A SIMULATION MODEL OF CONSUMER SPENDING AND HOUSING DEMAND

Peter F Westaway

Discussion Paper no. 15

A Simulation Model of Consumer Spending and Housing Demand

by

Peter F. Westaway

May 1992

This paper adopts a micro-to-macro approach to examine the determination of aggregate personal sector behaviour, notably of consumer spending and housing demand. Previous analysis of the impact of borrowing constraints on the simple finite horizon life cycle model of individual consumption is extended to include consideration of housing demand financed by mortgage loans and of bequests. The implied aggregate behaviour of consumer spending and housing demand is derived by simulating an overlapping generations model. Attention is focussed on the effects of the relaxation of borrowing constraints, on intergenerational transfers of wealth and on the implications of real growth in house prices. The consequences for the aggregate saving ratio and wealth to income ratios are described.

National Institute of Economic and Social Research
2 Dean Trench Street
Smith Square
London SW1P 3HE

A Simulation Model of Consumer Spending and Housing Demand

by P.F. Westaway†

1 Introduction

It is well known that any understanding of macroeconomic developments in the UK economy over the last decade depends crucially on the ability to explain personal sector behaviour over that time. Both in the boom in the second half of the 1980s and in the current recession, a central role was played by the changes in consumer spending behaviour and by associated developments in the housing market. Moreover, due to the shortcomings of the econometric models of this behaviour, policymakers and macroeconomic forecasters failed to predict adequately either the scale of the boom or the depth of the ensuing slump.

This paper is motivated by the view that these shortcomings are directly caused by the loosely defined theoretical framework on which conventional econometric models of personal sector behaviour are based. It is argued that this framework is particularly inadequate during periods when structural changes in borrowing behaviour are occuring. The conventional consumption function used by macroeconomic forecasters is based on the error correction formulation of Hendry and others. Its earliest manifestation in Davidson *et al* (1978) postulated a long run relationship between consumption and income which depended on long run growth and price inflation. Because this type of model ignored the role of cumulated savings on consumption, Hendry and Von Ungern Sternberg (1981) recognised the need to introduce wealth effects into the specification. This was justified theoretically on the grounds that consumers followed the life cycle hypothesis subject to an effect from wealth justified on precautionary grounds (see Deaton (1972)). Furthermore, consumers were assumed to adopt rule-of-thumb behaviour which implied error-correcting

† This paper presents preliminary research carried out under the project 'A micro-to-macro analysis of personal sector behaviour' financed by the Leverhulme foundation to whom support is gratefully acknowledged. It has been influenced by earlier work carried out with John Ermisch and by related work with Chris Tsoukis on an ESRC financed project, 'A forward-looking model of the UK housing market'. This paper has benefitted from helpful remarks from them and other members of the Institute staff but any remaining errors are my own responsibility. An earlier version of his paper was presented at a meeting of the Money, Macro and Finance research group at the London School of Economics in March 1992.

