{1+\eta},\\
d_2^i &= y_2 - c_2^i = \left(1-\frac{h_0^i}{H}\right)\frac{\eta y_2}{1+\eta},\\
1+r_1 &= \frac{y_2}{\beta y_1},\\
\rho_1 &= \eta\frac{y_1}{H},\\
P_2 &= \rho_2 = \eta\frac{y_2}{H},\\
P_1 &= \rho_1 + \frac{P_2}{1+r_1} = \frac{\eta}{H}\left(y_1+\frac{y_2}{1+r_1}\right) = \eta(1+\beta)\frac{y_1}{H}.
\end{aligned}$$

In line with our interpretation of the debt limit $\bar{d}_2$ as being set by a macro-prudential authority, we can interpret the equilibrium of our model with a non-binding debt limit as a laissez-faire equilibrium. When households choose how much to borrow, they take into account the impact their borrowing will have on their own future consumption. However, individual households do not internalize the fact that their contribution to aggregate debt may exacerbate the aggregate demand externality.

We derive restrictions on the values of the model parameters such that the laissez-faire equilibrium will suffer from an aggregate demand externality.

Proposition 5. *Provided the parameter restriction*

$$h_0^P \leq \left(1 - \frac{1+\eta}{\eta}\frac{\bar{D}}{\pi\bar{y}_2}\right) H$$

is satisfied, second-period output is reduced by an aggregate demand externality in the laissez-faire equilibrium with no binding debt limit.

Proof. Equilibrium conditions for aggregate debt and output are:

$$\begin{aligned}
D_2 &= \pi d_2^p\\
d_2^i &= \frac{\eta}{1+\eta}\left(1-\frac{h_0^i}{H}\right) y_2\\
y_2 &= \bar{y}_2 - \phi\left(\frac{D_2}{\bar{D}} - 1\right), \quad \text{for} \quad D_2 > \bar{D}
\end{aligned}$$

A fixed point for debt is therefore

$$D_2 = \frac{\pi\eta\bar{D}(H-h_0^P)(\bar{y}_2+\phi)}{H\bar{D}(1+\eta)+\phi\pi\eta(H-h_0^p)}$$

and

$$\frac{\pi\eta\bar{D}(H-h_0^P)(\bar{y}_2+\phi)}{H\bar{D}(1+\eta)+\phi\pi\eta(H-h_0^p)} > \bar{D}$$

when

$$h_0^P \leq \left(1 - \frac{1+\eta}{\eta}\frac{\bar{D}}{\pi\bar{y}_2}\right)H.$$

□

The condition in Proposition 5 says that the aggregate demand externality will be active in the laissez-faire equilibrium if poor households' inherited share of the aggregate stock of housing is low. The more unequal the inherited stock of housing, the more poor households will want to borrow. Similarly, the right-hand side of the condition is increasing in the housing preference parameter η because poor households' desire for borrowing is greater the stronger is their preference for housing occupancy over consumption of non-durables.

5 General equilibrium effects of the debt limit

The presence of an aggregate demand externality in the laissez-faire equilibrium of our model could motivate a macroprudential policymaker to impose a binding debt limit. However, as well as alleviating the aggregate demand externality, the debt limit has general equilibrium effects on relative prices in our model.

5.1 Rent

We have seen above that a binding debt limit means poor households purchase less housing from the rich than in the laissez-faire equilibrium, and the rental market becomes active. As we consider tighter debt limits, poor households purchase less housing and rent more of it from the rich. Rich households are unconstrained by the debt limit, and so must be indifferent between holding housing and bonds. All else equal, the rental rate in the first period must rise in order to compensate rich landlords for the increased deadweight rental cost.

5.2 Interest rate

There are two channels through which the debt limit can affect the equilibrium interest rate. Both channels operate through the market-clearing condition for bonds, which are in zero net supply.

The first is that when the debt limit tightens, the interest rate must fall so that rich households' desire to lend falls to match poor households' reduced ability to borrow. The second channel operates via the aggregate demand externality. If a tighter debt limit alleviates the aggregate demand externality, output in the second period increases relative to the first period. With income now relatively more abundant in the second period and scarce in the first period, all else equal the interest rate must rise to induce households to defer their consumption.

