

# Macroprudential Interventions in Liquidity Traps

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## Abstract

We characterize the joint optimal implementation of macroprudential and monetary policies in a New Keynesian model where endogenous supply-side financial frictions generate inflationary credit spreads. State-contingent macroprudential interventions help to stabilize volatile spreads, and substantially alter the transmission of optimal monetary policy under both discretion and commitment. In ‘normal times’, macroprudential policies replicate the first-best allocation. In liquidity traps, financial interventions remove the zero lower bound restriction on the nominal policy rate, thus minimizing output costs following both deflationary (inflationary) demand (financial) shocks. Discretionary and commitment policies with macroprudential taxes deliver equivalent welfare gains.

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# 1 Introduction

The zero lower bound (ZLB) on nominal interest rates has severely impeded the effectiveness of monetary policy during the recent liquidity trap episodes that have lingered since the Great Recession. Such challenges to traditional interest rate policy have called for the implementation of supplementary unconventional monetary and fiscal policies aimed at minimizing the social costs of macroeconomic and price fluctuations. Most of the literature has so far focused on increases in government spending (Eggertsson (2011) and Christiano, Eichenbaum and Rebelo (2011)), flexible adjustments in consumption and/or labour taxes (Correia, Farhi, Nicolini and Teles (2013) and Mertens and Ravn (2014)), and unconventional monetary policies involving credit easing and direct central bank lending (Cúrdia and Woodford (2011) and Gertler and Karadi (2011)). Less attention has been given to the normative implications of corrective financial policies, also referred to as macroprudential interventions, especially in *state-contingent* liquidity traps. Our paper fills this gap by developing a simple theoretical New Keynesian model that examines the stabilization roles of state-dependant macroprudential interventions - taking the form of private asset (deposit) taxation - in response to *both* demand-driven and supply-side-driven liquidity traps.

We characterize the optimal monetary-macroprudential policy mix under both discretion and commitment in a stylized textbook New Keynesian model à la Galí (2015). The basic framework is modified for: *i*) an inflationary credit spread arising from an endogenous supply-side collateral constraint and firm default risk; *ii*) financial (macroprudential) taxes; and *iii*) occasionally binding lower bound restrictions on the effective nominal interest rates faced by the economic agents. Our paper sheds new positive and normative insights to the ongoing debate around the role of unconventional financial policies, as well as to the benefits of macroprudential and monetary policy coordination. We argue that financial taxes should be activated in a state-dependent fashion based on the nature of the shock distorting the economy. Access to financial taxation substantially alters the transmission of optimal monetary policy under both discretion and commitment, and significantly alleviates the severity of liquidity trap episodes. Even in more ‘normal times’, when monetary policy is not constrained by the ZLB, variations in the macroprudential tax can replicate the first-best allocation by solving the policy trade-offs emerging from the existence of the cost-push financial frictions. Further, relative to the restricted regime involving only monetary policy, unconstrained optimal time-consistent (discretionary) and Ramsey (commitment) policies with macroprudential interventions produce identical welfare gains in response to both deflationary demand shocks and inflationary financial shocks. Thus, commitment policies are of secondary importance so long as the policy maker can optimally alter the macroprudential tax on loanable funds - deposits.

In our simple framework, and in line with Christiano and Eichenbaum (1992) and Ravenna and Walsh (2006), firms have to borrow in advance to finance their working-capital needs. Such a borrowing constraint gives rise to an inflationary cost channel effect due to the tight connection between borrowing costs and marginal production costs. This modeling approach is motivated by Christiano, Eichenbaum and Trabandt (2015), who show that the working-capital cost channel is important in explaining business cycle fluctuations, as well as the ‘missing deflation’ phenomenon observed during the Great Recession. Moreover, Gilchrist, Schoenle, Sim and Zakrajšek (2017) have shown that firms with limited internal liquidity and high leverage significantly increased prices in response to the 2008 financial market crash that corresponded with a steep output contraction and extremely high credit spreads. The output-inflation trade-off inflicted by the cost-push credit frictions and the financial shock is at the heart of our model. We demonstrate that optimal

macroprudential interventions act as natural remedies against these financial imperfections and the liquidity traps that occasionally follow.

Compared to the benchmark Ravenna and Walsh (2006) frictionless monetary policy cost channel model, in our setup the loan rate relates to both the nominal policy (deposit) rate, and to an endogenous finance premium that arises due to the possibility of firm default. Credit default risk, the finance premium and the ensuing risk-adjusted lending rate are positively related to the inflationary marginal production costs that, in turn, are proxied by leverage. In the absence of shocks, higher aggregate demand raises the demand for external working-capital finance used to support production. With output pinning down the level of collateral, higher leverage (as measured by the total cost of debt to GDP) results in greater risk and inflated borrowing costs. This financial market supply-side friction leads to a distorted long-run allocation, and to inefficient economic dynamics, both of which justify macroprudential interventions. Although we share the view of Farhi and Werning (2016) and Korinek and Simsek (2016) regarding the importance of financial asset taxes in alleviating credit market inefficiencies and liquidity traps, the source of distortion in our framework is of a *supply-side* nature rather than an aggregate demand externality. In particular, the inefficiency here stems from firm default risk and the endogenous credit spread that are inflationary, and that exacerbate macroeconomic fluctuations unless unconventional financial measures are implemented. The mechanism that links between leverage, risk, spreads, inflation and output is referred to as the (risk-adjusted) credit cost channel.

In the face of cost-push financial shocks that directly raise credit spreads and thus inflation through the credit cost channel, unrestricted optimal policy necessitates an *equal reduction* in both the macroprudential tax and the nominal risk-free policy rate. In this way, the *effective* nominal deposit rate faced by households, adjusted for the financial tax, is completely stabilized at its long-run positive level. The lower bound constraint on the effective nominal deposit rate is therefore entirely removed with the first-best allocation attained at all times. This bliss outcome holds regardless of whether the economy is in a liquidity trap or not, and does not require any policy commitments. Interestingly, despite the inflationary nature of the financial shock, restricted optimal monetary policy under commitment triggers the ZLB due to the large inefficient and persistent slump in output. Under unrestricted optimal policy with macroprudential subsidies, the policy maker can freely set a *negative* nominal interest rate without breaching the effective ZLB constraint. Such policy mix limits cost-push inflationary pressures as well demand-pull inflation that would transpire in the absence of financial policies. Furthermore, in the presence of a cash-in-advance constraint, this optimal expansionary macroprudential-monetary policy combination satisfies the households no-arbitrage condition between deposits and cash-financed consumption. Negative nominal deposit rates and financial subsidies echo some of the non-standard policy measures undertaken by several central banks in advanced economies.

Following adverse demand shocks to the natural rate of interest that push prices and output in the same direction, the financial tax stands out as the most effective policy instrument when the economy is away from the ZLB. Intuitively, raising the tax on deposits induces households to save less and consume more through a standard intertemporal substitution effect. The improvement in aggregate demand, associated with the demand-driven increase in borrowing costs, helps to secure full price and output stability via the standard demand and credit cost channels.<sup>1</sup> Optimal policy is

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<sup>1</sup>A procyclical reaction of borrowing costs relative to GDP following *demand-driven* shocks is accordant with the models of Carlstrom and Fuerst (1997), De Fiore and Tristani (2013) and Eggertsson, Juelsrud, Summers and Wold (2019). In fact, the latter also show that demand shocks resemble the situation further into the crisis (since 2011),

conducted solely with the macroprudential tax due the prevailing output-inflation trade-off caused by interest rate changes; a key feature in a cost channel environment. However, when the economy enters a liquidity trap triggered by large demand shocks, optimal policy warrants a hike in the financial tax rate and a more modest increase in the nominal policy rate. Such policy combination *lowers* the effective nominal and real interest rates, which, in turn, limit the shrinkage in GDP. At the same time, the monetary contraction lifts borrowing costs, and generates a sufficient cost-push inflationary force that fosters price stability. These qualitative results hold under both discretion and commitment policies. Both regimes with macroprudential interventions yield an analogous welfare gain relative to the constrained optimal monetary policy plan, despite marginal differences in the implied optimal dynamics that emerge due to policy promises under commitment. The recent attempts by the European Central Bank (ECB) to lower deposit rates by paying negative rates on bank reserves are consistent with the implications of a higher and inflationary tax on deposits that we advocate for in this model when the liquidity trap is demand-driven.<sup>2</sup>

Despite abstracting from more complex macroprudential and unconventional monetary policy instruments used in the world of policy making, the generic specification of the private financial asset tax in our theoretical model preserves the transmission channels of credit market interventions and enhances analytical tractability (see also Farhi and Werning (2016) and Bianchi and Mendoza (2018)). Arguably, a tax on deposits may not be the first tool to spring to minds of policy makers, but its relative simplicity, effectiveness and resemblance to other more complicated policies, could and should bring about extra consideration to this unique automatic stabilizer policy measure.

Our model benefits from nesting the prototypical New Keynesian model as a particular case, and from a tractable introduction of financial frictions, macroprudential taxes and occasionally binding ZLB constraints to an otherwise standard Ravenna and Walsh (2006)-type cost channel setup. This stylized framework enables us to derive analytical optimal target rules, and to examine the normative and positive properties of unconventional taxation policies. The small-scale nature of the model allows for a transparent analysis of macroprudential and monetary policy interactions under both time-consistent and Ramsey plans derived using the linear-quadratic approach. Cúrdia and Woodford (2016) and Benigno, Eggertsson and Romei (2019) also develop simple, yet insightful, New Keynesian models with financial frictions to examine the optimal conduct of monetary policy, but posit a reduced-form intermediation technology to justify the existence of default and spreads. In our setup, borrowing costs are endogenous. Similar to our approach, Demiral (2009) and De Fiore and Tristani (2013) also derive a micro-founded risk premium, yet focus solely on optimal monetary policy away from the ZLB. Here, optimal monetary and financial tax policies are examined against the backdrop of state-contingent liquidity traps.

The paper is related to the literature that examines the effects of financial recessions and the joint optimal implementation of monetary and macroprudential policies (see Collard, Dellas, Diba and Loisel (2017), De Paoli and Paustian (2017) and Silvo (2019), among others). However, these papers abstract from the lower bound, implying different state-contingent policy implications in relation to ours. Optimal tax policies when interest rates are at the zero bound have been studied in the New Keynesian models of Eggertsson and Woodford (2006) and Correia, Farhi, Nicolini and Teles (2013). The former illustrate how consumption taxation can be used to partially offset the adverse effects of the policy rate reaching the ZLB, while the latter show that adjusting commodity

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wherein loan rates have reached historical low levels. In contrast, financial shocks in their model and ours always produce a countercyclical response of credit spreads, and resemble more the onset of the financial crisis.

<sup>2</sup>Conti, Neri and Nobili (2017) find that adverse aggregate demand shocks have been the most important contributors to the disinflation and the lower real GDP growth experienced in the Eurozone since 2014.

taxes can circumvent the zero bound constraint and always attain the first-best outcome. We also emphasize the need for tax flexibility to neutralize various shocks, although our motivation is different. First, our focus is on the short-run cyclical properties of financial taxation as opposed to more conventional labour and consumption taxes. Second, we highlight the role of supply-side financial frictions, which prove to be imperative to the state-dependant optimal policy plans. In a recent contribution, Correia, De Fiore, Teles and Tristani (2019) show that credit subsidies to firms can prevent the economy from entering a liquidity trap. Unlike their classic monetary economy framework, we develop a New Keynesian model with an explicit analysis of optimal discretionary versus commitment policies, and study the role of a financial tax on deposits that directly influences the households intertemporal consumption patterns.

Our work is also linked with the literature investigating the multiplier effects of various fiscal policies and in particular taxation policies in a liquidity trap and away from it. Away from the ZLB and in the context of a financial accelerator-type model, Fernández-Villaverde (2010) shows that an exogenous tax cut on private assets produces positive effects on output. In contrast, Eggertsson (2011) demonstrates that savings tax cuts could actually be contractionary, especially in a liquidity trap. Contributing to these papers, we explore the normative properties of optimal financial taxation policies. To the best of our knowledge, the welfare and business cycle implications of novel financial tax policies, and their interactions with monetary policy during normal and abnormal times, have not been fully addressed in the literature; especially regarding the impact of this unconventional policy instrument on the multiple interest rates decisions within a tractable workhorse New Keynesian model.

The remainder of the paper proceeds as follows. Section 2 describes the model. Section 3 characterizes the long-run and short-run equilibrium properties. In Section 4, we explain the parameterization of the model and the solution strategy. In Section 5, we derive the state-contingent optimal policy target rules and study their dynamics and welfare implications. Section 6 concludes.

## 2 The Model

The economy is populated by households, a continuum of monopolistic intermediate good (IG) firms, a final good (FG) firm, a competitive commercial bank (the bank), and a benevolent public authority that is responsible for monetary, fiscal and financial policies.<sup>3</sup> Following the realization of aggregate shocks, households lend their deposits (private financial assets / savings) to the bank, and are paid the after-tax effective deposit rate. The bank uses households deposits in order to supply working-capital loans to IG firms, and sets the loan rate as a risk premium over the risk-free nominal deposit rate. For the going lending rate, IG firms decide on their demand for loans, set prices subject to Calvo (1983)-type nominal price rigidities, and face end of period idiosyncratic productivity shocks that give rise to default risk. Using a standard Dixit-Stiglitz (1977) technology, the FG firm combines all intermediate goods to produce a homogeneous final good used only for consumption purposes. We now turn to a more detailed exposition of the economic environment and equilibrium properties.

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<sup>3</sup>In this setup, there is no distinction between the central bank, government and the financial authority, all of whom operate under full coordination with the same objective function. All these entities therefore fall under the category of the “public authority”, “policy maker” or “social planner”.