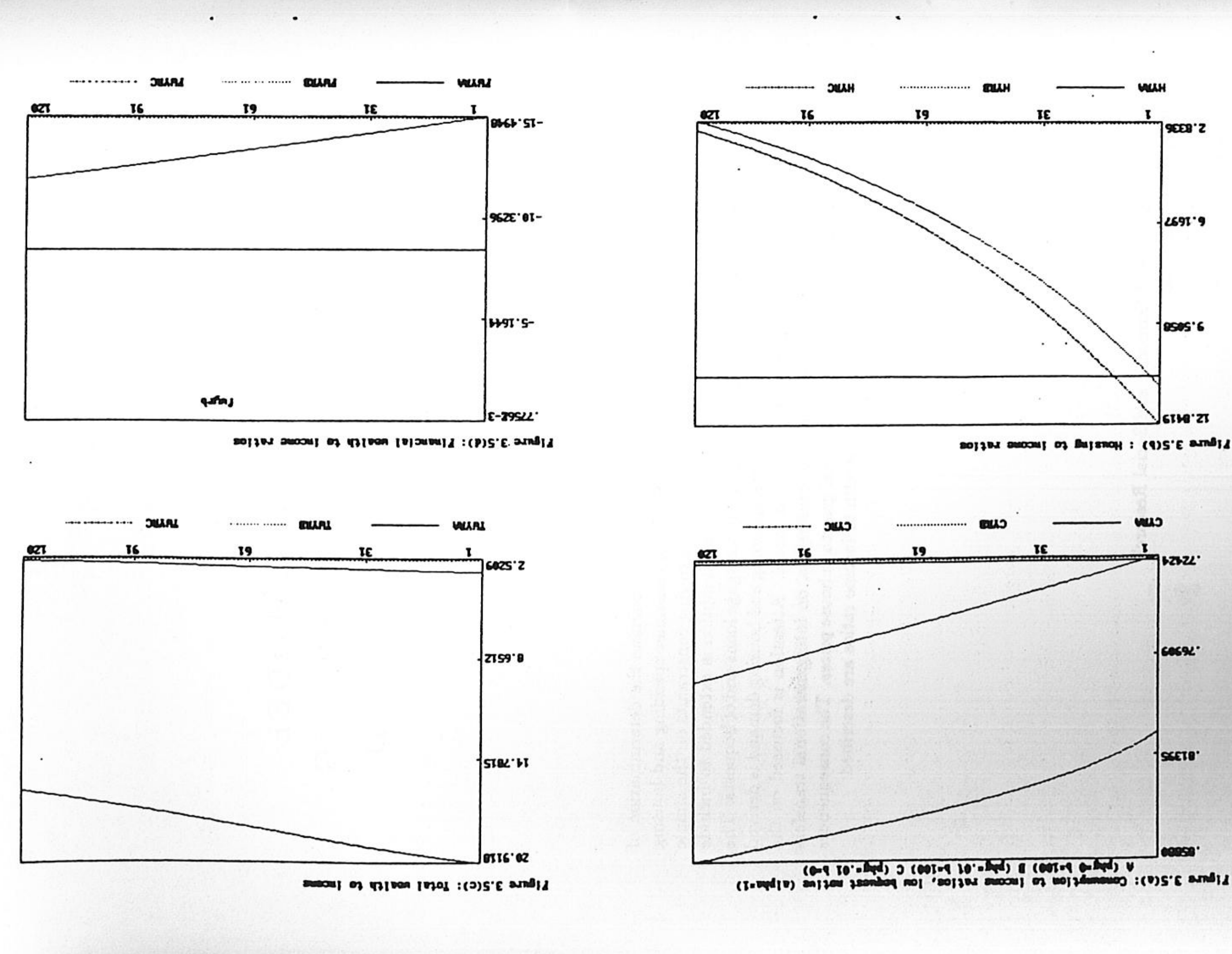

Figure 3.5(a): Consumption to income ratios, low bequest motive (alpha=1)

Figure 3.5(b) : Housing to income ratios

Figure 3.5(c): Total wealth to income

Figure 3.5(d): Financial wealth to income ratios behaviour around a long run relationship between income and wealth. Indeed, this form of integral correction mechanism (see Patterson (1984)) provides useful stabilising properties to the model and is one of the main reasons for the widespread adoption of this type of equation by macromodellers. In fact, the implied constancy of this long run wealth-income ratio is not typically emphasised by the advocates of this econometric approach partly because the target ratio implied by a given specification might not correspond at all closely to historical experience.

The relative fragility of this theoretical framework was further emphasised in the last 1980s when these types of equation failed to predict the fall in the saving ratio. The argument of this paper is that this failure is precisely what one would expect to see at a time when financial liberalisation was altering the relationship between net wealth and incomes. Various models (eg Westaway (1989), Muellbauer and Murphy (1989)) have attempted to capture these changes explicitly. Many researchers, however, have attempted to maintain the same theoretical approach by widening the definition of wealth to include housing wealth (see for example Currie *et al* (1990)). While this is theoretically correct in the sense that it explicitly acknowledges the previously ignored link between consumption and housing demand, it does not address the key question of what determines the 'target relationship' between wealth and income, nor the role that housing wealth and changing house prices play within this.

The rest of this paper attempts to address these questions directly. No attempt is made to present an aggregate consumption function but the results of the theoretical model are intended to have direct implications for the way aggregate consumption functions are estimated. The next section of the paper sets up the standard life-cycle utility maximisation problem facing the individual consumer, first for the conventional simple model where all goods are assumed to be non-durable, then for a more realistic model where durable goods, for example housing, are important and where consumers have a bequest motive. Following Ermisch and Westaway (1990), examples are given which illustrate the impact of borrowing constraints on lifetime spending behaviour. Section 3 derives the implied aggregate behaviour, and investigates the dynamics of the response of aggregate consumption to the relaxation of borrowing constraints. Most investigations of the impact of wealth

constraints on aggregate consumption have been restricted to steady-state comparisons (Russell 1977). One important exception is Tobin and Dolde's (1971) simulation study, but while their analysis assumed that the wealth constraint is only binding in the current period, the approach taken here is able to take into account the impact of expecting to be bound by such constraints in future periods on the optimal consumption plan. An important extension to the results of Ermisch and Westaway (1990) is facilitated not only by the inclusion of housing but also by the treatment of bequests which allows an investigation of the effects of intergenerational transfers of wealth; the potential importance of these types of effect have been emphasised recently by Miles (1992) and Holmans (1991). The results in section 3 also illustrate the effect of real house price growth under different assumptions. Clearly, however, house prices are endogenous, especially at a time when borrowing constraints are being removed. Section 4 introduces a simple market clearing model of house price determination which discusses this issue briefly.

Section 5 draws out the implications of the results for the conventional approach to econometric analysis of consumption. To anticipate the main conclusions, the results suggest that financial liberalisation, increasing home ownership and the associated implications for intergenerational wealth transfers are likely to have large, uncertain and long-lasting effects on the relationship between consumption, incomes and wealth. This is likely to render the margin of error associated with simple 'reduced form' econometric explanations of this behaviour to be even wider than usual.

2 A theoretical model of individual household behaviour

2.1 A simple model of non-durables consumption

It is useful to begin with the simplest life-cycle model of individual behaviour since it is this which underlies all of the literature initiated by Hall (1978) which seeks to test whether consumption conforms to the joint hypotheses of the life cycle permanent income theory and the rational expectations assumption. The model to be used is characterised as follows;

1. Each consumer has no initial wealth and identical preferences and earnings/pension profile.
2. Length of life and earnings profiles are certain.
3. Consumers are selfish; thus, with a certain length of life, they do not leave bequests.
4. Each consumer's lifetime utility function is:

$$U(\mathbf{c}) = \sum_{i=1}^{T} \frac{u(c_i)}{(1+\rho)^i}$$

   where c_i is consumption at age i, ρ is the rate of time preference, $u(c_i)$ is concave and the length of life (after becoming an adult) is T years.

5. The consumer is assumed to choose $c_1, \ldots, c_T$ to maximize $U(\mathbf{c})$ subject to the following constraints:

$$c_i = y_i - (w_i - (1+r)w_{i-1}), \qquad i = 1, \ldots, T \text{ and } w_0 = 0$$

$$w_i \geq b, \qquad i = 1, \ldots, T$$

   where y_i is labour (or pension) income at age i, w_i is wealth at age i, r is the (real) interest rate and b is the wealth 'floor' (b may be negative). The latter is the wealth constraint. While we have formulated it purely in terms of quantity, our analysis also applies if the interest rate on borrowing beyond b is large enough to preclude a consumer ever being on that part of the budget constraint.