These two channels have opposite effects, so for debt limits that do not completely eliminate the aggregate demand externality, the net effect on the interest rate of a tighter debt limit is ambiguous. For debt limits tight enough to completely eliminate the aggregate demand externality, only the first channel will operate and so the interest rate will fall as the debt limit tightens.

5.3 House price

As with the interest rate, there are two channels with opposite effects through which the debt limit can affect the equilibrium purchase price of housing.

The first of these works directly through the tightening of poor households' affordability constraints. As they can no longer afford to purchase as much housing, all else equal the price of housing must fall to induce rich households to hold more of it. The second channel operates via the deadweight cost of rental housing. A tighter debt limit means poor households rent more housing. This means the marginal deadweight cost they incur is higher, so all else equal the marginal value of housing ownership is higher for them.

5.4 Incidence on rich and poor households

Although the signs of the impacts of a tighter debt limit on the interest rate and the house price are ambiguous, the incidence of all the general equilibrium effects on rich and poor households are clear. In equilibrium, rich households are landlords, lenders, and net sellers of housing, whereas poor households are tenants, borrowers and net buyers of housing. It is therefore in the interests of rich households for the rental rate on housing, the interest rate, and the purchase price of housing to be high, and vice versa for poor households. A tighter debt limit also affects poor households' welfare directly by constraining their choices. Together, these effects mean that the tightness of the debt limit may have distributional consequences as well as alleviating the aggregate demand externality. Since the sign of the general equilibrium effects on the interest rate and the house price are ambiguous, the net consequences of a tighter debt for the welfare of rich and poor households will depend on the calibration of the parameters.

6 Calibration

We interpret each period in the model as lasting for 25 years and calibrate the model accordingly. We solve the model in such a way as to match the following targets, which are based on UK data, as closely as possible:

- An annualized growth rate of 3%.
- An annualized interest rate of 4%.
- The value of the housing stock is 3.5 times annual GDP.
- An owner-occupancy rate of 66%.
- A share of imputed rent in GDP of 10%.
- A private debt-to-GDP ratio of 150%.

As Table 1 shows, depending on whether we calibrate the model to match these targets in the first period only, or on average across the two periods (to capture the life-cycle dimension), the calibrated values of some of the parameters differ. In particular, the latter calibration features a higher degree of inequality,

		Calibration (a)	Calibration (b)
Discount factor	β	0.77	0.78
Housing preference	η	0.1	0.11
Rental friction	μ	0.01	0.01
Poor population share	π	0.52	0.86
Total housing stock per capita	H	1	1
Housing endowment of rich	h_0^R	2.11	7.3
Housing endowment of poor	h_0^P	0	0
Output in period 1	y_1	1	1
Potential output in period 2	$\bar{y}_2$	1.67	1.67
AD externality debt threshold	$\bar{D}$	0.056	0.0947
AD externality severity	ϕ	0.09	0.043
Debt limit	$\bar{d}_2$	0.05	0.0522

Table 1: 25-year calibration. Targets met in first period (a) or on average across both periods (b)

with the same aggregate housing endowment concentrated in a smaller share of the population.