## 2.1 Households

The objective of the representative household is to maximize the following expected lifetime utility,

$$\mathbb{U}_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] Z_t, \quad (1)$$

where  $N_t = \int_0^1 N_{j,t} dj$  are the total hours supplied to the production sector, and  $C_t$  is a Dixit-Stiglitz (1977) consumption aggregator of intermediate goods ( $Y_{j,t}$ ) combined by a perfectly competitive FG firm. Specifically,  $C_t = \left[ \int_0^1 C_{j,t}^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon-1)}$ , with  $\epsilon > 1$  denoting the constant elasticity of substitution between intermediate goods, and  $C_{j,t} = Y_{j,t}$  for all  $j \in (0, 1)$ . The relative consumption demand for intermediate good  $j$  is then given by  $C_{j,t} = (P_{j,t}/P_t)^{-\epsilon} C_t$ , where  $P_t = \left[ \int_0^1 P_{j,t}^{1-\epsilon} dj \right]^{1/(1-\epsilon)}$  is the aggregate price index such that  $P_t C_t = \int_0^1 P_{j,t} C_{j,t} dj$ . Moreover,  $\beta \in (0, 1)$  is the discount factor,  $\sigma$  is the inverse of the intertemporal elasticity of substitution in consumption, and  $\varphi$  is the inverse of the Frisch elasticity of labour supply. The preference (demand) shock follows an  $AR(1)$  process,

$$Z_t = (Z)^{1-\rho_Z} (Z_{t-1})^{\rho_Z} \exp(s.d(\alpha^Z) \cdot \alpha_t^Z), \quad (2)$$

where  $Z$  is the steady state value of the demand shock,  $\rho_Z$  is the degree of persistence, and  $\alpha_t^Z$  is a random shock distributed as standard normal with a constant standard deviation given by  $s.d(\alpha^Z)$ .<sup>4</sup>

Households start period  $t$  with real wealth  $\mathbb{W}_t$ . In the assets market, households decide on money balances ( $M_t$ ), as well as on one-period safe deposits ( $D_t$ ). At the start of period  $t+1$ , real wealth  $\mathbb{W}_{t+1}$  includes the zero-interest bearing money holdings ( $M_t$ ), and the after-tax gross return on deposits,  $(1 - \tau_t^D) R_t^D D_t$ , where  $R_t^D$  is the gross nominal deposit rate, and  $\tau_t^D$  is the tax rate on deposits. Importantly,  $\tau_t^D$  serves as a state-contingent financial policy instrument that can be used to stabilize the economy following various shocks resulting potentially in liquidity trap episodes. Note that we could either have  $\tau_t^D > 0$ , corresponding to a tax, or  $\tau_t^D \leq 0$  representing a subsidy to private asset income. In line with Farhi and Werning (2016), we simply refer to  $\tau_t^D$  as a financial tax / subsidy, or as a macroprudential intervention.

Households also receive their wage bill  $W_t N_t$  paid as cash at the beginning of period  $t+1$ , with  $W_t$  denoting real wages. Cash and salaries are then used to pay for consumption goods ( $C_t$ ) subject to the following cash-in-advance constraint,

$$C_t \leq M_t + W_t N_t. \quad (3)$$

Finally, households receive a lump-sum transfer from the public authority ( $T_t$ ), and all profits from the production sector ( $J_t$ ).<sup>5</sup> Thus, the flow of funds constraints in real terms are,

$$\mathbb{W}_t \geq M_t + D_t, \quad (4)$$

and,

$$\mathbb{W}_{t+1} \frac{P_{t+1}}{P_t} = (1 - \tau_t^D) R_t^D D_t + M_t + W_t N_t - C_t + J_t + T_t. \quad (5)$$

<sup>4</sup>Steady state values are denoted by dropping the time subscript.

<sup>5</sup>The financial intermediary, which is also owned by households, earns zero profits in equilibrium.

For  $(1 - \tau_t^D) R_t^D \geq 1$ , and taking real wages ( $W_t$ ), prices ( $P_t$ ) and financial taxes ( $\tau_t^D$ ) as given, the first-order conditions of the household's problem with respect to  $C_t, D_t, \mathbb{W}_{t+1}, M_t$  and  $N_t$  can be summarized as,

$$C_t^{-\sigma} = \beta \mathbb{E}_t \frac{Z_{t+1}}{Z_t} C_{t+1}^{-\sigma} \frac{(1 - \tau_t^D) R_t^D}{\pi_{t+1}}, \quad (6)$$

$$N_t^\varphi C_t^\sigma = W_t, \quad (7)$$

where  $\pi_{t+1} \equiv P_{t+1}/P_t$  is defined as the gross inflation rate. Equation (6) is the Euler equation augmented for the financial tax. The effective real interest rate is thus  $(1 - \tau_t^D) R_t^D / \mathbb{E}_t \pi_{t+1}$ , implying that macroprudential interventions directly distort the household's intertemporal consumption-savings pattern. Furthermore, with households deposits used to facilitate working-capital loans supplied by the financial intermediary, a tax on deposit returns can also be treated as a tax / subsidy on bank liquidity. Equation (7) determines the optimal labour supply.

The optimality conditions and flow of funds constraints are written under the assumption that  $(1 - \tau_t^D) R_t^D \geq 1$ . This is the lower bound constraint on the *effective nominal* deposit rate which is an equilibrium restriction. Without macroprudential policies nor a cash-in-advance constraint, the non-negativity bound is attached only to the nominal risk-free interest rate (as in Eggertsson and Woodford (2003, 2006) and Eggertsson (2011)). However, the actual observed savings rate that enters the household's Euler equation accounts for any potential changes in the financial tax, and serves as the opportunity cost to money holdings. Cash, in turn, carries a zero nominal interest rate and is used to purchase consumption goods subject to (3). Therefore, the effective lower bound that satisfies the household's no-arbitrage condition between cash-financed consumption and deposits must apply to  $(1 - \tau_t^D) R_t^D \geq 1$ .

## 2.2 Production

There is a continuum of measure one of monopolistically competitive IG firms, indexed by  $j \in (0, 1)$ , who produce a differentiated good  $Y_{j,t}$  using the following linear production function,

$$Y_{j,t} = \varepsilon_{j,t} N_{j,t}, \quad (8)$$

where  $N_{j,t}$  is the employment demand by firm  $j$ , and  $\varepsilon_{j,t}$  represents an idiosyncratic shock that occurs as period  $t + 1$  comes to a close. This shock is distributed uniformly over the interval  $(\underline{\varepsilon}, \bar{\varepsilon})$  with a constant variance and a mean of unity.<sup>6</sup> Each firm has to borrow in advance in order to finance the household's wage bill in the subsequent period. Specifically, working-capital loans decided in period  $t$  are held in zero-interest bearing cash accounts, and are then used to pay for the household's wage bill at the start of period  $t + 1$ . Loans are paid back with interest at the end of period  $t + 1$ , with the gross lending rate determined by  $R_t^L$ . Let  $L_{j,t}$  be the amount borrowed by firm  $j$ , then the borrowing constraint is,

$$W_t N_{j,t} \leq L_{j,t}. \quad (9)$$

The pricing decision takes place at the start of date  $t + 1$  and consists of two stages. In the first stage, each borrowing producer minimizes the cost of employing labour, taking its effective

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<sup>6</sup>We use the uniform distribution in order to generate a plausible data-consistent steady state credit spread, and to obtain simple closed-form solutions to the model without loss of generality.

costs as given. Defining profits in the first stage as  $Y_{j,t} - W_t N_{j,t} - (R_t^L - 1) L_{j,t}$  then the first order conditions, accounting for the expectations with respect to the idiosyncratic shock, yield the real marginal cost,<sup>7</sup>

$$mc_t = \frac{R_t^L W_t}{\mathbb{E}_t \varepsilon_{j,t}}. \quad (10)$$

For  $R_t^L \geq 1$ , the borrowing constraint (9) is always binding. This non-negativity restriction on the equilibrium lending rate represents a no-arbitrage condition ensuring that firms cannot make large profits by keeping their working-capital loans in the form of zero-interest cash accounts. Therefore,  $R_t^L \geq 1$  is the lower bound constraint on the cost of borrowing. In the specific case of  $R_t^L = 1$ , firms are indifferent between keeping their loans in cash accounts and paying back these loans at the prevailing lending rate. Under this marginal case, firms choose the latter.

In the second stage, each producer chooses the optimal price for its good. Here Calvo (1983)-type contracts are employed, where a portion of  $\theta$  firms keep their prices fixed while a portion of  $1 - \theta$  producers reset prices optimally. Denoting  $P_t^*$  as the optimal price set by IG producers who can reset prices, then the standard maximization problem under symmetry yields the optimal price setting rule,

$$\frac{P_t^*}{P_t} = \left( \frac{\epsilon}{\epsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \theta^s \beta^s C_{t+s}^{-\sigma} mc_{t+s} \left( \frac{P_{t+s}}{P_t} \right)^\epsilon Y_{t+s}}{E_t \sum_{s=0}^{\infty} \theta^s \beta^s C_{t+s}^{-\sigma} \left( \frac{P_{t+s}}{P_t} \right)^{\epsilon-1} Y_{t+s}}, \quad (11)$$

with  $pm \equiv \left( \frac{\epsilon}{\epsilon-1} \right)$  representing the price mark-up, and  $mc_t$  given by (10).

To model credit default, we assume that in each period a fraction  $\chi_t$  of the firm's expected output ( $Y_{j,t}$ ) must be pledged as collateral in order to secure working-capital loans. Moreover, the borrowing firm has the option to 'run away' and default on its debt. In the good states of nature, each firm pays back the bank principal plus interest on credit. Default occurs if the firm's value after non-payment is greater than its expected value after repaying back the loan in full,

$$(1 - \chi_t) Y_{j,t} > Y_{j,t} - R_t^L L_{j,t}, \quad (12)$$

with  $(1 - \chi_t) Y_{j,t}$  denoting the expected value of the firm after 'running away', and  $\chi_t Y_{j,t}$  representing the share of collateralized output the bank is able to retain in case of default. It is further assumed that  $\chi_t$  follows the  $AR(1)$  shock process,

$$\chi_t = (\chi)^{1-\rho_\chi} (\chi_{t-1})^{\rho_\chi} \exp(s.d(\alpha^\chi) \cdot \alpha_t^\chi), \quad (13)$$

where  $\chi \in (0, 1)$  is the mean value of this fraction,  $\rho_\chi$  is the degree of persistence, and  $\alpha_t^\chi$  is a white-noise process with constant standard deviation  $s.d(\alpha^\chi)$ . A shock to the probability of collateral recovery ( $\chi_t$ ) represents a financial shock in this model, as it directly impacts credit risk at the firm level as well as bank credit spreads, as shown below.<sup>8</sup>

<sup>7</sup>Note that  $\mathbb{E}_t \varepsilon_{j,t}^F$  is identical across all firms in the pricing decision stage that takes place at the *beginning* of period  $t + 1$ , just after the realization of *aggregate* shocks and before the idiosyncratic shock that occurs at the very end of the period. Hence, under symmetry, the subscript  $j$  can be dropped from the marginal cost and consequently from the optimal price level derived below.

<sup>8</sup>Jermann and Quadrini (2012) and Tayler and Zilberman (2016) also motivate a similar type of financial / credit



Using (8), (9), and re-arranging (13) results in the threshold value ( $\varepsilon_{j,t}^M$ ) below which the firm defaults,

$$\varepsilon_{j,t}^M = \varepsilon_t^M = \frac{R_t^L W_t}{\chi_t}. \quad (14)$$

The cut-off point is related to aggregate credit shocks, borrowing costs and real wages, and is identical across all firms.<sup>9</sup> As real wages are approximated by the loan to output ratio (with  $Y_{j,t} = N_{j,t}$  for  $\mathbb{E}_t \varepsilon_{j,t} = 1$ , and output serving as collateral), then higher leverage ( $R_t^L L_{j,t}/Y_{j,t}$ ) driven by increased demand for working-capital loans raise the firms' marginal costs and translate into elevated financial risk. Later in the text we show that this supply-side credit friction leads to an inefficient level of output in the the long-run, and can destabilize the economy in the short-run. Thus, macroprudential policies are warranted to alleviate these inefficiencies. Given the uniform properties of  $\varepsilon_t$ , the probability of default is,

$$\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^M} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^M - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}. \quad (15)$$

### 2.3 Financial Intermediation

The banking sector is perfectly competitive. The bank raises  $D_t$  funds via the households in order to finance the working-capital requirements of IG firms. The bank's balance sheet satisfies,

$$L_t = D_t, \quad (16)$$

where  $L_t = \int_0^1 L_{j,t} dj = W_t N_t$  is the total lending to the production industry, and  $N_t = \int_0^1 N_{j,t} dj$ .

The loan rate is set at the very beginning of period  $t + 1$ , just after the realization of aggregate shocks, but before firms engage in production and pricing decisions. The bank breaks-even from its intermediation activity, such that the expected income from lending to a continuum of firms is equal to the total costs of borrowing these funds. The bank's expected intra-period zero-profit condition from lending is,

$$\int_{\varepsilon_{j,t}^M}^{\bar{\varepsilon}} R_t^L L_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^M} \chi_t Y_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} = R_t^D D_t, \quad (17)$$

where  $f(\varepsilon_{j,t})$  is the probability density function of  $\varepsilon_{j,t}$ . The first element on the left hand side is the expected repayment to the bank in the non-default states, while the second element is the expected return in the default states, measured in terms of collateralized output ( $\chi_t Y_{j,t}$ ). The term  $R_t^D D_t$  is the overall gross interest payment on deposit liabilities. To derive the lending rate, we use the balance sheet equation (16), the binding constraint (12) for  $\chi_t \varepsilon_{j,t}^M N_{j,t} = R_t^L L_{j,t}$ , the production function (8), divide by  $L_{j,t}$ , and apply the characteristics of the uniform distribution. After some

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/ risk shock that directly hits the collateral constraint.

<sup>9</sup>As we solve explicitly for the risk of default using a threshold condition, the collateral constraint in this model (12), from which we derive the cut-off point, is always binding.

algebra, the loan rate equation reads,<sup>10</sup>

$$R_t^L = \nu_t R_t^D, \quad (18)$$

with  $\nu_t \equiv \left[1 - \left(\frac{\bar{\varepsilon} - \varepsilon}{2\varepsilon_t^M}\right) \Phi_t^2\right]^{-1} > 1$  defined as the risk premium,  $\varepsilon_t^M$  given by (14), and  $\Phi_t$  determined by (15). The loan rate is therefore set as a finance premium over the risk-free policy rate due to the possibility of credit default.

## 2.4 Public Authority

The public authority targets the short-term risk-free policy rate  $R_t^D$  and the financial tax rate  $\tau_t^D$  that respect the ZLB constraint on the effective nominal deposit rate,

$$(1 - \tau_t^D) R_t^D \geq 1. \quad (19)$$

Furthermore, to maintain the firms' no-arbitrage condition between cash holdings and loan repayments, policy must be set such that the cost of borrowing cannot fall below zero,

$$R_t^L = \nu_t R_t^D \geq 1. \quad (20)$$

Finally, the public authority's budget constraint satisfies,

$$\mathbb{W}_{t+1} \frac{P_{t+1}}{P_t} - \mathbb{W}_t + \tau_t^D R_t^D D_t = T_t. \quad (21)$$

## 2.5 Market Clearing

On the production side, market clearing requires  $Y_t \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} dj = N_t \int_0^1 \varepsilon_{j,t} dj$ , where  $N_t = N_{j,t}$  in a symmetric equilibrium. Using the distribution properties of the idiosyncratic shocks, which satisfy  $\int_0^1 \varepsilon_{j,t} dj = 1$  and have a mean of unity, we obtain the following equilibrium condition,

$$Y_t \Delta_t = N_t, \quad (22)$$

where  $\Delta_t \equiv \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} dj$  is defined as the usual price dispersion index. Using (10) and (18), the marginal costs faced by IG firms are,

$$mc_t = R_t^L W_t = \nu_t R_t^D W_t. \quad (23)$$

Moreover, for  $R_t^L \geq 1$  the borrowing constraint (9) is binding and is identical across all firms,

$$L_t = W_t N_t = D_t. \quad (24)$$

Finally, the aggregate resource constraint is,

$$Y_t = C_t. \quad (25)$$

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<sup>10</sup>The cut-off value  $\varepsilon_{j,t}^M$  depends on the state of the economy and is therefore identical across all firms (see (14)). Similarly, real wages and labour employed by each firm are identical such that the volume of demand-determined loans is also the same. Thus, the subscript  $j$  is dropped in what follows.