6. $u(c_i) = c_i^{1-\gamma}/(1-\gamma)$; that is, $u(c_i)$ displays 'constant relative risk aversion' and $1/\gamma$ is the intertemporal elasticity of substitution.
7. The economy is a small open economy which faces fixed exchange rates and an interest rate r given by international capital markets. Under these assumptions, the interest rate is *not* constrained to be at least as large as the rate of time preference, and even in a closed economy, the equilibrium interest rate may be less than the rate of time preference when consumers have finite horizons (*e.g.* see Blanchard (1985), as is the case in our life cycle model. Production only uses labour with productivity given by the age-earnings profile and general productivity growth over time. Thus, total labour income is total production in the economy.

The general solution to the maximization problem entails

$$\frac{c_i^{-\gamma}}{(1+\rho)^i} = \lambda(1+r)^{T-i} + \sum_{j=i}^{T-1} \mu_j (1+r)^{j-i} \tag{1}$$

where λ is the Lagrange multiplier associated with the terminal wealth constraint and μ_i are the Kuhn Tucker constraints associated with the borrowing restrictions at age i such that

$$\mu_i > 0 \text{ when } w_i = b \text{ and } \mu_i = 0 \text{ when } w_i > b$$

When the wealth constraint is not binding at age i nor age $i-1$,

$$\frac{c_i}{c_{i-1}} = \left(\frac{1+r}{1+\rho}\right)^{1/\gamma} \tag{2}$$

Thus, when the wealth constraint is never binding, the consumption profile is rising if $r > \rho$, falling if $r < \rho$ and flat if $r = \rho$. A higher value of γ (*i.e.* a lower intertemporal elasticity of substitution) reduces the absolute value of the slope of the consumption profile when $r \neq \rho$. In the absence of binding wealth constraints, it is easy to solve for c_i ($i = 1, \ldots, T$) using (2) and the lifetime wealth constraint (i.e. that terminal wealth is zero in the absence of bequests). This is done by re-expressing W_T in terms of c_i and hence in terms of c_1 alone, thus giving an analytic expression for first period consumption as

$$c_1 = [W_0 + \sum_{i=1}^{T} y_i (1+r)^{-i}] / \sum_{i=1}^{T} \frac{(1+r)^{(i(1-\gamma)-1)/\gamma}}{(1+\rho)^{(i-1)/\gamma}} \tag{3}$$

When the wealth constraint does bind during the lifetime of the consumer, the computational problem is slightly more complicated and prevents the usual straightforward analytical solution described above. Two points are noteworthy however;

- Even if the wealth constraint is not binding in a particular period (i.e if $\mu_i \geq 0$), the level of consumption in that period may still be affected by future binding constraints.
- If the wealth constraint is known to bind in a particular period i, then the constrained maximisation problem decomposes into two problems, one determining consumption and wealth up to period i when the terminal value of wealth is given by its constrained value, the second determining consumption from period i+1 onwards.

Successive application of an algorithm based on these properties allows the constrained optimisation problem to be solved; for more details, see Westaway (1992).

In order to derive illustrative solutions to the problem, it is necessary to make some assumptions about the lifetime of the consumer and the shape of the age-earnings profiles. In all of the examples to be used in this paper, we assume an 'economic working life' of 50 years (for simplicity, these will be referred to as ages 1 to 50 although more realistically it may represent ages 21 to 70 for example), during which time an individual's labour income y increases by 0.02×100 per cent a year during the ages 1-40 (we shall see below that this growth of labour income for the individual comprises the sum of general productivity growth plus the individual's 'seniority' increase). At the end of year 40, retirement then occurs, at which time a pension equal to half of earnings at age 40 is paid for the last ten years of a person's life. We assume that these developments are perfectly anticipated by consumers.

Examples In this and all subsequent examples to be given in this paper, we assume that $r = 0.025$, $\rho = 0.025$ and $\gamma = 0.8$. The assumption for γ may appear low, but it is consistent with recent micro-econometric estimates from a sample of elderly people by Hurd (1989). Figure 2.1(a) shows the consumption profiles relative to (labour) income when consumers are unconstrained in their ability to borrow (b=∞), totally constrained (b=0) and partly constrained (b=1.5). Figure 2.1(b) shows the corresponding profiles for lifetime wealth. The assumption that $r = \rho$ produces the familiar flat 'consumption

smoothed' age-consumption profile when wealth constraints are never binding. In fact, such a profile is not consistent with observed cross-section patterns of household consumption with age of the household head, but the observed profiles may reflect borrowing constraints. When $b = 0$, our representative consumer faces a binding wealth constraint over the first 28 years. In this case, consumption rises in line with income[1] until age 28 and then stays flat thereafter.[2]. The partially constrained case (b=1.5) illustrates the point made above that, for the first 13 years, the consumer does not face a binding constraint (i.e $w \geq b$) but consumption still differs from the completely unconstrained profile because of the future constraint which binds from periods 14 to 26.

[1] This type of consumption behaviour is often referred to as 'Keynesian' behaviour

[2] If we had assumed that the discount factor was higher than the interest rate then consumption would fall from then on. Such a pattern would be consistent with the hump-shaped cross-section profile of household consumption observed in the U.K.