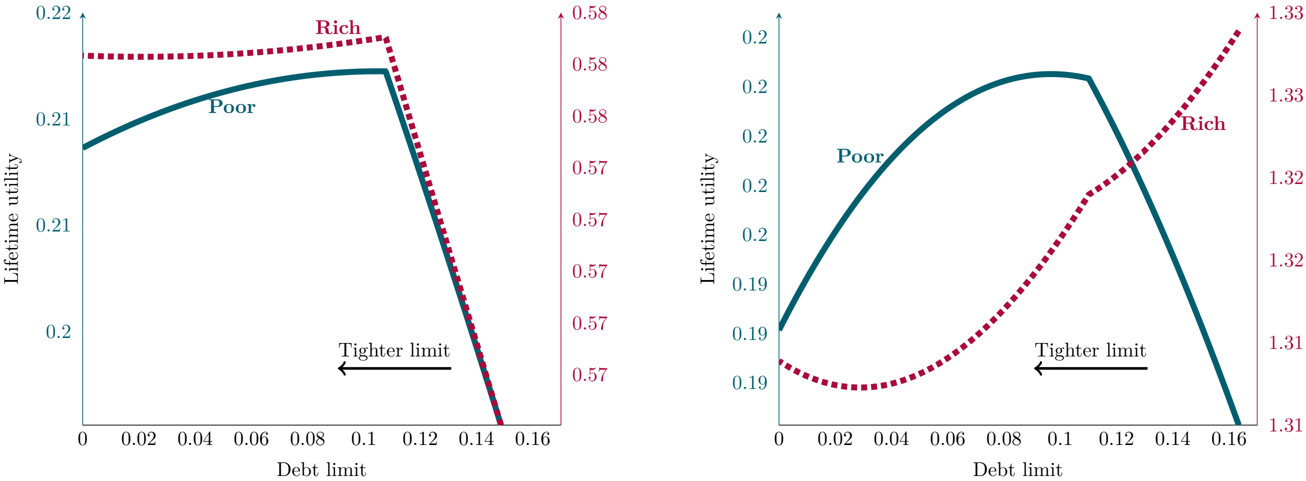
As discussed above, the calibrated values of the parameters affect the relative strength of the general equilibrium effects of tighter debt limits. We find in both calibrations that the interest rate is lower for tighter debt limits, which implies that the first channel identified above dominates. We also find in both calibrations that the affordability effect on the house price dominates, so that the equilibrium house price is lower for tighter debt limits. This latter finding is in line with those of Gete and Reher (2018), who find that a reduction in mortgage availability since the Great Recession has led to higher rents.

These general equilibrium effects in turn determine the distributional impact of macroprudential policy. As shown in Figure 1, under calibration (a) the interests of rich and poor households are aligned. The welfare of both types of household is maximized around a debt limit just tight enough to eliminate the aggregate demand externality, but no tighter.

By contrast, under calibration (b) the interests of rich and poor households diverge. Poor households again prefer that the macroprudential authority sets a debt limit in the region of the point where the aggregate demand externality is eliminated. Rich households, however, prefer the laissez-faire equilibrium. Having observed that house prices and the interest rate both increase with less restrictive debt limits under this calibration, we can infer that this latter effect is what drives rich households to prefer the laissez-faire equilibrium. This is because all the other effects of a looser debt limit, namely a worse aggregate demand externality, lower rents, and a lower house price, are detrimental to rich households' welfare.

7 Conclusion

In this paper we have solved a simple two-period, two-agent model with housing and debt, and used it to study the impact of macroprudential debt limits motivated by an aggregate demand externality. We have shown that in our model



(a) Calibration with $\pi = 0.52$.

(b) Calibration with $\pi = 0.86$.

Figure 1: Distributional impact of macroprudential policy depends on strength and incidence of general equilibrium effects.

such limits can have subtle general equilibrium effects on prices, which have distributional effects. In future, we aim to extend our model by introducing nominal rigidities and an effective lower bound on the nominal interest rate.

References

Aikman, David, Jonathan Bridges, Anil Kashyap and Caspar Siegert (2018), "Would macroprudential regulation have prevented the last crisis?", Tech. Rep. 747, Bank of England, https://ideas.repec.org/p/boe/boeewp/0747.html.

Alter, Adrian, Alan Xiaochen Feng and Nico Valckx (2018), "Understanding the Macro-Financial Effects of Household Debt: A Global Perspective", *IMF Working Papers*, 18(76): 1, DOI:10.5089/9781484349861.001.

Bank of Canada (2017), "Financial System Review - June 2017", Tech. rep., https://www.bankofcanada.ca/2017/06/fsr-june-2017/.

Bank of England (2017), "The FPC's approach to addressing risks from the UK mortgage market", Financial Stability Report 41, Bank of England.