### 3 Equilibrium

To solve the model, we log-linearize the behavioral equations and the resource constraint around the non-stochastic, zero inflation ( $\pi = 1$ ) steady state. Using the log-linear versions of (7), (11), (22), (23) and (25) together with the assumption of a symmetric equilibrium, allows to write the New Keynesian Phillips Curve (NKPC) as,

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \lambda \left[ (\sigma + \varphi) \hat{Y}_t + \hat{R}_t^L \right], \quad (26)$$

with  $\lambda \equiv (1 - \theta)(1 - \theta\beta)/\theta$ .

The aggregate level of lending is obtained from the log-linear versions of (7), (22), (24) and (25), and is given by,

$$\hat{L}_t = (1 + \sigma + \varphi) \hat{Y}_t. \quad (27)$$

To derive the loan rate, we first log-linearize equations (7), (14), (15), (22) and (25) to obtain the log-linearized risk of default,

$$\hat{\Phi}_t = \left( \frac{\varepsilon^M}{\varepsilon^M - \underline{\varepsilon}} \right) \left[ \hat{R}_t^L + (\sigma + \varphi) \hat{Y}_t - \hat{\chi}_t \right]. \quad (28)$$

By log-linearizing (18) and using (28), the equation determining the credit spread is,

$$\hat{R}_t^L - \hat{R}_t^D = \left( \frac{\Psi}{1 - \Psi} \right) \left[ \hat{R}_t^D + (\sigma + \varphi) \hat{Y}_t - \hat{\chi}_t \right], \quad (29)$$

with  $\Psi \equiv \frac{(\varepsilon^M + \underline{\varepsilon})(\varepsilon^M - \underline{\varepsilon})}{[2\varepsilon^M(\underline{\varepsilon} - \underline{\varepsilon}) - (\varepsilon^M - \underline{\varepsilon})^2]} \in (0, 1)$  measuring the degree of financial market imperfections.

The term  $\varepsilon^M = mc/\chi$  is the steady state threshold value below which the IG firm defaults (see (10) and (14)), where  $mc = (\varepsilon - 1)/\varepsilon \equiv (pm)^{-1}$  from equation (11). The steady state risk of default is therefore  $\Phi = \frac{(mc/\chi) - \underline{\varepsilon}}{\underline{\varepsilon} - \underline{\varepsilon}}$  while the long-run credit spread is  $R^L/R^D = \nu$ , with  $\nu \equiv \left[ 1 - \left( \frac{\underline{\varepsilon} - \underline{\varepsilon}}{2\varepsilon^M} \right) \Phi^2 \right]^{-1} > 1$  and,

$$R^D = \frac{1}{(1 - \tau^D)\beta}. \quad (30)$$

Equations (28) and (29) show that the credit spread increases with aggregate demand, the policy rate, and in response to an adverse financial shock. Intuitively, a rise in the demand for goods, all else equal, raises the firms demand for external working-capital finance used to support production. With production pinning down the level of collateral, higher leverage elevates the firms marginal costs, the probability of default and thus the credit spread charged by the bank. Furthermore, a rise in  $\hat{R}_t^D$  pushes up  $\hat{R}_t^L$  through a standard monetary policy cost channel effect. In the absence of the financial friction,  $\Psi = 0$ , the loan rate tracks only the risk-free policy rate,  $\hat{R}_t^L = \hat{R}_t^D$ , as in the basic cost channel framework of Ravenna and Walsh (2006). Also, an exogenous decline in the collateral recovery rate,  $\hat{\chi}_t < 0$ , translates directly to a hike in default risk, leading to a higher credit spread. Finally, observe that from (27), (28) and (29), the credit spread and risk are positively related to variations in the loan to GDP (leverage) ratio as  $(\sigma + \varphi) \hat{Y}_t = \hat{L}_t - \hat{Y}_t$ . Contributing to Cúrdia and Woodford (2016) and Benigno, Eggertsson and Romei (2019), this positive relationship is micro-founded, and does not hamper the analytical tractability of the model.

Long-run output is calculated from the steady state versions of equations (7), (11), (22), (23)

and (25) as well as from (30),

$$Y^{\sigma+\varphi} = \frac{1}{\nu R^D} (pm)^{-1} = \frac{\beta (1 - \tau^D)}{\nu} (pm)^{-1}, \quad (31)$$

where  $Y^{\sigma+\varphi}$  is the long-run marginal rate of substitution between consumption and hours worked. The unconstrained first-best allocation, absent financial frictions and the price mark-up, corresponds to  $Y^{\sigma+\varphi} = 1$ . This efficiency condition can be supported through the implementation of the following long-run corrective *hypothetical* macroprudential subsidy,

$$\tau^{D,I} = 1 - \frac{(pm)\nu}{\beta} < 0, \quad (32)$$

where superscript  $I$  denotes unrestrained first-best policy. Under standard parameterization with  $\beta < 1 < (pm)\nu$  and  $pm, \nu > 1$ , a subsidy on private financial assets can therefore completely offset both the financial friction stemming from ex-ante default, and the price mark-up resulting from monopolistic competition in the deterministic steady state. The negative relationship between output ( $Y$ ) and the loan rate ( $\nu R^D = \nu / (1 - \tau^D) \beta$ ) arising from the risk-adjusted cost channel generates an additional degree of freedom that enables the policy maker to directly intervene in the credit markets and consequently eliminate steady state distortions using a macroprudential subsidy.

However, this theoretical unconstrained first-best policy is *not* feasible as it must be accompanied by a negative loan rate. Specifically, substituting (32) in (30) and using the steady state equation for  $R^L$  yields  $R^L = \nu R^D = (pm)^{-1} < 1$  for  $\nu > 1$ . Such outcome violates the firms no-arbitrage condition between loan repayments and storing loans in zero-interest bearing cash accounts. As optimal policy in the deterministic steady state pushes the loan rate to non-viable negative territory, the constrained-efficient long-run policy that respects the non-negativity constraint on borrowing costs is obtained by setting  $\nu R^D = 1$ . Combining (30) with  $\nu R^D = 1$  results in,

$$\tau^{D,II} = 1 - \frac{\nu}{\beta} < 0, \quad (33)$$

with superscript  $II$  standing for the constrained-efficient long-run policy and  $|\tau^{D,I}| > |\tau^{D,II}|$ . In contrast to the unconstrained first-best policy, the more modest and restricted financial subsidy is feasible, and serves as a natural policy instrument that can remove the long-run inefficiency induced by the supply-side credit friction,  $\nu = f(\Phi(\chi))$ . A higher value of  $\nu$  (or a lower  $\chi$ ) calls for a larger financial subsidy which helps to alleviate the credit friction by lowering the cost of loanable funds. Importantly, the implementation of the constrained-efficient financial policy enables the public authority to set a *negative* policy rate,  $R^D = \nu^{-1} < 1$ , which together with  $\tau^{D,II} < 0$ , satisfy also the households no-arbitrage condition between deposits and cash-financed consumption, i.e.,  $(1 - \tau^{D,II}) R^D = \beta^{-1}$ . In this way, there exists a single combined policy implementation of the financial tax and the nominal policy rate set to their effective lower bounds. This policy prescription represents a modified Friedman (1969) rule. Without seeking a rate of deflation as implied by the original Friedman proposal, zero effective savings and loan rates can be accomplished through the enactment of financial subsidies. Macroprudential interventions thus provide a rationale for adopting a prolonged negative nominal deposit rate; a policy measure that echoes some of the recent practices undertaken by several central banks in advanced economies.

To capture the ZLB constraint on the effective savings rate in the short-run, we log-linearize

(19) to obtain,

$$\hat{R}_t^D - \hat{r}_t^D \geq -r^d, \quad (34)$$

with  $r^d = -(\beta - 1)$  denoting the steady state net real rate of interest expressed in log-deviations, and  $\hat{r}_t^D = \ln \frac{(1-\tau_t^D)}{(1-\tau^D)}$ . Log-linearizing (20) and the expression for the risk premium ( $\nu_t$ ) yields the lower bound constraint on the lending rate represented in terms of deviations from steady state,

$$\hat{R}_t^L = \frac{1}{(1-\Psi)} \left[ \hat{R}_t^D + \Psi \left( (\sigma + \varphi) \hat{Y}_t - \hat{\chi}_t \right) \right] \geq -r^l, \quad (35)$$

where  $r^l \equiv -((1-\tau^D)\beta\nu^{-1} - 1)$  is the long-run net loan rate. Notice that in steady state where each variable satisfies  $\hat{X}_t = 0$ , the tax level that brings the loan rate to its ZLB is set to  $(1-\tau^D)\beta\nu^{-1} - 1 = 0$  or  $\tau^{D,H} = 1 - \nu\beta^{-1}$ , as suggested by the constrained-efficient policy (33). Consequently, a macroprudential subsidy in steady state brings the economy closer to its constrained-efficient long-run equilibrium.

To simplify the subsequent short-run optimal policy analysis, we examine the normative policy implications following large shocks that cause  $\hat{R}_t^D$  and potentially  $\hat{R}_t^D - \hat{r}_t^D$  to hit their lower bounds, but not significant enough to drive  $\hat{R}_t^L$  to its floor. The analysis below is therefore conducted with one occasionally binding constraint. Indeed, in the aftermath of the Great Recession, both deposit and lending rates have been hovering at historically low levels. Despite the downward pressures placed on borrowing costs, especially in response to adverse demand-driven disturbances, the data does not suggest loan rates being set to their effective lower bounds (see also Eggertsson, Juelsrud, Summers and Wold (2019)). Lending rates have remained consistently elevated relative to the policy rate and the various risk-free market savings rates.<sup>11</sup>

Substituting (29) in (26), and log-linearizing (6), the model can then be expressed in terms of the following equations,

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\lambda}{(1-\Psi)} \left[ \hat{R}_t^D + (\sigma + \varphi) \hat{Y}_t - \Psi \hat{\chi}_t \right], \quad (36)$$

$$\hat{Y}_t = \mathbb{E}_t \hat{Y}_{t+1} - \sigma^{-1} \left( \hat{R}_t^D - \hat{r}_t^D - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n \right), \quad (37)$$

with  $\hat{r}_t^n \equiv \hat{Z}_t - \mathbb{E}_t \hat{Z}_{t+1}$  defined as the natural rate of interest that is a function only of the preference shock. Equation (36) is the extended NKPC establishing the short-run aggregate supply (AS) relation between inflation and output, augmented for the degree of credit frictions,  $\Psi = f(\varepsilon^M(\chi))$ , and the financial shock,  $\hat{\chi}_t$ . The financial shock, which has a structural interpretation in our model as explained above, manifests itself in a direct cost-push disturbance without altering the efficient level of output.<sup>12</sup> The cost-push component of the financial friction and the credit shock quantify

<sup>11</sup>The theoretical prospect of  $\hat{R}_t^L$  hitting its lower bound acts as mere amplification mechanism following *only* negative demand shocks that produce a procyclical relationship between credit spreads and GDP (see also Eggertsson, Juelsrud, Summers and Wold (2019)). In contrast, an adverse financial shock inherent in a lending rate spike can be completely contained with an appropriate combination of state-contingent monetary and financial subsidy policies. In other words, optimal policy results in  $\hat{R}_t^L$  being stabilized at its long-run *positive* level. Therefore, little economic insight is gained from the introduction of the short-run non-negativity constraint on the loan rate.

<sup>12</sup>Without Total Factor Productivity (TFP) shocks, the efficient level of output is set to unity, implying that cyclical output is equal to the output gap. Moreover, without the financial friction ( $\Psi = 0$ ), the cost-push financial shock disappears from the model. Indeed, it is the financial market imperfection that gives rise to the inflationary cost-push shock in this framework.

and reflect the nature of the risk-adjusted credit cost channel, in which higher risk and credit spreads push up marginal costs and thus inflation. These inflationary pressures arise independently from the direct monetary policy cost channel effect linking  $\hat{R}_t^D$  to  $\hat{\pi}_t$ . While the direct effect of an increase in  $\hat{R}_t^D$  is to raise  $\hat{\pi}_t$ , the overall impact, that takes into account the standard demand channel of monetary policy, is calculated by  $\frac{\partial \hat{\pi}_t}{\partial \hat{R}_t^D} = \frac{\lambda}{(1-\Psi)} - \frac{\lambda}{(1-\Psi)} \frac{(\sigma+\varphi)}{\sigma}$  or  $\frac{\partial \hat{\pi}_t}{\partial \hat{R}_t^D} = -\frac{\lambda}{(1-\Psi)} \frac{\varphi}{\sigma} < 0$ . Conditional on inflation expectations, a rise in the nominal policy rate therefore lowers inflation, with a higher  $\Psi$  amplifying the decline in  $\hat{\pi}_t$  following the monetary contraction.

Equation (37) is the Euler equation that determines the aggregate demand (*AD*) schedule, augmented for the preference shock and the financial tax. A lower tax on deposits increases desired savings such that in equilibrium output falls more than in the absence of tax changes. Nevertheless, in response to inflationary shocks, implementing a macroprudential subsidy can act to stabilize inflation and consequently be welfare improving. The optimal state-contingent policy plans against financial and demand shocks are investigated later in the text.

A novel aspect of our model is that output and debt (proxies for the marginal costs) largely determine the finance premium and the ensuing credit spread (see equations (28) and (29)). This risk premium effect operates via the wider credit cost channel, and provides additional means through which monetary and financial tax policies can alter the economic activity. Finally, our model nests the frictionless cost channel framework of Ravenna and Walsh (2006) by setting  $\Psi = 0$  and  $\hat{\tau}_t^D = 0, \forall t$ , as well as the Galí (2015) textbook New Keynesian setup by ignoring the term  $\lambda(1-\Psi)^{-1} \hat{R}_t^D$  in equation (36) and setting again  $\Psi = 0$  and  $\hat{\tau}_t^D = 0, \forall t$ .