2.2 An extended model of individual household behaviour

The model of consumer behaviour described in the last section has been very influential since it has been used to test the validity of the life cycle permanent income rational expectations hypothesis, and variants on it have been used to test for the presence of liquidity constraints (see Zeldes (1989) for example). Yet there are a number of important respects in which this model is unrealistic. In this section, the basic model is extended in a way which overcomes many of these shortcomings. The extensions made are as follows;

(i) The role of durable consumption goods is introduced into the model (as in Mankiw (1982), Bernanke(1985)) Clearly, the most important durable good purchased by a household is, for owner occupiers at least, the house in which they live. Since we are particularly interested in the influence of housing on personal sector behaviour, the mathematical exposition will refer to housing (h) from now on although most of the arguments apply in a similar manner to other durable goods. Durable goods have a number of distinguishing features which we build into our model;

- They provide a flow of services which will be proportional to the stock of durable goods held,[3] i.e.

$$U(h_t) = \frac{(\theta h_t)^{1-\gamma}}{1-\gamma} \tag{4}$$

- Durable goods will depreciate through time (at a rate δ, which will be assumed to be zero in our example)
- The price of durable goods (ph) may be different from that for non-durables and may diverge continually (i.e. they have a different growth rate).
- Borrowing conditions will differ from that for non-durables consumption. First, the principal as well as the interest on the loan must be paid. Second, the loan may be paid over an extended period (eg 25 year mortgage). Third, there may be down payment restrictions (ie D per cent of the purchase price must be met from the individuals assets).

[3] In this exposition, it is assumed that the intertemporal elasticity of substitution for housing is the same as for consumption. It is quite possible to assume a different value, but at the expense of computational complexity.

- Rental markets may exist which offer the consumer advantages over outright purchase of the durable good. In the model described below, the rental market for housing is introduced as a safety net which prevents consumers from consuming 'too low' a level of housing services when borrowing constraints are binding.

(ii) Consumers are assumed to derive utility from their leaving bequests, thus introducing an extra term into the lifetime utility function of the form

$$\frac{U(B_{T+1})}{(1+\rho)^{T+1}} \tag{5}$$

where

$$B_{T+1} = W_{T+1} + ph_{T+1}h_T \tag{6}$$

and

$$U(B_{T+1}) = \alpha\frac{(W_{T+1} + ph_{T+1}h_T)^{1-\gamma}}{1-\gamma} \tag{7}$$

Note we assume that the utility derived from the bequest is proportional to that received from consumption itself with the same intertemporal elasticity of substitution. Importantly, we do not assume that an individual's heir's utility appears in the utility function, thus avoiding the infinite regress problem usually associated with overlapping generations models (see Blanchard and Fisher (1989)).

(iii) Following (ii), we allow individuals to inherit wealth (BEQ) which they receive NB periods into their own life-times. We assume, for simplicity that all bequests are anticipated.

To summarise, the utility maximisation problem facing the consumer is now as follows

$$\text{Maximise } U(\mathbf{c},\mathbf{h}) = \sum_{i=1}^{T}\frac{U_i(c,h)}{(1+\rho)^i} + \frac{U(B_{T+1})}{(1+\rho)^{T+1}} \tag{8}$$

where

$$U_i(c,h) = \frac{c_i^{1-\gamma}}{1-\gamma} + \frac{(\theta h_i)^{1-\gamma}}{1-\gamma} \tag{9}$$

and

$$U(B_{T+1}) = \alpha\frac{(W_{T+1} + ph_{T+1}h_T)^{1-\gamma}}{1-\gamma} \tag{10}$$

If constant interest rates are assumed and if mortgages are assumed to be paid off over the remaining years of the individuals life, then the financial (non-mortgage) wealth evolution equation is given by

$$\begin{aligned} W_T =& (1+r)^T W_0 + BEQ(1+r)^{T-NB} + \sum_{i=1}^{T}(y_i - c_i)(1+r)^{T-i} \\ & - \sum_{i=1}^{T-1}\left[\frac{(1+r)^{T+1-i}-1}{r}R_i ph_i - \frac{(1+r)^{T-i}-1}{r}R_{i+1}ph_{i+1}\right]h_i \\ & - R_T ph_T h_T \end{aligned} \tag{11}$$

where

$$R_i = \frac{r(1+r)^{T-i}}{(1+r)^{T+1-i}-1} \tag{12}$$

which is the repayment on a (T+1-i) period mortgage.

The unconstrained maximisation problem yields the first order conditions

$$\frac{dL}{dc_i} = \frac{c_i^{-\gamma}}{(1+\rho)^i} - \alpha'\frac{(1+r)^{T-i}}{(1+\rho)^T}[W_T + \frac{ph_{T+1}}{(1+r)}h_T]^{-\gamma} \tag{13}$$

and

$$\frac{dL}{dh_i} = \frac{\theta^{(1-\gamma)}h_i^{-\gamma}}{(1+\rho)^i} - \alpha' x_i \frac{(1+r)^{T-i}}{(1+\rho)^T}[W_T + \frac{ph_{T+1}}{(1+r)}h_T]^{-\gamma} \tag{14}$$

where

$$\alpha' = \alpha(1+r)^{1-\gamma}/(1+\rho)$$

and

$$x_i = \left[\frac{(1+r)^{T+1-i}-1}{r}R_i ph_i - \frac{(1+r)^{T-i}-1}{r}R_{i+1}ph_{i+1}\right]/(1+r)^{T-i}$$

x_i can be interpreted as the user cost of housing which simplifies to

$$x_i = ph_i\frac{r - phg}{(1-r)}$$

where phg is the growth rate of real house prices.