Cateau, Gino, Tom Roberts and Jie Zhou (2015), "Indebted Households and Potential Vulnerabilities for the Canadian Financial System: A Microdata Analysis", p. 10, https://www.bankofcanada.ca/wp-content/uploads/2015/12/fsr-december2015-cateau.pdf.

Eggertsson, Gauti B. and Paul Krugman (2012), "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach", *The Quarterly Journal of Economics*, 127(3): 1469–1513, DOI:10.1093/qje/qjs023.

Farhi, Emmanuel and Iván Werning (2016), "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities", *Econometrica*, 84(5): 1645–1704, DOI:10.3982/ECTA11883.

Gete, Pedro and Michael Reher (2018), "Mortgage Supply and Housing Rents", *The Review of Financial Studies*, 31(12): 4884–4911, DOI:10.1093/rfs/hhx145.

Guerrieri, Veronica and Guido Lorenzoni (2017), "Credit Crises, Precautionary Savings, and the Liquidity Trap", *The Quarterly Journal of Economics*, 132(3): 1427–1467, DOI:10.1093/qje/qjx005.

Iacoviello, Matteo and Marina Pavan (2013), "Housing and debt over the life cycle and over the business cycle", *Journal of Monetary Economics*, 60(2): 221–238, DOI:10.1016/j.jmoneco.2012.10.020.

Jordà, Òscar, Moritz Schularick and Alan M. Taylor (2016), "The great mortgaging: Housing finance, crises and business cycles", *Economic Policy*, 31(85): 107–152, DOI:10.1093/epolic/eiv017.

Kaplan, Greg, Kurt Mitman and Giovanni L. Violante (2017), "The Housing Boom and Bust: Model Meets Evidence", Working Paper 23694, National Bureau of Economic Research, DOI:10.3386/w23694.

Kiyotaki, Nobuhiro, Alexander Michaelides and Kalin Nikolov (2011), "Winners and Losers in Housing Markets", *Journal of Money, Credit and Banking*, 43(2-3): 255–296, DOI:10.1111/j.1538-4616.2011.00374.x.

Korinek, Anton and Alp Simsek (2016), "Liquidity Trap and Excessive Leverage", *American Economic Review*, 106(3): 699–738, DOI:10.1257/aer.20140289.

Mian, Atif, Kamalesh Rao and Amir Sufi (2013), "Household Balance Sheets, Consumption, and the Economic Slump", *The Quarterly Journal of Economics*, 128(4): 1687–1726, DOI:10.1093/qje/qjt020.

Mian, Atif and Amir Sufi (2017), "Finance and Business Cycles: The Role of Credit Supply Expansion and Household Demand", https://initiative.princeton.edu/sites/princetoninitiative/files/09a_mian_empirical_credit_amplification.pdf.

Mian, Atif and Amir Sufi (2018), "Finance and Business Cycles: The Credit-Driven Household Demand Channel", *Journal of Economic Perspectives*, 32(3): 31–58, DOI:10.1257/jep.32.3.31.

Mian, Atif, Amir Sufi and Emil Verner (2017), "Household Debt and Business Cycles Worldwide", *The Quarterly Journal of Economics*, 132(4): 1755–1817, DOI:10.1093/qje/qjx017.

Reserve Bank of Australia (2017), "Characteristics of Highly Indebted Households", Financial Stability Review, https://www.rba.gov.au/publications/fsr/2017/apr/box-c.html.

Schularick, Moritz and Alan M. Taylor (2012), "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008", *American Economic Review*, 102(2): 1029–1061, DOI:10.1257/aer.102.2.1029.

Sinai, Todd and Nicholas S. Souleles (2005), "Owner-Occupied Housing as a Hedge Against Rent Risk", *The Quarterly Journal of Economics*, 120(2): 763–789, DOI:10.1093/qje/120.2.763.

Zabai, Anna (2017), "Household debt: Recent developments and challenges", https://www.bis.org/publ/qtrpdf/r_qt1712f.htm.