The competitive approximate equilibrium is defined as a collection of real allocations  $\{\hat{Y}_t\}_{t=0}^{\infty}$ , prices  $\{\hat{\pi}_t\}_{t=0}^{\infty}$ , interest rates  $\{\hat{R}_t^D\}_{t=0}^{\infty}$  and macroprudential policies  $\{\hat{\tau}_t^D\}_{t=0}^{\infty}$  such that for a given sequence of exogenous *AR*(1) shock processes  $\{\hat{Z}_t, \hat{\chi}_t\}_{t=0}^{\infty}$ , conditions (34), (35), (36) and (37) are satisfied.

## 4 Parameterization and Solution Strategy

Although most of our results are shown analytically, in order to illuminate the implications of the state-contingent optimal policies for welfare and economic dynamics, we also solve the model numerically. We employ parameterization largely used in the New Keynesian literature. The parameter values are summarized in Table 1 and serve as the baseline calibration of the model.

Table 1: Benchmark Parameterization

| Parameter              | Value | Description   |
|------------------------|-------|---|
| $\beta$                | 0.994 | Discount factor                                     |
| $\sigma$               | 1.00  | Inverse of elasticity of intertemporal substitution |
| $\varphi$              | 0.50  | Inverse of the Frisch elasticity of labour supply   |
| $Z$                    | 1.00  | Average preference shock value                      |
| $\epsilon$             | 6.00  | Elasticity of demand for intermediate goods         |
| $\theta$               | 0.80  | Degree of price stickiness                          |
| $\bar{\epsilon}$       | 1.20  | Idiosyncratic productivity shock upper range        |
| $\underline{\epsilon}$ | 0.80  | Idiosyncratic productivity shock lower range        |
| $\chi$                 | 0.97  | Fraction of collateral seized in default states     |
| $\rho_Z$               | 0.90  | Degree of persistence - Demand shock                |
| $\rho_\chi$            | 0.90  | Degree of persistence - Financial shock             |
| $s.d(\alpha^Z)$        | 0.01  | Standard deviation - Demand shock                   |
| $s.d(\alpha^\chi)$     | 0.05  | Standard deviation - Financial shock                |

Elaborating on some of the unique parameters to this framework. The subjective discount factor is set to  $\beta = 0.994$ , while the financial tax in the benchmark case is  $\tau^D = 0$ . The implied long-run risk-free interest rate for this parameterization is 2.4 percent, consistent with the low interest rates environment that predated the Great Recession. Furthermore, we set the range of the idiosyncratic shock to  $(0.8, 1.2)$ , and the fraction of output received in case of default to  $\chi = 0.97$ . These values, together with a price mark-up of 20 percent ( $\epsilon = 6$ ) and  $\varphi = 0.5$ , yield an annual credit spread of  $\nu = 2.04$  percent and a loan to GDP ratio of 82.41 percent. These financial market estimates roughly correspond with the long-run U.S. data.

As for the main shocks examined in our paper, we fix the persistence parameters governing the evolution of financial and demand shocks,  $\rho_\chi$  and  $\rho_Z$ , both to 0.90, while the standard deviations associated with these shocks are  $s.d(\alpha^\chi) = 0.05$  and  $s.d(\alpha^Z) = 0.01$ , respectively. Our shock moments are within range of the calibrated values obtained in Gilchrist, Schoenle, Sim and Zakrajšek (2017) and Jermann and Quadrini (2012) - for financial shocks; and Eggertsson (2011) - for demand shocks. Finally, to quantitatively solve the model with occasionally binding constraints, we implement the piecewise-linear methodology developed in Guerrieri and Iacoviello (2015), and confirm the results using Holden's (2016) algorithm for the perfect foresight solution.

## 5 Optimal Macroprudential Interventions

The presence of nominal rigidities, the supply-side credit distortions and the various shocks generate inefficient economic dynamics. Moreover, as shown above, in the deterministic steady state macroprudential interventions cannot fully correct for the long-run price mark-up friction despite being able to offset the credit market imperfection. As our main focus is on financial taxation and its interaction with monetary policy in the short-run, we introduce a labour subsidy that can eliminate all average distortions.<sup>13</sup> Therefore, we take a second-order approximation of the household's ex-ante utility function around the efficient deterministic steady state. The public authority's

<sup>13</sup>Such fiscal policy instruments may not be available for stabilizing credit frictions, inflation and output in a state-contingent fashion. However, Correia, Farhi, Nicolini and Teles (2013) show that the implementation of commodity taxes results in an efficient equilibrium, thereby circumventing the ZLB problem. Unlike their paper, our focus is on

objective welfare function is then given by,<sup>14</sup>

$$\mathcal{W}_t = \sum_{t=0}^{\infty} \beta^t \frac{(\mathbb{U}_t - \mathbb{U})}{\mathbb{U}_C C} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[ \frac{\epsilon}{\lambda} \hat{\pi}_t^2 + (1 + \varphi) \hat{Y}_t^2 \right]. \quad (38)$$

The period losses read,

$$\left( \frac{\lambda}{\epsilon} \right) \mathcal{L}_t = \hat{\pi}_t^2 + \vartheta \hat{Y}_t^2, \quad (39)$$

where  $\vartheta \equiv \kappa/\epsilon$ ,  $\kappa \equiv \lambda(1 + \varphi)$  and  $\lambda \equiv (1 - \theta)(1 - \theta\beta)/\theta$ .

We now follow the linear-quadratic approach employed in Adam and Billi (2006, 2007) in order to characterize optimal monetary and macroprudential policies subject to the unique lower bound constraints of our model. The optimal policy analysis considers both discretionary (time-consistent) and commitment (Ramsey) policies.

### 5.1 Optimal Policy under Discretion

Under discretion, the social planner takes private sector expectations as given when solving its optimization problem. Each period, the policy maker chooses  $\hat{\pi}_t$ ,  $\hat{Y}_t$ ,  $\hat{R}_t^D$  and  $\hat{\tau}_t^D$  to maximize its objective function (38) subject to the constraints (34)-(37), taking  $\hat{r}_t^n, \hat{\chi}_t$  and  $\left\{ \hat{\pi}_{t+i}, \hat{Y}_{t+i}, \hat{R}_{t+i}^D, \hat{\tau}_{t+i}^D \right\}_{i=1}^{\infty}$  as given. The Lagrangian for this problem takes the form,

$$\begin{aligned} \mathfrak{L}_t = & -\frac{1}{2} \left( \hat{\pi}_t^2 + \vartheta \hat{Y}_t^2 \right) - \hat{\zeta}_{1,t} \left[ \hat{\pi}_t - \beta \mathbb{E}_t \hat{\pi}_{t+1} - \frac{\lambda}{(1 - \Psi)} \left[ \hat{R}_t^D + (1 + \varphi) \hat{Y}_t - \Psi \hat{\chi}_t \right] \right] \\ & - \hat{\zeta}_{2,t} \left[ \hat{Y}_t - \mathbb{E}_t \hat{Y}_{t+1} + \hat{R}_t^D - \hat{\tau}_t^D - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n \right] - \hat{\zeta}_{3,t} \left[ -\hat{R}_t^D + \hat{\tau}_t^D - r^d \right], \end{aligned}$$

where  $\hat{\zeta}_{1,t}$ ,  $\hat{\zeta}_{2,t}$  and  $\hat{\zeta}_{3,t}$  are the Lagrange multipliers on constraints (36), (37) and (34), respectively. Under discretion, first-order conditions are given by,

$$-\hat{\pi}_t = \hat{\zeta}_{1,t}, \quad (40)$$

$$-\vartheta \hat{Y}_t + \frac{\kappa}{(1 - \Psi)} \hat{\zeta}_{1,t} = \hat{\zeta}_{2,t}, \quad (41)$$

$$\frac{\lambda}{(1 - \Psi)} \hat{\zeta}_{1,t} - \hat{\zeta}_{2,t} + \hat{\zeta}_{3,t} = 0, \quad (42)$$

$$\hat{\zeta}_{2,t} = \hat{\zeta}_{3,t}, \quad (43)$$

and the slackness condition,

$$\hat{\zeta}_{3,t} \left( -\hat{R}_t^D + \hat{\tau}_t^D - r^d \right) = 0. \quad (44)$$

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state-dependant financial taxation in a model featuring both supply-side credit frictions, and effective lower bound constraints that stem from the unique financial market structure of our setup.

<sup>14</sup>The full derivation of the welfare function is provided in the Appendix.



### 5.1.1 Policy Away from the Effective Lower Bounds

To highlight the implications of macroprudential interventions, it is useful to start from the examination of the optimal target rule that emerges when the public authority has no access to financial taxation ( $\hat{\tau}_t^D = 0, \forall t$ ), and when constraint (34) is slack ( $\hat{\zeta}_{3,t} = 0$ ). The optimal target rule in this case is characterized by,

$$\vartheta \hat{Y}_t = -\frac{(\kappa - \lambda)}{(1 - \Psi)} \hat{\pi}_t, \quad (45)$$

The policy maker thus finds it optimal to engineer a fall in output in order to curb inflationary pressures provoked by a cost-push financial shock. A marked credit friction ( $\Psi > 0$ ) aggravates the variability of output for given movements in inflation. Intuitively, a higher degree of financial market imperfections (as also measured by a lower  $\chi$ ) escalates the hike in the risk premium and  $\hat{R}_t^L$  following an adverse financial disturbance. This upshot leads to a more pronounced increase in inflation, and forces the optimizing policy maker to adopt a stricter anti-inflationary policy stance. The more hawkish policy response accelerates the contraction in aggregate demand, which, in turn, dampens the rise in the credit spread and inflation via both the standard demand channel of monetary policy as well as the risk-adjusted credit cost channel.

In the Ravenna and Walsh (2006) frictionless cost channel setup where  $\Psi = 0$ , the coefficient on  $\hat{\pi}_t$  in the targeting rule would be  $\frac{(\kappa - \lambda)}{\vartheta} < (1 - \Psi)^{-1} \frac{(\kappa - \lambda)}{\vartheta}$ , implying a more muted output adjustment for a given inflation deviation relative to our setup. In Ravenna and Walsh (2006), variability in inflation is larger because a rise in  $\hat{R}_t^D$  not only acts to reduce  $\hat{Y}_t$  and  $\hat{\pi}_t$  through a standard demand effect, but also serves to increase  $\hat{\pi}_t$  and amplify the fall in  $\hat{Y}_t$  via the monetary policy cost channel. These effects make inflation stabilization more costly in terms of output stability, triggering a monetary policy trade-off. In our model, this policy trade-off is intensified due to the existence of the financial friction that warrants a more aggressive monetary policy.<sup>15</sup>

Let's now introduce macroprudential policies. Observe that the presence of financial taxation adds the first-order condition  $\hat{\zeta}_{2,t} = 0$ , which, together with  $\lambda(1 - \Psi)^{-1} \hat{\zeta}_{1,t} - \hat{\zeta}_{2,t} = 0$  or  $\hat{\zeta}_{1,t} = 0$ , removes the constraints imposed by *both* the *AS* and *AD* schedules (see conditions (40)-(43) when  $\hat{\zeta}_{3,t} = 0$ ). Optimal stabilization policy that simultaneously adjusts  $\hat{R}_t^D$  and  $\hat{\tau}_t^D$  thus results in the first-best allocation where inflation and output are given by  $\hat{\pi}_t = 0$  and  $\hat{Y}_t = 0$ . The efficient equilibrium is achieved through macroprudential interventions that eliminate the monetary policy trade-off induced by the working-capital constraint.

A more direct proof exemplifies this point even further. Suppose the policy maker sets  $\hat{\pi}_t = \hat{Y}_t = 0, \forall t$ . Then, from the *AS* curve (36) we have  $\hat{R}_t^D = \Psi \hat{\chi}_t$ . To satisfy the *AD* curve (37), the tax instrument should be set to  $\hat{\tau}_t^D = \Psi \hat{\chi}_t - \hat{r}_t^n$  in order to undo any effect of  $\hat{R}_t^D$  on  $\hat{Y}_t$ . Such policy rules would seem natural candidates to implement the optimal policy allocation. After substituting  $\hat{R}_t^D$  and  $\hat{\tau}_t^D$  in (36) and (37), the equilibrium conditions under the above rules can be represented by the system,

$$\begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbb{A}_0 \begin{bmatrix} \mathbb{E}_t \hat{Y}_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \end{bmatrix}, \quad (46)$$

<sup>15</sup>De Fiore and Tristani (2013) also examine optimal monetary policy (under commitment only) in a risk-adjusted cost channel model where a standard costly state verification problem gives rise to an inflationary financial friction. Unlike their model, our focus is on optimal financial interventions in both 'normal times' and in liquidity traps, with an explicit analysis of discretion versus commitment policies.

where,

$$\mathbb{A}_{\mathbb{O}} \equiv \begin{bmatrix} 1 & 1 \\ \frac{\kappa}{(1-\Psi)} & \beta + \frac{\kappa}{(1-\Psi)} \end{bmatrix}.$$

One solution to (46) is indeed the bliss point where  $\hat{\pi}_t = \hat{Y}_t = 0$ . This outcome, however, is not unique, as it can be shown that both eigenvalues of  $\mathbb{A}_{\mathbb{O}}$  cannot lie inside the unit circle. Thus, by the Blanchard and Kahn (1980) conditions, there exists a multiplicity of equilibria because the number of eigenvalues inside the unit circle is smaller than the number of non-predetermined variables. The optimal first-best allocation is only one solution, and there is nothing in the above tax and monetary policy rules that drives the economy back to this desired equilibrium.