This results in the usual intertemporal 'random walk' equation for consumption given previously by equation (2) and an analagous intertemporal allocation equation for the demand for housing services given by

$$\frac{h_i}{h_{i-1}} = \left(\frac{x_{i-1}}{x_i}\frac{1+r}{1+\rho}\right)^{1/\gamma} \tag{15}$$

Alternatively, consumption and housing demand are linked by the intratemporal Euler condition,

$$h_i = x_i^{-1/\gamma}\theta^{(1-\gamma)/\gamma}c_i \tag{16}$$

Finally, the marginal utility of the bequest is determined by the equation

$$\alpha'^{1/\gamma}\frac{(1+r)}{(1+\rho)}^{(T-1)/\gamma}c_1 = W_T + \frac{ph_{T+1}}{(1+r)}h_T \tag{17}$$

From this equation, W_T can be re-expressed in terms of c_i and h_i and hence in terms of c_1 alone, thus giving an analytic expression for first period consumption as

$$c_1 = [W_0 + \frac{ph_0}{(1+r)}h_0 + \sum_{i=1}^{T}y_i(1+r)^{-i} + BEQ(1+r)^{-NB}]/\text{Denom} \tag{18}$$

where

$$\text{Denom} = \alpha'^{1/\gamma}\frac{(1+r)^{(T(1-\gamma)-1)/\gamma}}{(1+\rho)^{(T-1)/\gamma}} + \sum_{i=1}^{T}(1 + x^{(\gamma-1)/\gamma}\theta^{(1-\gamma)/\gamma})\frac{(1+r)^{(i(1-\gamma)-1)/\gamma}}{(1+\rho)^{(i-1)/\gamma}} \tag{19}$$

As with the simple model, the existence of wealth constraints introduce the Kuhn-Tucker inequality conditions into the problem which imply the modified first order conditions

$$\frac{dL}{dc_i} = \frac{c_i^{-\gamma}}{(1+\rho)^i} - \alpha'\frac{(1+r)^{T-i}}{(1+\rho)^T}[W_T + \frac{ph_{T+1}}{(1+r)}h_T]^{-\gamma} - \sum_{j=i}^{T}\mu_j(1+r)^{i-j} \tag{13'}$$

and

$$\frac{dL}{dh_i} = \frac{\theta^{(1-\gamma)}h_i^{-\gamma}}{(1+\rho)^i} - \alpha' x_i\frac{(1+r)^{T-i}}{(1+\rho)^T}[W_T + \frac{ph_{T+1}}{(1+r)}h_T]^{-\gamma} - \sum_{j=i}^{T}\mu_j xw_j(1+r)^{i-j} \tag{14'}$$

where

$$xw_j = \frac{dW_j}{dh_i}/(1+r)^{i-j}$$

Now the computational problem is further complicated by the fact that any binding wealth constraint not only distorts the intertemporal Euler condition but also the intratemporal relationship between consumption and housing. In general, it is still possible to solve this constrained maximisation problem by exploiting the fact that the early periods of the life-cycle are likely to be when wealth constraints bind (for more details, see Westaway (1992)).

Examples Figures 2.2(a)-(d) give details for this extended model assuming the same parameters as with the simple model, but now including a bequest motive, α. A generous α of 20 is compared with a low motive of 1. Any mortgage loan is assumed to be repaid over the remaining lifetime of the consumer and no downpayments are assumed to be necessary.[4] Figures 2.2(a) and (b) show consumption and housing to have flat lifetime profiles for both values of α, with consumption and housing around 20 per cent lower when the bequest motive is large. 2.2(d) shows financial wealth (ie net assets plus mortgage debt) turning positive after 27 years in the large bequest case, but interestingly remaining negative up to death in the low bequest case. Total wealth (2.2(c)) comprising financial wealth plus housing equity, grows steadily throughout the 'altruistic' consumer's life, delivering the required large bequest at the end of 50 years. By contrast, in the low bequest case, the rise in housing equity towards the end of life is mostly offset by an increase in non-mortgage debt, resulting in higher consumption but a lower bequest as intended.

Now the effects of borrowing constraints on individual behaviour are considered. Financial debt is restricted to be at most three times current labour income at any point in the individuals life cycle. Figures 2.3(a)-(d) show the comparison with the unconstrained case for α=20. The constraint binds for the first 16 years and the resulting bequest is some 5 per cent higher. The effect of the same constraint on the low bequest case is more substantial, as shown in Figures 2.4(a)-(d). Now, because consumers are not able to die with a large debt, their bequest is necessarily close to the value of their housing equity , since they still need to live in a house up to the point when they die. This implication of financial liberalisation has been emphasised by Miles (1992) and as the next section shows, can have important implications for the intergenerational transfer of wealth.[5]

One potentially unrealistic feature of these constrained solutions is that consumption

[4] The computer simulation programme has been set up to allow mortgages to be paid off over a shorter period, over the working life for example. Similarly, non-zero downpayment requirements can be imposed. Neither of these refinements are discussed in this paper.

[5] The size of this effect may be mitigated by people trading down to the rental sector but the limited supply of this type of housing of the required quality may prevent this from occurring.

and housing are very low in the early years of the life cycle. In practice, there may be a minimum level of housing below which it is not possible to fall (because houses are indivisible). This effect can be captured by assuming that if, *ex ante*, housing does fall below this minimum threshold, consumers will rent a house of this minimum size (in the public sector for example) until they have saved enough to enter the housing market as owner occupiers. Case H in Figure 2.4 illustrates this for a floor on housing set at 750, showing how consumers save over the first ten years, then becoming owner occupiers but at a lower level than in the previous case.

The examples so far have all assumed constant relative house prices. As will be explained in section 4, this is likely to be an unrealistic assumption. Figures 2.5(a)-(d) illustrate the implications of real house price growth of 1 per cent per year. The profile of housing in the unconstrained case shows a much higher starting level followed by a steady decline or 'trading down' throughout the life cycle. While surprising at first glance, this profile is completely explained by the user cost of housing which, for a given level of house prices, will be lower when house price inflation is positive (because of the capital gain element) but which will rise in line with house prices thereafter, thus causing consumers to substitute away from housing. This does not mean that consumers are ignoring the effects of the later capital gains, but rather that they buy a large house as early as possible, to exploit the capital gains to the full. Of course, in reality, we do not typically see this type of profile but this may be because we only observe behaviour constrained by borrowing constraints. Such a constrained case is also shown in Figure 2.5 which confirms that the more usual rise in housing is observed when borrowing constraints bind.

3 The Determination of Aggregate Behaviour

3.1 Aggregate behaviour in the simple model

Having derived the implications of the model for individual personal sector behaviour, it is, in principal, straightforward to generate notional series for aggregate behaviour. In our example, since individuals are assumed to live for 50 years, we require 50 generations, or cohorts of data to construct a single aggregate observation. Aggregate consumption in year t is given by $C_t = \sum_{i=1}^{50} c_{it}$, aggregate income is given by $Y_t = \sum_{i=1}^{50} y_{it}$ and aggregate wealth by $W_t = \sum_{i=1}^{50} w_{it}$. The consumption-income ratio is given by C_t/Y_t and the different wealth-income ratios are defined analogously. Two considerations are relevant when moving from individual to aggregate behaviour;

- general productivity growth will cause the labour income of successive generations to rise, as well as causing the earnings profile of the individual to rise more quickly. Thus, if the starting income of successive generations grow by g per cent per year due to general productivity then career income will grow by (s+g) per cent per year during the individuals working life where s is seniority - related growth of earnings as described earlier. In this example, s and g are each set at 0.01 combining to give career income growth of 2 per cent a year.
- changes in the structure of population will cause cohort sizes to vary, thus affecting aggregate behaviour. More extended discussion of these effects are described in Ermisch and Westaway (1990).

First it is instructive to show the effects on aggregate behaviour of the complete removal of a borrowing constraint in the simple model without durable goods; these results are similar to those in Ermisch and Westaway (1990). The model is run for 100 years (150 generations), with the borrowing constraint ($b = 0$) removed in the 26th year. Figure 3.1(a) shows the resulting profile of the aggregate consumption-income ratio, jumping by 10 per cent in the year that the wealth constraint is removed, and then gradually falling back to a higher steady-state level than that obtained under the wealth constraint. Figure 3.1(b) shows that the wealth-income ratio falls after removing the constraint, reaching a

much lower level 50 years later. The fifty year adjustment period and near linear fall in the consumption-income ratio is particularly interesting. It is not until every person who has spent some of their life in the wealth-constrained regime dies that the consumption and wealth to income ratios reach their new steady-state values. Although the youngest generation at the time of the constraint relaxation only spent one year in the constrained regime, they register the biggest jump in consumption, and it is not until they die that the aggregate consumption-income ratio stops falling. This explains why the fall is nearly linear. Thus, the economy's period of adjustment to a relaxed wealth constraint is a long one, even though there are no costs of adjustment in the model. The length purely reflects the aggregation across generations, as a diminishing proportion of the population lived any part of their life in the constrained regime.

Of course, the precise nature of the steady state consumption to income ratios and wealth to income ratios will vary depending on the parameters chosen. This question is investigated for this simple model in more detail in Ermisch and Westaway (1990). The main findings there may be summarised as follows;

- A higher rate of time preference (ρ) produces a larger jump in consumption when the wealth constraint is removed, a higher steady-state consumption-income ratio (C/Y), and a larger fall in the wealth income ratio (W/Y). Raising the intertemporal elasticity of substitution (lowering γ) has similar effects.
- A steeper age-earnings profile (higher s) or a faster rate of general productivity growth (g) also increase desired consumption and borrowing when young, thereby producing similar effects of relaxing the wealth constraint.
- Raising the interest rate (r) has the opposite effects, reducing the jump in C/Y, lowering the steady-state C/Y and raising the steady-state W/Y.
- As Russell (1977, Theorem 1) has shown, when $r = g$, consumption will always equal production (labour income), in both the wealth-constrained and unconstrained steady-states. In this case, the ratio of external debt (assets) to income is stabilised because the debt (assets) increases each year by the amount of annual foreign interest payments (receipts) and production increases by the same percentage. The country becomes a

net foreign debtor after removing the wealth constraint if the rate of time preference exceeds the interest rate in most of the simulations (see also Blanchard (1985)).

- As originally shown by Russell(1977), the steady-state ratio of aggregate consumption to *labour* income will increase when the wealth constraint is removed, if $g > r$, but the opposite will be the case if $g < r$. If, however, $g = 0$, then, as Modigliani and Brumberg (1954) first showed, $C/Y = 1$.
- In the original formulation of the life cycle model by Modigliani and Brumberg (1954), they concluded that a higher rate of general productivity growth (g) would reduce the steady-state C/Y. Farrell (1970) demonstrated that this need not be the case when people foresee the resulting increase in future earnings. This model confirms that higher g usually raises the consumption-income ratio when wealth constraints.

Thus, parameters of the consumer's utility function are very important in assessing the impact of relaxing borrowing constraints on consumption, the current account and the level of foreign assets. There is not, however, a consensus on the appropriate values for these parameters. Surveys of evidence reported in Davies (1981) and Scott (1990) suggest low values for the intertemporal elasticity of substitution, thus high values of γ. If correct, this suggests that relaxation of borrowing constraints could have relatively large effects on the aggregate consumption-income ratio, independent of the relative values of the rates of interest and time preference. As noted above however, Hurd's (1989) estimates suggest a much higher elasticity of substitution.