This shortcoming leads us to consider the following monetary and macroprudential policy rules,

$$\hat{R}_t^D = \Psi \hat{\chi}_t, \quad (47)$$

$$\hat{\tau}_t^D = \Psi \hat{\chi}_t - \hat{r}_t^n - \phi_\pi \mathbb{E}_t \hat{\pi}_{t+1}, \quad (48)$$

where  $\phi_\pi$  is a coefficient that measures the strength of the financial tax response to variations in *expected* inflation. Under (47) and (48), the implied dynamics are described by,

$$\begin{bmatrix} \hat{Y}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbb{A}_{\mathbb{F}} \begin{bmatrix} \mathbb{E}_t \hat{Y}_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \end{bmatrix}, \quad (49)$$

where,

$$\mathbb{A}_{\mathbb{F}} \equiv \begin{bmatrix} 1 & -(\phi_\pi - 1) \\ \frac{\kappa}{(1-\Psi)} & \beta - \frac{\kappa(\phi_\pi - 1)}{(1-\Psi)} \end{bmatrix}.$$

In this case, an optimal macroprudential policy rule with a forward-looking inflation target satisfying  $\phi_\pi > 1$  guarantees equilibrium uniqueness. For  $\phi_\pi > 1$ , the constrained-efficient allocation is attained as the distinct equilibrium outcome because the eigenvalues of  $\mathbb{A}_{\mathbb{F}}$  lie both inside the unit circle. Unlike the basic New Keynesian model, the Taylor principle is applied to the macroprudential instrument, and is independent of the parameter values. Moreover, for  $\hat{\pi}_t = \hat{Y}_t = 0$ ,  $\forall t$ , and from an *ex-post* perspective, the policy rate and the financial tax satisfy  $\hat{R}_t^D = \Psi \hat{\chi}_t$  and  $\hat{\tau}_t^D = \Psi \hat{\chi}_t - \hat{r}_t^n$ . The presence of a “threat” to adjust the macroprudential tax in reaction to deviations in expected future inflation leads to a determinate equilibrium outcome, and is sufficient to rule out any variations in equilibrium. According to the optimal macroprudential policy rule, a rise in expected inflation warrants a more than one-to-one financial tax cut. The latter, in turn, acts to raise the real interest rate and thus limit fluctuations in output, which would otherwise result in inefficient variations in inflation. In this way, full access to monetary and macroprudential policies, which include a credible signal to modify taxes in response to any deviations in expected inflation, yields the first-best *time-consistent* allocation.

What are the transmission channels of time-consistent optimal policy? Consider first the effects of only an adverse financial shock ( $\hat{\chi}_t < 0$  and  $\hat{r}_t^n = 0$ ). According to (47), the social planner should *lower* the policy rate. In this state of the world, the nominal interest rate curtails the cost-push inflationary impact of the shock, and alleviates the drop in output via a standard intertemporal substitution effect. To prevent inflation escalating due to the monetary expansion, the macroprudential instrument should track the short-run contemporaneous movements in the policy rate (see (48)). All else equal, a financial tax *cut* raises the effective interest rate and incentivizes savings, both of which result in a short-run GDP contraction. The decline in output attributed

to the macroprudential subsidy exerts downward pressure on borrowing costs and consequently on prices due to the credit cost channel. Setting  $\hat{\tau}_t^D = \hat{R}_t^D - \phi_\pi \mathbb{E}_t \hat{\pi}_{t+1}$  keeps the effective savings rate unchanged and thus output at its long-run level. These effects neutralize demand-pull inflation. Overall, the implementation of both macroprudential and monetary policies is crucial for achieving complete inflation and output stability against the backdrop of a financial recession.

Optimal policy following only an adverse shock to the natural rate of interest ( $\hat{r}_t^n < 0$  and  $\hat{\chi}_t = 0$ ) calls for *increasing* the financial tax rate to perfectly offset the negative demand shock, and keeping the nominal policy rate *constant* (see (47) and (48)). Raising the financial tax and keeping the nominal policy rate unchanged lowers the effective real savings rate, thereby disincentivizing savings and encouraging an expansion in output. The improvement in aggregate demand then places upward pressure on the firms marginal and borrowing costs, two intertwined mechanisms that eliminate price deflation. In the basic New Keynesian model without a cost channel, complete output and inflation stabilization could be replicated through the adjustment of only the nominal policy rate, a result also known as the ‘divine coincidence’ (see Blanchard and Galí (2007)). However, in our model, lowering  $\hat{R}_t^D$  in response to a contractionary demand shock creates greater price instability by amplifying deflationary pressures via the credit cost channel, thus worsening the short-run trade-off between output and inflation. Optimal discretionary monetary policy in this state of the world is thus characterized by  $\hat{R}_t^D = 0$ . Overall, a financial tax stands out as a natural and sole policy instrument that can offset the friction and policy trade-offs generated by the working-capital constraint.

These results represent the advantages of macroprudential interventions even in more ‘normal times’ and away from liquidity traps. Not only does unrestricted optimal policy completely stabilize economic shocks and deliver the unique efficient equilibrium, but it also has the benefit of generating a credible signal of reacting to expected inflationary pressures in a time-consistent manner.

### 5.1.2 Policy at the Effective Lower Bounds

We now turn to study the role of discretionary state-contingent macroprudential interventions when the economy enters a liquidity trap. In our framework, the lower bound constraint is imposed on the effective nominal deposit rate,  $\hat{R}_t^D - \hat{\tau}_t^D$ , which may account for the financial tax.

Consider the first case where the effective deposit rate is at its lower bound ( $\hat{\zeta}_{3,t} > 0$ ), and where macroprudential policy is initially not used,  $\hat{\tau}_t^D = 0, \forall t$ . The optimal discretionary target rule in this scenario is,

$$\vartheta \hat{Y}_t = -\frac{(\kappa - \lambda)}{(1 - \Psi)} \hat{\pi}_t - \hat{\zeta}_{3,t}. \quad (50)$$

Thus, for a given variation in inflation, a tighter constraint on  $\hat{R}_t^D = -r^d$ , as measured by  $\hat{\zeta}_{3,t} > 0$ , leads to a more substantial fall in output following a deflationary demand shock. Once at the ZLB and using the slackness constraint (44), the policy rate follows,

$$\hat{R}_t^D = -r^d; \hat{\zeta}_{3,t} > 0. \quad (51)$$

The equilibrium path for inflation and output during the ZLB episode are obtained by substituting (51) in (36) and (37),

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\lambda}{(1 - \Psi)} \left[ (1 + \varphi) \hat{Y}_t - \Psi \hat{\chi}_t - r^d \right], \quad (52)$$

$$\hat{Y}_t = \mathbb{E}_t \hat{Y}_{t+1} + \mathbb{E}_t \hat{\pi}_{t+1} + \hat{r}_t^n + r^d. \quad (53)$$

The rational expectations equilibrium in a liquidity trap that is instigated by only a zero nominal interest rate is then given by equations (50), (52) and (53), taking expectations and the  $AR(1)$  shocks as given.

A few results are worth emphasizing. First, at the ZLB and without macroprudential policy, it is no longer possible to set inflation and output to their optimal first-best levels. Such outcome would require a negative nominal interest rate when the natural rate falls below zero. Second, the direct monetary policy cost channel is irrelevant in a liquidity trap as any dynamic effects of  $\hat{R}_t^D$  disappear from both the  $AD$  and  $AS$  equations. Third, following a large negative demand shock that triggers the ZLB, output, the marginal cost and prices plummet. Beyond this direct *demand-pull* deflationary consequence, the slump in the marginal cost and aggregate demand exerts downward pressure on borrowing costs via the risk premium effect.<sup>16</sup> The fall in the credit spread then magnifies the deflationary impact of the shock and deepens the economic recession by keeping the real policy rate at elevated levels. This amplification effect is captured by  $\Psi$ , as can be inferred from (52), and serves as a *cost-push* deflationary by-product. Finally, re-arranging (50) after substituting in (52) and (53) reveals that  $\hat{\zeta}_{3,t}$  is a negative function of  $\hat{r}_t^n$ , and a positive function of  $\hat{\chi}_t$ . Intuitively, a sizeable negative demand shock that pushes output and inflation in the same direction lowers the natural rate of interest and increases the risk of entering a liquidity trap, hence tightening the ZLB constraint. In contrast, a positive financial shock that lowers inflation acts to lift the real interest rate and further depress aggregate demand. Our model therefore gives rise to a variant of the *paradox of toil* (as popularized by Eggertsson (2010)), wherein otherwise expansionary supply-side shocks can paradoxically lead to lower welfare by amplifying deflationary pressures and keeping the nominal policy rate at its effective lower bound. In our setup, this paradox stems from the existence of the risk-adjusted credit cost channel.

Now suppose the public authority has access to the macroprudential tax. Using conditions (40)-(43), the optimal target rule with  $\hat{\zeta}_{3,t} > 0$  becomes,

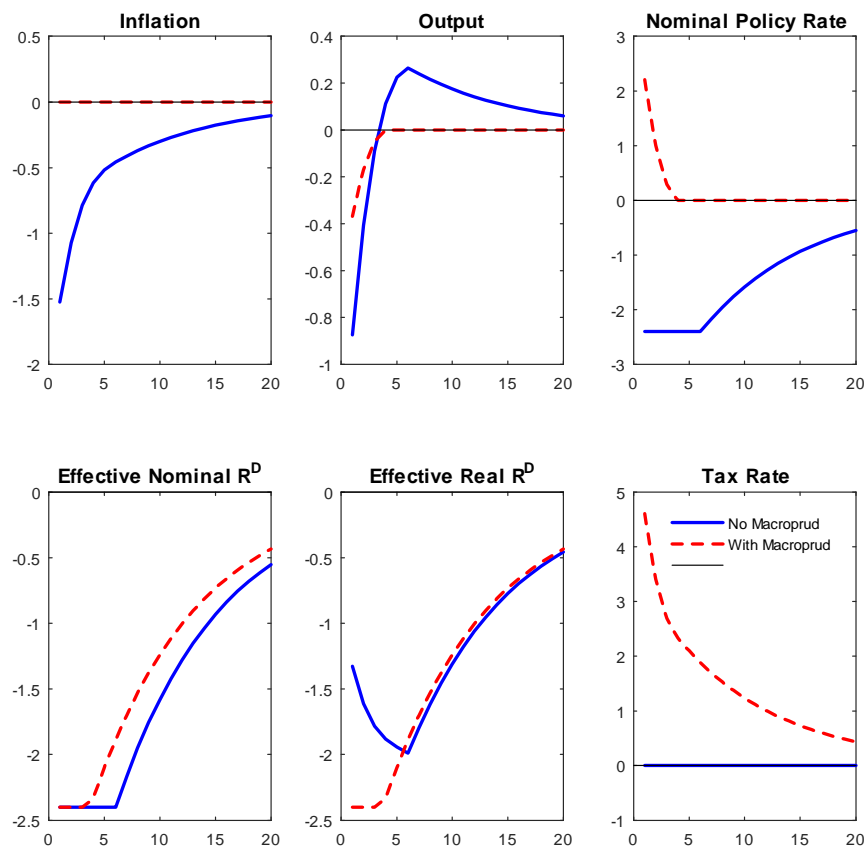
$$\vartheta \hat{Y}_t = -\hat{\zeta}_{3,t}. \quad (54)$$

The financial tax adds the first-order condition  $\hat{\zeta}_{2,t} = \hat{\zeta}_{3,t}$ , which together with (42), removes the policy restriction imposed by the  $AS$  curve ( $\hat{\zeta}_{1,t} = 0$ ). Complete price stability ( $\hat{\pi}_t = 0$ ) is therefore attained with the introduction of  $\hat{\tau}_t^D$ . To build the intuition for this result, we plot the impulse response functions following a sizeable adverse demand shock of magnitude  $8 \times s.d(\alpha^Z)$ , with  $s.d(\alpha^Z) = 0.01$  remaining constant. The comparison is made between the dynamics of the model as implied from target rule (50) with  $\hat{\tau}_t^D = 0, \forall t$  (labeled “No Macroprud”), and the short-run fluctuations arising from the implementation of target rule (54) when  $\hat{\tau}_t^D$  is set optimally (labeled “With Macroprud”). The results are shown in Figure 1.

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<sup>16</sup>Similar to De Fiore and Tristani (2013) and Eggertsson, Juelsrud, Summers and Wold (2019), without credit or supply-side shocks that lead to a rise in borrowing costs, demand shocks generate a positive comovement between credit spreads and output.

Figure 1 - Adverse Demand Shock with Discretionary Policy and the ZLB



Note: Figure 1 displays the equilibrium responses to an adverse demand shock under optimal discretionary policy. Interest rates, inflation and the financial tax rate are measured in annualized percentage point deviations from steady state. Output is measured in annualized percentage deviations.

Enacting macroprudential policies renders the policy maker an extra degree of freedom in stimulating aggregate demand by hiking the financial tax rate that feeds into the household's intertemporal consumption-savings decision. Full price stability is then fostered by a more modest alteration of the nominal policy rate compared to the tax adjustment. In particular, raising  $\hat{\tau}_t^D$  by around 4.5 percentage points, and increasing the nominal policy rate  $\hat{R}_t^D$  by approximately 2 percentage points, *lowers* the effective nominal savings rate ( $\hat{R}_t^D - \hat{\tau}_t^D$ ), thus considerably mitigating the decline in  $\hat{Y}_t$ . At the same time, raising  $\hat{R}_t^D$  lifts borrowing costs, and generates a sufficient cost-push inflationary force that yields complete price stability. Despite the lack of commitment technology, private agents understand that the social planner has both the incentive and now the means to deliver zero inflation each period without creating any additional output distortions. Hence, the expected path of the *real* effective deposit rate tracks the movements of the *nominal* effective deposit rate,

allowing the public authority to completely stabilize output once the economy escapes the liquidity trap. The implementation of the state-contingent financial tax at the ZLB helps to reduce both the scale and the duration of the downturn relative to the benchmark case without macroprudential policy.

To obtain the reduced form expressions for  $\hat{Y}_t$  and  $\hat{\zeta}_{3,t}$  under optimal discretion with financial taxation at the ZLB, combine the optimality condition (54) with (34), (36) and (37), and then impose that private sector expectations are rational,

$$\hat{Y}_t = \frac{1}{(1-p)} \left( \hat{r}_t^n + r^d \right), \quad (55)$$

$$\hat{\zeta}_{3,t} = -\frac{\vartheta}{(1-p)} \left( \hat{r}_t^n + r^d \right), \quad (56)$$

where  $p$  satisfies  $\mathbb{E}_t \hat{Y}_{t+1} = p \hat{Y}_t$ . Unconstrained discretionary policy with macroprudential interventions eliminates the risk of entering a liquidity trap following a financial shock as  $\hat{\chi}_t$  does not enter neither (55) nor (56). Intuitively, without  $\hat{\tau}_t^D$  and away from the ZLB, optimal time-consistent policy in the face of an adverse financial shock warrants a rise in  $\hat{R}_t^D$  to tackle the inflationary component of the credit spread (see discussion below equation (45)). Hence, the ZLB constraint is less consequential. With macroprudential policy, both  $\hat{R}_t^D$  and  $\hat{\tau}_t^D$  must fall in order to bring about complete output and inflation stabilization. For  $\hat{\pi}_t = 0, \forall t$ , the optimal effective savings rate satisfies  $\hat{R}_t^D - \hat{\tau}_t^D = 0$ , insulating the real economy from the inflationary effect that would otherwise follow from the expansionary monetary policy. Given that the effective deposit rate is optimally set to its positive steady state value, the ZLB constraint is removed.