3.2 Aggregate behaviour in the extended model

It is straightforward to extend the aggregation procedure described above to incorporate durable goods such as housing. The introduction of bequests however complicates the problem in an interesting and important manner since it allows intergenerational transfers of wealth to affect personal sector behaviour, not just for the giver, as was described in the examples of section 2.2, but also for the heir who is receiving the bequest. In the examples that follow, bequests are assumed to be reveived 25 periods into the working life of the consumer; this implies that an individual's parents die twenty-five years before the individual's own demise. Given these transfers, it is immediately interesting to investigate

whether the system will settle down to a meaningful steady-state. Figure 3.2 illustrates how quickly the consumption to income ratio reaches its steady state if bequests are initialised at zero. In Figure 3.2(a) when α =20, the system takes almost 100 years to reach its steady state level. Having computed this steady state path, it is then straightforward to put the system on its steady state immediately by appropriate initialisation of bequests (as done for CYRssu in Figure 3.2(a)). The presence of borrowing constraints lengthens the dynamics still further since bequests are larger in this case. This influence of the size of bequests on the dynamics is emphasised in Figure 3.2(b) which shows the analagous swift convergence of the consumption ratio in the low bequest case.

Having developed the methodology for computing the steady state path in the presence of intergenerational transfers, it is now possible to examine the impact of the relaxation of borrowing constraints in this extended model. At this stage, analysis will be confined to the case where real house prices are constant. Figures 3.3(a) and (b) show the results for the consumption to income ratio and the total wealth to income ratio. As in the simple model, the consumption ratio rises sharply. The effect is more marked in the low bequest case since all consumers including those in the latter stages of their life cycle find it advantageous to exploit the new borrowing opportunities. In the high bequest case, by contrast, only those younger consumers whose wealth constraint is binding increase their consumption, the rest sticking to their original plans. It is also interesting to compare the speed of the dynamic response in this extended model compared to the simple case. Now, as illustrated in Figures 3.2 above, the high bequest economy takes over 100 years to reach its new steady state path, while the low bequest consumers reach their long run in just over 50 years, almost as quickly as the simple no bequest case. Again, it is worth emphasising that this protracted aggregate response has nothing to do with adjustment costs, but instead is explained by aggregation dynamics as before, and now also by the long lasting effects of the intergenerational transfers.

Now consider the case where real house prices are growing at a constant rate of 1 per cent a year; Figures 3.4(a)-(d) show the aggregate outcome in the case when the bequest motive is high, and 3.5(a)-(d) when the motive is low. The main features of these examples is that the steady state is no longer charactersied by constant relationships between consumption, incomes, housing and wealth. As was seen in the individual case, the rising user cost of housing causes consumers to substitute away from housing through time. This not only causes the housing (nominal and real) to income ratio to fall but also causes the consumption ratio to rise through time. This long run effect occurs despite the fact that each individual is following a flat consumption profile in their own lifetimes. The ratio of total wealth to income stays relatively flat in the unconstrained case but falls by around 10 per cent in the constrained steady state.

It is worth pausing at this stage to emphasise the implications of these model properties because they have important implications for conventional approaches to modelling consumption. First, conventional tests of the Hall random walk model of consumption on aggregate data have tended to ignore this substitution effect arising from house prices. Second, it illustrates starkly that any approach to modelling consumption based on the notion of a constant long run wealth to income ratio is likely to be mis-specified, first because this ratio will shift whenever borrowing conditions alter, second because the ratio may itself trend if real house prices are changing in the steady state.

As with the previous examples, it is possible to demonstrate the dynamic effects of a relaxation of borrowing constraints when house prices are growing. Figures 3.6(a)-(b) show the effects of changing the borrowing constraint from b=2 to b=3 first under the usual assumption that house prices are unaffected by the change, then under the assumption that the resultant increase in demand pushes the level of house prices up by 3 per cent and six per cent. This represents a crude attempt to capture the endogenous response of house prices to a sudden surge in demand. Indeed, since housing supply is likely to be very sluggish in the short term, some jump in house prices of this nature is certain. However, Figure 3.6(b) shows clearly that despite the increase in price, housing demand is hardly choked off at all, partly because the substitution effect is outweighed by the wealth effect arising from the higher house price; this wealth effect causes higher consumption as shown in Figure 3.6(a). Clearly, any attempt to gauge the effects of financial liberalisation must make some attempt to capture the endogenous response of the housing market, not just in the period when the change takes place as is done here, but also by taking into account the implications for the future rate of change of house prices. This can only be attempted

within a structural model of the housing market itself.

4 A structural model of house prices and the housing market

This section presents preliminary work on a structural model of the housing market which seems necessary to address the questions raised in the last section. No final results are presented but a number of the problems encountered and issues raised by the work done so far are discussed.

The simple model of the housing market to be adopted assumes that house prices move, period by period, to equalise the demand for housing and the stock supply. Given the forward-looking nature of the demand for housing, house prices will behave as a 'jump variable', reacting to any current or future developments relating to the housing market (see Poterba (1984) for the first application of this type of model to the housing market).

The analysis of section 3 has allowed us to identify the demand function, which may be written as

$$HD_t = \sum_{i=t-49}^{t} hd(i) \qquad (20)$$

where $hd(i)$ represents the demand for housing services by consumers in the i'th period of their life cycle. This seemingly innocuous expression in fact implies that in a full rational expectations equilibrium, current period housing demand depends on house prices 50 periods previous (because of the decisions made then by the consumer now in the last year of life) and on all intervening house prices up to fifty periods into the future (because of the youngest consumer in the first year of life).