## 5.2 Optimal Policy under Commitment

This section determines the optimal policy plans under commitment. As shown earlier, away from liquidity traps, optimal discretionary policies always achieve the first-best outcome when macroprudential policies are activated. Thus, examining optimal commitment policies with access to  $\hat{\tau}_t^D$  in more ‘normal times’ becomes redundant. The focus of this section is therefore on credible commitment policies with macroprudential interventions when the effective deposit rate is occasionally constrained by the ZLB.

Under commitment, the benevolent public authority chooses state-contingent paths for inflation, output, the nominal policy rate and the financial tax to maximize its objective function (38) subject to constraints (34), (36) and (37), taking  $\hat{r}_t^n$  and  $\hat{\chi}_t$  as given. The associated Lagrangian is,

$$\mathfrak{L}_t = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} -\frac{1}{2} \left( \hat{\pi}_t^2 + \vartheta \hat{Y}_t^2 \right) - \hat{\zeta}_{1,t} \left[ \hat{\pi}_t - \beta \mathbb{E}_t \hat{\pi}_{t+1} - \frac{\lambda}{(1-\Psi)} \left[ \hat{R}_t^D + (1+\varphi) \hat{Y}_t - \Psi \hat{\chi}_t \right] \right] \\ -\hat{\zeta}_{2,t} \left[ \hat{Y}_t - \mathbb{E}_t \hat{Y}_{t+1} + \hat{R}_t^D - \hat{\tau}_t^D - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n \right] - \hat{\zeta}_{3,t} \left[ -\hat{R}_t^D + \hat{\tau}_t^D - r^d \right] \end{array} \right\},$$

Under commitment, the resulting first-order conditions read,

$$-\hat{\pi}_t - \hat{\zeta}_{1,t} + \hat{\zeta}_{1,t-1} + \frac{1}{\beta} \hat{\zeta}_{2,t-1} = 0, \quad (57)$$

$$-\vartheta \hat{Y}_t + \frac{\kappa}{(1-\Psi)} \hat{\zeta}_{1,t} - \hat{\zeta}_{2,t} + \frac{1}{\beta} \hat{\zeta}_{2,t-1} = 0, \quad (58)$$

$$\frac{\lambda}{(1-\Psi)}\hat{\zeta}_{1,t} - \hat{\zeta}_{2,t} + \hat{\zeta}_{3,t} = 0, \quad (59)$$

$$\hat{\zeta}_{2,t} = \hat{\zeta}_{3,t}. \quad (60)$$

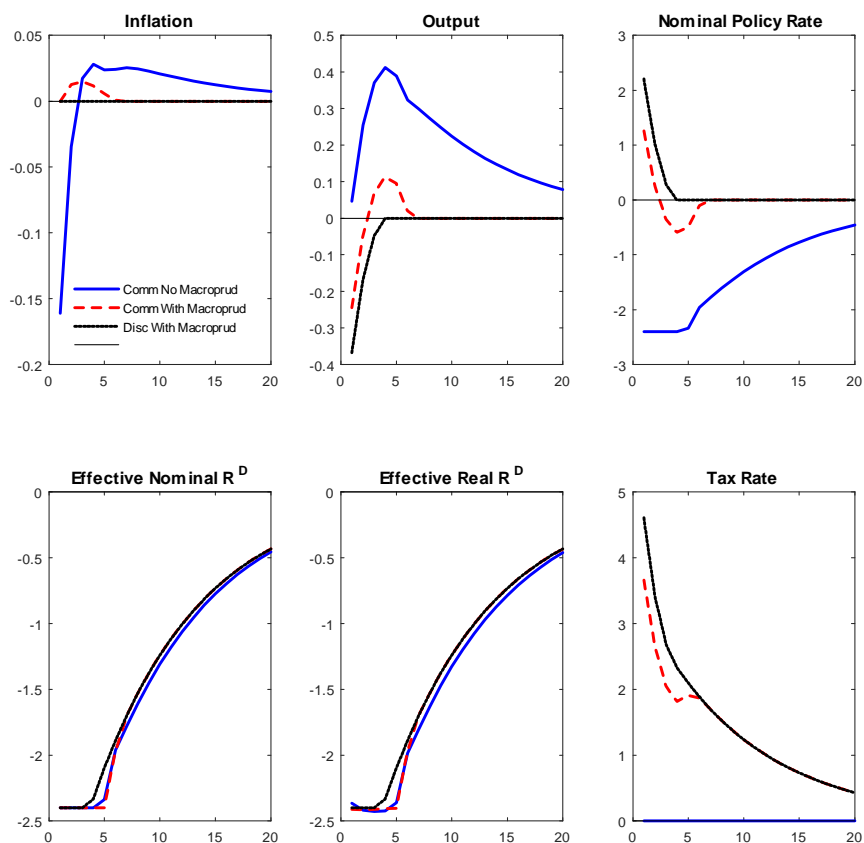
The complementary slackness condition is,

$$\hat{\zeta}_{3,t} \left( -\hat{R}_t^D + \hat{\tau}_t^D - r^d \right) = 0, \quad \hat{\zeta}_{3,t} \geq 0, \quad (61)$$

where the initial conditions satisfy  $\hat{\zeta}_{1,-1} = \hat{\zeta}_{2,-1} = \hat{\zeta}_{3,-1} = 0$ . The optimal state-contingent evolution of the endogenous variables  $\left\{ \hat{\pi}_t, \hat{Y}_t, \hat{R}_t^D, \hat{\tau}_t^D \right\}$  is then characterized by the above first-order conditions together with constraints (36) and (37), as well as the slackness condition (61). Under commitment, optimal policy becomes history-dependent as reflected by the lagged Lagrange multipliers in (57) and (58). These additional state variables reflect “promises” that must be kept from past commitments.

**Demand Shocks.-** Figure 2 presents the optimal responses of the key variables of the model to a negative demand shock of size  $8 \times s.d(\alpha^Z)$ , with the unconditional standard deviation given by  $s.d(\alpha^Z) = 0.01$ . The joint optimal monetary and macroprudential policy plan under commitment (labeled “Comm with Macroprud”) is compared with the corresponding discretionary regime (labeled “Disc with Macroprud”), as well as with the constrained commitment regime that involves only monetary policy as a stabilization tool (labeled “Comm no Macroprud”).

Figure 2 - Adverse Demand Shock with Commitment Policy and the ZLB



Note: Figure 2 displays the equilibrium responses to an adverse demand shock under optimal commitment policies compared with the unconstrained optimal discretionary policy. Interest rates, inflation and the financial tax rate are measured in annualized percentage point deviations from steady state. Output is measured in annualized percentage deviations.

Starting from the examination of optimal monetary policy under commitment ( $\hat{\tau}_t^D = 0$ ), a negative demand shock provokes the policy maker to slash the nominal interest rate and keep it at its lower bound for 4 periods in order to induce a persistent, yet gradual, economic expansion from the first period. At the same time, the interest rate reduction places downward pressure on inflation due to the presence of the risk-adjusted credit cost channel. Compared to Adam and Billi (2006), the existence of the credit cost channel prompts the public authority to drive output above its steady state level for a longer period of time, with the objective of dampening the fall in prices. Because the demand channel dominates the cost channel mechanism, such policy prescription is optimal. In our environment, deflation is amplified due to the drop in both aggregate demand and the credit spread. As a result, deviations in output and inflation persist beyond the life of the shock. By committing to present and future positive output gaps, the policy maker can mitigate



the current and expected deflationary impact of the demand disturbance, thereby improving the output gap-inflation trade-off already from the period of the shock. The added stimulus to the system generated by the promise to keep  $\hat{\pi}_t$  and  $\hat{Y}_t$  positive even after the economy escapes the liquidity trap allows for an earlier lift off of  $\hat{R}_t^D$  from its floor.<sup>17</sup> Overall, the presence of financial frictions warrants a rather substantial and much more persistent output boom compared to the dynamics of output that arises in a standard New Keynesian model without a credit cost channel.

With a credible commitment to adjust both the nominal policy rate and the macroprudential instrument, the dynamics of output and inflation are considerably subdued compared to the case where  $\hat{\tau}_t^D$  is not available. In the scenario where the financial tax is implemented, the adverse demand shock does not require the nominal policy rate to hit its lower bound. Instead, and similar to the discretion case, optimal policy involves an increase in the tax rate and a more subtle initial hike in the nominal policy rate such that only the *effective* savings rate reaches its floor. This policy combination stimulates output via a standard demand channel, and limits deflationary pressures through the credit cost channel effect. The latter mechanism, in turn, is driven by both the relative rise in  $\hat{R}_t^D$  and  $\hat{Y}_t$  which raise borrowing costs and consequently inflation. Moreover, conditions (57)-(61) reveal that once  $\hat{\tau}_t^D$  is accessible, the policy maker commits to future inflation as a substitute for the inability to further lower the effective savings rate. Specifically, for  $\hat{\zeta}_{2,t} = \hat{\zeta}_{3,t}$  and  $\hat{\zeta}_{1,t} = 0$ , promised inflation is positive as  $\hat{\pi}_t = \hat{\zeta}_{1,t-1} + \beta^{-1}\hat{\zeta}_{2,t-1}$ . Note that compared to the discretionary case with macroprudential policy, the effective deposit rate is kept at its floor for 2 additional periods under the unconstrained optimal commitment regime with  $\hat{\tau}_t^D$ . Importantly, the longer and looser anticipated policy mix, involving a modest nominal interest rate cut from the third period, dampens the initial decline in output and inflation but requires a small rise in these two variables for a short period of time in the future. Comparing discretion versus commitment from a welfare perspective, the first few periods more cushioned drop in output under the commitment case offsets the optimal amount of costly above-target promised inflation and output. Quantitatively, unconstrained commitment and discretionary policies with macroprudential interventions yield an *identical* welfare gain of 0.015 percent relative to the constrained commitment policy comprising only of monetary policy.<sup>18</sup>

A macroprudential tax in a liquidity trap, as we advocate for in this model, is in line with the recent unconventional policy attempts taken by the ECB to lower effective deposit rates and to increase credit spreads in light of the persistent low inflation experienced in the Eurozone. We show that a tax on deposits stands out as a natural policy tool to address the inefficiencies associated with liquidity traps instigated by deflationary shocks.

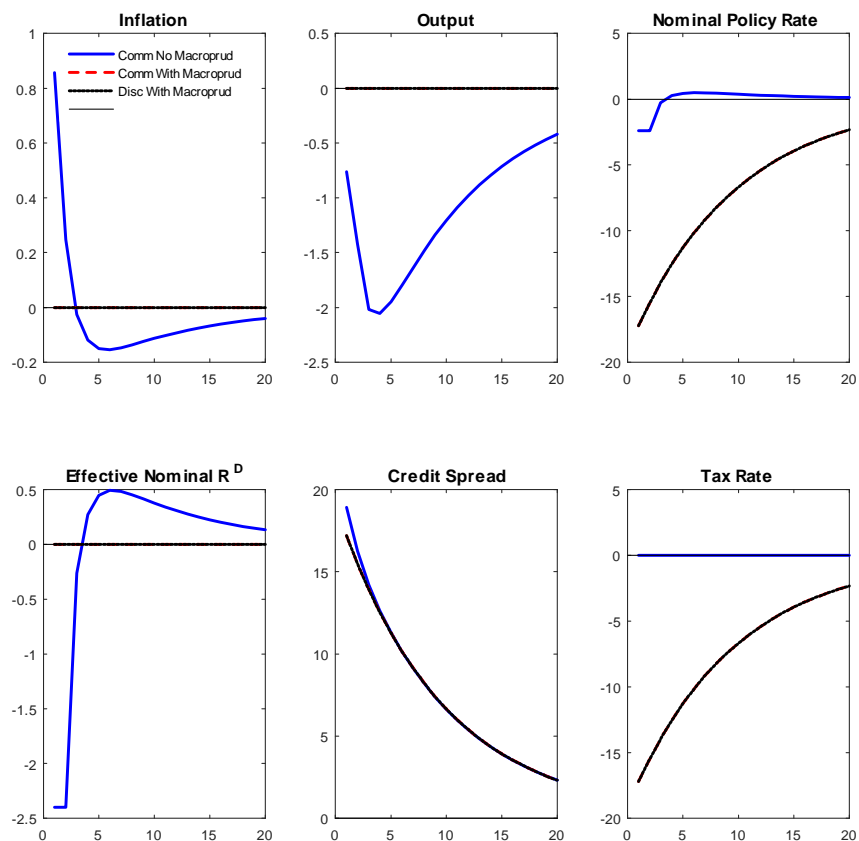
**Financial Shocks.-** Figure 3 displays the optimal responses of the key variables of the model to a significant adverse financial shock of size  $6 \times s.d(\alpha^X)$ , with  $s.d(\alpha^X) = 0.05$  remaining constant. We compare the optimal commitment policy with monetary policy used as the sole stabilization instrument (labeled “Comm no Macroprud”) with the commitment regime involving macroprudential interventions (labeled “Comm with Macroprud”), as well as with the discretionary case that also includes macroprudential policy (labeled “Disc with Macroprud”).

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<sup>17</sup>Applying our calibration values and shock moments to a standard New Keynesian model with the ZLB and no cost channel (i.e., Eggertsson and Woodford (2003) and Adam and Billi (2006)), would result in the policy rate being released from its lower bound after 6 periods, as opposed to 4 periods in our model. This extra simulation is available upon request.

<sup>18</sup>The welfare gain is measured in terms of the equivalent permanent increase in consumption. Furthermore, these welfare gains are the same up to the 8<sup>th</sup> decimal point. Thus, we comfortably argue that time-consistent and Ramsey plans with financial taxation are coequal from a *quantitative welfare* perspective.

Figure 3 - Adverse Financial Shock with Commitment Policy and the ZLB



Note: Figure 3 displays the equilibrium responses to an adverse financial shock under optimal commitment policies compared with the unconstrained optimal discretionary policy. Interest rates, inflation and the financial tax rate are measured in annualized percentage point deviations from steady state. Output is measured in annualized percentage deviations.

Under constrained commitment with monetary policy, an adverse cost-push financial shock requires a *cut* in the nominal interest rate despite the inflationary pressures precipitated by the rise in the credit spread. This result comes in stark contrast to the constrained discretionary policy outcome that warrants a hike in the nominal interest rate, as discussed above. Similar to De Fiore and Tristani (2013), a credit shock that directly raises borrowing costs leads to an inefficient and entirely undesirable slump in output. The downward pressure on real wages generated by the spike in inflation discourages both labour supply and the demand for consumption goods, resulting in a persistent contraction in the output gap. A large inefficient credit disturbance therefore sends the nominal policy rate to its lower bound for 2 periods, with the accommodative monetary policy helping to smooth the adjustment of output at the expense of short-lived inflationary pressures.

As soon as the economy starts recovering, the social planner promises to generate mild deflation in the future, which helps to alleviate the cost-push inflationary force in the first few periods. Thus, our model can explain why nominal policy rates have been hovering around their lower bounds also in response to inflationary financial shocks, as well as the ‘missing deflation puzzle’ observed during the Great Recession. The peculiar nature of the supply-side financial shock in our framework generates both a countercyclical credit spread and a negative comovement between inflation and output.

Under unconstrained commitment policies, direct macroprudential interventions allow for an *unrestricted* reduction in the nominal policy rate that, in combination, insulate the economy from the adverse repercussions of the credit shock. As in the case of discretion,  $\hat{R}_t^D$  should be lowered one-to-one with respect to the cut in  $\hat{\tau}_t^D$  such that the effective savings rate remains constant and at its positive steady state level. In this way, both output and inflation are completely stabilized despite the high credit spread. Notice that while the loan rate is completely stabilized at its long-run positive value ( $\hat{R}_t^L = 0$ ), the credit spread remains elevated due to the sharp fall in  $\hat{R}_t^D$ . The monetary expansion directly cushions the cost-push effects generated by the otherwise higher borrowing costs, while the financial tax cut prevents any demand-pull inflationary pressures. This optimal policy prescription holds regardless of whether the economy enters a liquidity trap or not. Furthermore, the identical optimal dynamics implied from the unconstrained Ramsey and time-consistent policies with macroprudential interventions yield an equivalent welfare gain of 0.066 percent relative to the constrained optimal monetary policy commitment case. Unconventional financial policies remove the ZLB constraint for monetary policy, and enable the policy maker to set negative nominal interest rates *without* violating the household’s no-arbitrage condition between deposits and holding cash for consumption purposes. Such policies are not inconsistent with the practices of some central banks in advanced economies which have set unprecedented negative nominal interest rates with the aim to stimulate aggregate demand. Our model shows that these policies are indeed feasible so long as financial policies are implemented correctly and in a state-contingent fashion.

## 6 Concluding Remarks

This paper has studied the properties of optimal time-consistent and Ramsey macroprudential policies in the context of a stylized New Keynesian model modified for a credit cost channel, endogenous credit spreads and effective lower bound constraints. The model sheds new insights on the stabilization roles and transmission mechanisms of macroprudential interventions both in a liquidity trap and away from it. We have shown that varying the financial tax according to the state of the business and financial cycles has meaningful effects on the behaviour of key macroeconomic variables, and substantially alters the transmission of optimal monetary policy under both discretion and commitment.

The distinctive supply-side financial frictions highlighted in this paper present an additional motivation for activating state-contingent macroprudential policies. In ‘normal circumstances’, financial interventions solve the policy trade-offs that arise due the presence of the credit cost channel, and help to accomplish full stabilization of output and inflation. In a liquidity trap, financial policies unleash the restrictions imposed on the nominal policy rate, and substantially diminish the adverse consequences of both negative demand and financial shocks. Finally, the positive and normative implications of optimal unconstrained time-consistent policies with macroprudential interventions are remarkably similar to their Ramsey counterparts. These results suggest that commitment poli-

cies are of secondary importance so long as the policy maker can optimally alter the financial tax on loanable funds.

Like Correia, Farhi, Nicolini and Teles (2013), Correia, De Fiore, Teles and Tristani (2019) and Eichenbaum (2019), our state-dependent policy recommendations require taxes to be highly adaptable in times of economic uncertainty. It is well known that fiscal and financial policy tools are not as versatile as monetary policy instruments, and require a long legislative process until they can actually be executed. Writing automatic stabilizer financial programs into law during tranquil times could circumvent these political and economic challenges faced by policy makers in the midst of a crisis. The recent Great Recession nonetheless has led to somewhat more flexibility in terms of implementing fiscal and financial policies, despite the main focus still placed on government spending since the American Recovery and Reinvestment Act (ARRA) of 2009, and countercyclical regulation associated with the gradual imposition of Basel III. Either way, we make a normative point that financial tax policies should be at least as proactive as monetary policy, so long as the policy maker can correctly identify the source and the size of the shock distorting the economy.

## References

- [1] Adam, Klaus, and Roberto M. Billi. (2006). “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates.” *Journal of Money, Credit and Banking*, 38(7), 1877-1905.
- [2] Adam, Klaus, and Roberto M. Billi. (2007). “Discretionary monetary policy and the zero lower bound on nominal interest rates.” *Journal of Monetary Economics*, 54(3), 728-752.
- [3] Benigno, Pierpaolo, Gauti B. Eggertsson, and Federica Romei. (2019). “Dynamic Debt Deleveraging and Optimal Monetary Policy.” *American Economic Journal: Macroeconomics*; forthcoming.
- [4] Bianchi, Javier, and Enrique G. Mendoza. (2018). “Optimal Time-Consistent Macroprudential Policy.” *Journal of Political Economy* 126(2), 588-634.
- [5] Blanchard, Olivier, and Jordi Galí. (2007). “Real Wage Rigidities and the New Keynesian Model.” *Journal of Money, Credit and Banking*, 39(s1), 35-65.
- [6] Blanchard, Olivier, and Charles M. Kahn. (1980). “The solution of linear difference models under rational expectations.” *Econometrica*, 48(5), 1305-1311.
- [7] Calvo, Guillermo A. (1983). “Staggered Prices in a Utility-Maximizing Framework.” *Journal of Monetary Economics*, 12, 383-398.
- [8] Carlstrom, Charles T., and Timothy S. Fuerst. (1997). “Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis.” *American Economic Review*, 87, 893-910.
- [9] Christiano, Lawrence J. and Martin S. Eichenbaum. (1992). “Liquidity Effects and the Monetary Transmission Mechanism.” *American Economic Review Papers and Proceedings*, 82, 346-353.

- [10] Christiano, Lawrence J., Martin S. Eichenbaum, and Sergio Rebelo. (2011). “When is the government spending multiplier large?.” *Journal of Political Economy*, 119(1), 78-121.
- [11] Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. (2015). “Understanding the Great Recession.” *American Economic Journal: Macroeconomics*, 7(1), 110–167.
- [12] Collard, Fabrice, Harris Dellas, Behzad Diba, and Olivier Loisel. (2017). “Optimal monetary and prudential policies.” *American Economic Journal: Macroeconomics*, 9(1), 40-87.
- [13] Conti, Antonio Maria, Stefano Neri, and Andrea Nobili. (2017). “Low Inflation and monetary policy in the euro area.” Working Paper Series 2005, European Central Bank.
- [14] Correia, Isabel, Fiorella De Fiore, Pedro Teles, and Oreste Tristani. (2019). “Credit Subsidies.” *Journal of Monetary Economics*; forthcoming.
- [15] Correia, Isabel, Emmanuel Farhi, Juan Pablo Nicolini, and Pedro Teles. (2013). “Unconventional Fiscal Policy at the Zero Bound.” *American Economic Review*, 103(4), 1172-1211.
- [16] Cúrdia, Vasco, and Michael Woodford. (2016). “Credit Frictions and Optimal Monetary Policy.” *Journal of Monetary Economics*, 84, 30-65.
- [17] Cúrdia, Vasco, and Michael Woodford. (2011). “The central-bank balance sheet as an instrument of monetary policy.” *Journal of Monetary Economics*, 58(1), 54-79.
- [18] De Fiore, Fiorella, and Oreste Tristani. (2013). “Optimal Monetary Policy in a Model of the Credit Channel.” *The Economic Journal*, 123(571), 906-931.
- [19] De Paoli, Bianca, and Matthias Paustian. (2017). “Coordinating Monetary and Macroprudential Policies.” *Journal of Money, Credit and Banking*, 49 (2-3), 319-349.
- [20] Demirel, Ufuk D. (2009). “Optimal monetary policy in a financially fragile economy.” *The B.E. Journal of Macroeconomics*, 9(1), Article 15.
- [21] Dixit, Avinash K., and Joseph E. Stiglitz. (1977). “Monopolistic Competition and Optimum Product Diversity.” *American Economic Review*, 67, 297-308.
- [22] Eggertsson, Gauti B. (2010). “The Paradox of Toil.” Federal Reserve Bank of New York Staff Report 443.
- [23] Eggertsson, Gauti B. (2011). “What fiscal policy is effective at zero interest rates?.” *NBER Macroeconomics Annual 2010*, 25, 59-112.
- [24] Eggertsson, Gauti B., Ragnar E. Juelsrud, Lawrence H. Summers, and Ella G. Wold. (2019). “Negative Nominal Interest Rates and the Bank Lending Channel.” National Bureau of Economic Research (NBER) working paper number w25416.
- [25] Eggertsson, Gauti B., and Michael Woodford. (2003). “The Zero Bound on Interest Rates and Optimal Monetary Policy.” *Brookings Papers on Economic Activity*, 1, 139–233.
- [26] Eggertsson, Gauti B., and Michael Woodford. (2006). “Optimal Monetary and Fiscal Policy in a Liquidity Trap.” In *NBER International Seminar on Macroeconomics 2004*, edited by Richard H. Clarida, Jeffrey Frankel, Francesco Giavazzi, and Kenneth D. West, 75–144. Cambridge, MA: MIT Press.

- [27] Eichenbaum, Martin S. (2019). “Rethinking Fiscal Policy in an Era of Low Interest Rates.” Northwestern University, mimeo.
- [28] Farhi, Emmanuel, and Iván Werning. (2016). “A Theory of Macroprudential Policies in the Presence of Nominal Rigidities.” *Econometrica*, 84(5), 1645-1704.
- [29] Fernández-Villaverde, Jesús. (2010). “Fiscal policy in a model with financial frictions.” *American Economic Review Papers and Proceedings*, 100(2), 35-40.
- [30] Friedman, Milton. (1969). “The Optimum Quantity of Money.” Aldine, Chicago.
- [31] Galí, Jordi. (2015). “Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications.” Princeton University Press.
- [32] Gertler, Mark, and Peter Karadi. (2011). “A model of unconventional monetary policy.” *Journal of Monetary Economics*, 58(1), 17-34.
- [33] Gilchrist, Simon, Raphael Schoenle, Jae Sim, and Egon Zakrajšek. (2017). “Inflation Dynamics During the Financial Crisis.” *American Economic Review*, 107(3), 785-823.
- [34] Guerrieri, Luca, and Matteo Iacoviello. (2015). “OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily.” *Journal of Monetary Economics*, 70, 22-38.
- [35] Holden, Tom. (2016). “Computation of solutions to dynamic models with occasionally binding constraints.” EconStor Preprints 144569, ZBW - Leibniz Information Centre for Economics.
- [36] Jermann, Urban, and Vincenzo Quadrini. (2012). “Macroeconomic effects of financial shocks.” *American Economic Review*, 102(1), 238-71.
- [37] Korinek, Anton, and Alp Simsek. (2016). “Liquidity Trap and Excessive Leverage.” *American Economic Review*, 106(3), 699-738.
- [38] Mertens, Karel RSM, and Morten O. Ravn. (2014). “Fiscal Policy in an Expectations-Driven Liquidity Trap.” *The Review of Economic Studies*, 81(4), 1637-1667.
- [39] Ravenna, Federico, and Carl E. Walsh. (2006). “Optimal Monetary Policy with the Cost Channel.” *Journal of Monetary Economics*, 53, 199-216.
- [40] Silvo, Aino. (2019). “The Interaction of Monetary and Macroprudential Policies.” *Journal of Money, Credit and Banking*, 51(4), 859-894.
- [41] Taylor, William J., and Roy Zilberman. (2016). “Macroprudential regulation, credit spreads and the role of monetary policy.” *Journal of Financial Stability*, 26(c), 144-158.
- [42] Woodford, Michael. (2003). “Interest and Prices: Foundations of a Theory of Monetary Policy.” Princeton University Press.

## 7 Appendix - Welfare Function Derivation (Not for publication)

The derivation of the loss function as presented in the paper strictly follows Woodford (2003) and Galí (2015). To derive a second-order approximation of the representative utility function, it is first useful to clarify some additional notation. For any variable  $\mathbb{X}_t$ , let  $\mathbb{X}$  be its steady state value,  $\tilde{\mathbb{X}}_t = \mathbb{X}_t - \mathbb{X}$  be the deviation of  $\mathbb{X}_t$  around its steady state, and finally  $\hat{\mathbb{X}}_t = \log(\mathbb{X}_t/\mathbb{X})$  be the log-deviation of  $\mathbb{X}_t$  around its corresponding steady state. Using a second order Taylor approximation, the variables  $\tilde{\mathbb{X}}_t$  and  $\hat{\mathbb{X}}$  can be related through the following equation,

$$\frac{\mathbb{X}_t}{\mathbb{X}} = 1 + \log\left(\frac{\mathbb{X}_t}{\mathbb{X}}\right) + \frac{1}{2}\left[\log\left(\frac{\mathbb{X}_t}{\mathbb{X}}\right)\right]^2 = 1 + \hat{\mathbb{X}}_t + \frac{1}{2}\hat{\mathbb{X}}_t^2. \quad (62)$$

As we can write  $\tilde{\mathbb{X}}_t = \mathbb{X}\left(\frac{\mathbb{X}_t}{\mathbb{X}} - 1\right)$ , it follows that  $\tilde{\mathbb{X}}_t \approx \mathbb{X}\left(\hat{\mathbb{X}}_t + \frac{1}{2}\hat{\mathbb{X}}_t^2\right)$ .

Utility is assumed to be separable in consumption and hours worked,

$$\mathbb{U}_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{Z_t C_t^{1-\sigma}}{1-\sigma} - \frac{Z_t N_t^{1+\varphi}}{1+\varphi} \right\}. \quad (63)$$

where  $U_t(C_t, Z_t) \equiv \frac{Z_t C_t^{1-\sigma}}{1-\sigma}$  and  $V_t(N_t, Z_t) \equiv \frac{Z_t N_t^{1+\varphi}}{1+\varphi}$ . We start by approximating the utility from consumption. With the steady state value of the demand shock ( $Z$ ) equal to 1, the second order expansion for  $U(C_t, Z_t)$  yields,

$$\begin{aligned} U(C_t, Z_t) &\approx U(C, 1) + U_C(C, 1)\tilde{C}_t + \frac{1}{2}U_{CC}(C, 1)\tilde{C}_t^2 + \\ &U_Z(C, 1)\tilde{Z}_t + \frac{1}{2}U_{ZZ}\tilde{Z}_t^2 + U_{CZ}\tilde{Z}_t\tilde{C}_t. \end{aligned} \quad (64)$$

Using  $\tilde{Z}_t \approx \hat{Z}_t$ ,  $U_{CC}(C, 1) = -\sigma U_C(C, 1)C^{-1}$  and  $U_{CZ} = U_C(C, 1)$ , the above becomes,

$$\begin{aligned} U(C_t, Z_t) &\approx U(C, 1) + U_C(C, 1)C \left( \hat{C}_t + \frac{1}{2}\hat{C}_t^2 \right) - \frac{1}{2}\sigma U_C(C, 1)C \left( \hat{C}_t + \frac{1}{2}\hat{C}_t^2 \right)^2 + \\ &+ U_Z(C, 1)\hat{Z}_t + \frac{1}{2}U_{ZZ}\hat{Z}_t^2 + U_C(C, 1)\hat{Z}_tC \left( \hat{C}_t + \frac{1}{2}\hat{C}_t^2 \right). \end{aligned}$$

Ignoring the terms  $\mathbb{X}^i$  for  $i > 2$  yields,

$$\begin{aligned} U(C_t, Z_t) - U(C, 1) &\approx U_C(C, 1)C \left[ \left(1 + \hat{Z}_t\right) \hat{C}_t + \frac{(1-\sigma)}{2}\hat{C}_t^2 \right] + \\ &+ \left[ U_Z(C, 1)\hat{Z}_t + \frac{1}{2}U_{ZZ}\hat{Z}_t^2 \right]. \end{aligned} \quad (65)$$

We next derive an expression for the disutility from labour. The Taylor expansion for  $V(N_t, Z_t)$  gives,

$$\begin{aligned} V(N_t, Z_t) &\approx V(N, 1) + V_N(N, 1)\tilde{N}_t + \frac{1}{2}V_{NN}(N, 1)\tilde{N}_t^2 + \\ &+ V_Z(N, 1)\tilde{Z}_t + \frac{1}{2}V_{ZZ}\tilde{Z}_t^2 + V_{NZ}\tilde{Z}_t\tilde{N}_t. \end{aligned} \quad (66)$$

Applying  $\tilde{Z}_t \approx \hat{Z}_t$ , and ignoring the terms  $\mathbb{X}^i$  for  $i > 2$  results in,

$$\begin{aligned} V(N_t, Z_t) \approx & V(N, 1) + V_N(N, 1)N \left( \hat{N}_t + \frac{1}{2}\hat{N}_t^2 \right) + \frac{1}{2}V_{NN}(N, 1)N^2 \left( \hat{N}_t + \frac{1}{2}\hat{N}_t^2 \right)^2 + \\ & + V_Z(N, 1)\hat{Z}_t + \frac{1}{2}V_{ZZ}\hat{Z}_t^2 + V_{NZ}\hat{Z}_tN \left( \hat{N}_t + \frac{1}{2}\hat{N}_t^2 \right). \end{aligned}$$

Using  $V_{NN}(N, 1) = \varphi V_N(N, 1)N^{-1}$  and  $V_{NZ} = V_N(N, 1)$  yields,

$$\begin{aligned} V(N_t, Z_t) - V(N, 1) \approx & V_N(N, 1)N \left[ \left(1 + \hat{Z}_t\right) \hat{N}_t + \frac{(1 + \varphi)}{2}\hat{N}_t^2 \right] \\ & + V_Z(N, 1)\hat{Z}_t + \frac{1}{2}V_{ZZ}\hat{Z}_t^2. \end{aligned} \quad (67)$$

Subtracting (67) from (65) gives,

$$\begin{aligned} U(C_t, Z_t) - U(C, 1) - V(N_t, \vartheta_t) + V(N, 1) = & U_C(C, 1)C \left[ \left(1 + \hat{Z}_t\right) \hat{C}_t + \frac{(1 - \sigma)}{2}\hat{C}_t^2 \right] + \\ & + U_Z(C, 1)\hat{Z}_t + \frac{1}{2}U_{ZZ}\hat{Z}_t^2 - \\ & - V_N(N, 1)N \left[ \left(1 + \hat{Z}_t\right) \hat{N}_t + \frac{(1 + \varphi)}{2}\hat{N}_t^2 \right] - \\ & - V_Z(N, 1)\hat{Z}_t - \frac{1}{2}V_{ZZ}\hat{Z}_t^2, \end{aligned}$$

or using the notation for the utility function  $\mathbb{U}_t$ , and applying  $\hat{C}_t = \hat{Y}_t$  and  $C = H = Y$ ,

$$\frac{\mathbb{U}_t - \mathbb{U}}{U_C(C, 1)C} = \left(1 + \hat{Z}_t\right) \hat{Y}_t + \frac{(1 - \sigma)}{2}\hat{Y}_t^2 - \frac{V_N(N, 1)}{U_C(C, 1)} \left[ \left(1 + \hat{Z}_t\right) \hat{N}_t + \frac{(1 + \varphi)}{2}\hat{N}_t^2 \right] + tip, \quad (68)$$

where  $tip \equiv \frac{1}{U_C(C, 1)C} \left( U_Z(Y, 1)\hat{Z}_t - V_Z(N, 1)\hat{Z}_t + \frac{1}{2}U_{ZZ}\hat{Z}_t^2 - \frac{1}{2}V_{ZZ}\hat{Z}_t^2 \right)$  are terms independent of policy.

Recall that the price dispersion index is  $\Delta_t \equiv \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} dj$ , which up to a first-order is,

$$\hat{\Delta}_t = \ln \left[ \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} dj \right] \approx 0. \quad (69)$$

At the second-order, this result can no longer be used. The following second-order approximation of  $\left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon}$  will be useful, where  $\hat{p}_{j,t} \equiv \hat{P}_{j,t} - \hat{P}_t$  is approximated around zero:

$$\left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} \approx 1 + (1 - \epsilon)\hat{p}_{j,t} + \frac{1}{2}(1 - \epsilon)^2\hat{p}_{j,t}^2.$$

From  $P_t = \left[ \int_0^1 P_{j,t}^{1-\epsilon} dj \right]^{1/(1-\epsilon)}$ , we have that  $1 = \left[ \int_0^1 \left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} dj \right]^{\frac{1}{(1-\epsilon)}}$ . Thus, taking expectations



from both sides yields,

$$1 = \left[ 1 + (1 - \epsilon) \mathbb{E}_t \hat{p}_{j,t} + \frac{1}{2} (1 - \epsilon)^2 \mathbb{E}_t \hat{p}_{j,t}^2 \right],$$

or,

$$\mathbb{E}_t \hat{p}_{j,t} = -\frac{(1 - \epsilon)}{2} \mathbb{E}_t \hat{p}_{j,t}^2 = -\frac{(1 - \epsilon)}{2} \text{var}_j p_{j,t}, \quad (70)$$

where the price dispersion is denoted by  $\text{var}_j p_{j,t}$ . Now, the second-order approximation of  $\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon}$  is,

$$\left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} \approx 1 - \epsilon \hat{p}_{j,t} + \frac{1}{2} \epsilon^2 \hat{p}_{j,t}^2. \quad (71)$$

Finally, insert (71) and (70) into (69) to get the second-order approximation of  $\Delta_t$ ,

$$\hat{\Delta}_t = \ln \left[ \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} dj \right] \approx \ln \left\{ \int_0^1 \left[ 1 - \epsilon \hat{p}_{j,t} + \frac{1}{2} \epsilon^2 \hat{p}_{j,t}^2 \right] dj \right\}.$$

After some algebra, the above yields,

$$\hat{\Delta}_t \approx \frac{\epsilon}{2} \text{var}_j p_{j,t}. \quad (72)$$

Ignoring the terms  $\mathbb{X}^i$  for  $i > 2$ , and substituting (72) and  $\hat{N}_t = \hat{Y}_t + \hat{\Delta}_t$  in (68) yields,

$$\begin{aligned} \frac{\mathbb{U}_t - \mathbb{U}}{U_C(C, 1)C} &= \frac{(1 - \sigma)}{2} \hat{Y}_t^2 - \frac{V_N(N, 1)}{U_C(C, 1)} \left[ \frac{\epsilon}{2} \text{var}_j p_{j,t} + \frac{(1 + \varphi)}{2} \hat{Y}_t^2 \right] \\ &+ \left[ 1 - \frac{V_N(N, 1)}{U_C(C, 1)} \right] \hat{Y}_t + \left[ 1 - \frac{V_N(N, 1)}{U_C(C, 1)} \right] \hat{Z}_t \hat{Y}_t + \text{tip}. \end{aligned} \quad (73)$$

Note that the steady state labour market equilibrium condition is,

$$\frac{V_N(N, 1)}{U_C(C, 1)} = Y^{\sigma + \varphi} = \frac{\beta (1 - \tau^D)}{\nu} (pm)^{-1}, \quad (74)$$

where  $pm \equiv \frac{\epsilon}{(\epsilon - 1)}$  is the price mark-up. The efficient steady state implies that  $\frac{V_N}{U_C} = Y^{\sigma + \varphi} = MPN = 1$ . However, as explained in the main text under standard parameterization, the first-best optimal macroprudential policy ( $\tau^{D,I} < 0$ ) would require a non-feasible negative loan rate. As a result,  $\frac{V_H}{U_C} < 1$  so the first-best is not attainable with financial taxation. In order to make our welfare analysis transparent with a focus on the short-run stabilization roles of macroprudential policies, we set  $\tau^D = 0$  in the long-run, and introduce a labour subsidy  $\tau^N$  that satisfies,

$$Y^{\sigma + \varphi} = (1 - \tau^N) \frac{\beta}{\nu} (pm)^{-1}.$$

In this way, setting  $\tau^N = 1 - \frac{(pm)\nu}{\beta}$  eliminates all steady state distortions without violating any of the lower bound constraints of the model. We can then use  $\frac{V_N}{U_C} = Y^{\sigma + \varphi} = 1$  in (73) which yields,

$$\frac{\mathbb{U}_t - \mathbb{U}}{U_C(C, 1)C} = -\frac{\epsilon}{2} \text{var}_j p_{j,t} - \frac{1}{2} (\varphi + \sigma) \hat{Y}_t^2 + \text{tip}. \quad (75)$$

The next step consists in rewriting  $var_j p_{j,t}$  as a function of inflation. Note that because only a fraction of  $\theta$  firms keep their prices fixed while a portion of  $1 - \theta$  producers reset prices optimally. We can rewrite the expected price for good  $j$  as,

$$\mathbb{E}_{j,t} P_{j,t} = (1 - \theta) P_t^* + \theta P_{j,t-1},$$

or,

$$P_t^* = \frac{1}{(1 - \theta)} (\mathbb{E}_{j,t} P_{j,t} - \theta P_{j,t-1}). \quad (76)$$

From basic statistics, we can write the price dispersion measure as,

$$var_j p_{j,t} = \mathbb{E}_{j,t} \left[ (P_{j,t} - P_{j,t-1})^2 \right] - (\mathbb{E}_{j,t} P_{j,t} - P_{j,t-1})^2. \quad (77)$$

Moreover, because only  $(1 - \theta)$  firms are able to reset their prices,

$$\mathbb{E}_{j,t} \left[ (P_{j,t} - P_{j,t-1})^2 \right] = \theta (P_{j,t-1} - P_{j,t-1})^2 + (1 - \theta) (P_t^* - P_{j,t-1})^2. \quad (78)$$

Substitute (78) and (76) in (77) to get,

$$var_j p_{j,t} = \theta (P_{j,t-1} - P_{j,t-1})^2 + (1 - \theta) (P_t^* - P_{j,t-1})^2 - (\mathbb{E}_{j,t} P_{j,t} - P_{j,t-1})^2,$$

or,

$$var_j p_{j,t} = \theta (P_{j,t-1} - P_{j,t-1})^2 + \frac{\theta}{(1 - \theta)} (\mathbb{E}_{j,t} P_{j,t} - P_{j,t-1})^2,$$

or,

$$var_j p_{j,t} \approx \theta (var_j p_{j,t-1}) + \left( \frac{\theta}{1 - \theta} \right) \hat{\pi}_t^2.$$

Iterating backwards and collecting terms for every period  $s$  results in,

$$var_j p_{j,t} \approx \sum_{s=0}^t \theta^s \left( \frac{\theta}{1 - \theta} \right) \hat{\pi}_{t-s}^2.$$

Taking the discounted value of these terms over all periods yields,

$$\sum_{t=0}^{\infty} \beta^t var_j p_{j,t} = \sum_{t=0}^{\infty} \beta^t \theta^s \left( \frac{\theta}{1 - \theta} \right) \hat{\pi}_t^2 = \left[ \frac{\theta}{(1 - \theta)(1 - \theta\beta)} \right] \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2. \quad (79)$$

Inserting (79) into (75) and taking the sum over all discounted time periods,

$$\sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_C(C, 1)C} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{\epsilon}{\lambda} \hat{\pi}_t^2 + (\varphi + \sigma) \hat{Y}_t^2 \right] \right\} + tip, \quad (80)$$

where  $\lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ . With  $\sigma = 1$  and  $Y = C$ , the present discounted value of the representative household welfare is,

$$\mathcal{W}_t \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t \approx U - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{L}_t. \quad (81)$$

The period losses are given by,

$$\mathcal{L}_t = \frac{1}{2} \left[ \frac{\epsilon}{\lambda} \hat{\pi}_t^2 + (1 + \varphi) \hat{Y}_t^2 \right]. \quad (82)$$

### Welfare Measure

We measure the welfare benefit of a particular optimal policy  $j$  as a fraction of the consumption path under the benchmark optimal monetary policy case that must be given up in order to obtain the benefits of welfare associated with the various joint optimal macroprudential and monetary policy;  $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathbb{U} \left( C_t^j, N_t^j \right) = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathbb{U} \left( (1 - \Upsilon) C_t^I, N_t^I \right)$ . Given the utility function adopted and with  $\sigma = 1$ , the expression for  $\Upsilon$  in percentage terms is,

$$\Upsilon = \left\{ 1 - \exp \left[ (1 - \beta) \left( \mathcal{W}_t^j - \mathcal{W}_t^I \right) \right] \right\} \times 100,$$

where  $\mathcal{W}_t^j = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathbb{U} \left( C_t^j, N_t^j \right)$  represents the unconditional expectation of lifetime utility under joint optimal macroprudential and monetary policy  $j$ , and  $\mathcal{W}_t^I = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathbb{U} \left( C_t^I, N_t^I \right)$  is the welfare associated with the benchmark optimal monetary policy regime. Converting the loss function to the welfare measure gives,

$$\mathcal{W}_t \equiv \mathbb{U} - \frac{1}{2} \frac{\mathbb{U}_{CC}}{(1 - \beta)} \left[ \left( \frac{\epsilon}{\lambda} \right) \text{var}(\hat{\pi}_t) + (1 + \varphi) \text{var}(\hat{Y}_t) \right].$$