Housing supply is determined by the identity,

$$HS_t = (1-\delta)HS_{t-1} + HINV_t \qquad (21)$$

where $HINV$ is housing investment which is assumed to be driven by expected profitability of housebuilding[6] i.e

$$HINV_t = q(ph_t/cc_t - 1) \qquad (22)$$

[6] Strictly speaking, housing investment will depend on lagged starts, and it is starts which depend on a forward-looking measure of profitability; see Topel and Rosen (1988), and Tsoukis and Westaway (1992) for an application to UK data.

where cc_t are construction costs at time t. These will be defined as a weighted average of labour costs, raw materials and land, i.e.

$$cc_t = w_1 NLCC_t + w_2 pl_t \tag{23}$$

where NLCC are non land construction costs and pl is the price of land. The weights will be assumed to be given by the underlying fixed-coefficients production technology of housebuilding. It will be assumed that wage costs and raw material prices are determined exogenously to the building sector, but the price of residential land is clearly endogenous. This will be assumed to clear the market defined by the demand curve,

$$LD_t = k_d(HD_t)^{ld} \tag{24}$$

and the supply curve,

$$LS_t = k_s L(pl/p)_t^{ls} \tag{25}$$

This yields an expression for land prices of

$$(pl/p) = (\frac{k_d(HD_t)^{ld}}{k_s L})^{1/l_s} \tag{26}$$

Substitution of this expression into the housing investment equation gives an equation for the housing supply which depends on housing demand and house prices, as well as exogenous variables. In principle, this should allow the house price to be solved as a rationally expected price which equalises housing supply and housing demand. In practice, this problem is very complicated to solve for a number of reasons; first, the due to the length of the forward and backward effect of house prices in current housing demand; second because the effect of house prices in the market clearing equation is highly non-linear (for example via the user cost itself). Indeed, the complexity of this problem probably explains why most empirical studies have focussed on two special cases for house price behaviour, either an infinite housing supply price elasticity which delivers constant real house prices, or a zero supply elasticity which usually implies that house prices grow in line with real incomes. In fact, no parameterisation of the simple model of the housing market described above will imply constant house prices. Similarly, the often quoted result that house prices

should grow in line with incomes is also unlikely to be true since an increasing user cost will cause consumers to substitute away from housing. Having said that, preliminary simulations of the above model with a low supply elasticity imposed does suggest that growth in line with incomes provides a fairly close approximation to the market clearing solution.

5 Conclusion

This paper has not attempted to present an empirical model of personal sector behaviour which encompasses all previous work, or to give a detailed explanation of consumer behaviour in the UK over the last twenty years. Rather, this paper is intended to offer salutary warnings to macromodellers (which number the present author among them) who persist in using econometric models based on the conventional framework. The results which have emerged from this type of micro-to-macro analysis so far are predominantly negative ones. It has presented compelling reasons why traditional reduced form error correction consumption functions might not produce coherent medium term properties, based as they are on the premise that consumers have constant target wealth to income ratios. The relatively good econometric track record of these equations over much of the 70s and 80s might be used in their defence, but more recent experience would seem to be more damning. Moreover, the theoretical model presented in this paper might imply that a number of recent and prospective developments will cause large changes in previously 'constant' relationships; these changes include the general relaxation of borrowing constraints and the increased prevalence of owner occupation. This latter effect could possibly deliver greatly increased bequests in the near future, a trend which is already beginning to manifest itself in the data for mortgage equity withdrawal (see Holmans (1986),(1991)), but as emphasised by Miles (1992) and Hamnett *et al* (1991), these intergenerational transfers could continue to increase in the next decade.

All these important changes to the personal sector balance sheet suggest the need for a modelling strategy which attempts to identify the underlying structural processes rather than relying on reduced form models which rely on the assumption that the structure is constant. This paper is a preliminary attempt to approach these problems in this manner although there are obviously a large number of important areas of further research which are required if progress is to be made in improving our models of aggregate behaviour.

References

Bernanke, B. (1985), Adjustment costs, durables and aggregate consumption, *Journal of Monetary Economics*, Vol. 15, pp.41-68.

Blanchard, O.J. (1985), Debts, deficits, and finite horizons, *Journal of Political Economy* Vol. 93, pp.223-247.

Blanchard, O.J. and Fisher S. (1989), *Lectures on Macroeconomics*, MIT Press.

Currie, D., Holly, S. and Scott, A. (1990), A seasonal error correction model of UK aggregate consumption, *LBS Centre for Economic forecasting Discussion Paper* No. 90-22.

Davidson, J., Hendry, D., Srba, F. and Yeo, S. (1978) Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the UK, *Economic Journal*, Vol. 88.

Davies, J.B. (1981), Uncertain lifetime, consumption, and dissaving in retirement, *Journal of Political Economy* Vol. 89, pp.561-577.

Deaton, A.S. (1972), Wealth effects on consumption in a modified life-cycle model, *Review of Economic Studies*, Vol. 39, pp.899-910.

Ermisch, J. and Westaway, P.F. (1990), The dynamics of aggregate consumption in an open economy life cycle model, *National Institute Discussion Paper* No. 190.

Farrell, M.J. (1970), The magnitude of rate of growth effects on aggregate savings, *Economic Journal* Vol. 80, pp.873-894.

Hall, R.E. (1978), Stochastic Implications of the Life Cycle - Permanent Income Hypothesis: Theory and Evidence, *Journal of Political Economy*, Vol. 96.

Hamnett, C., Harmer, M. and Williams, P. (1991), *Safe as houses: Housing inheritance in Britain*, Paul Chapman, London.

Hendry, D.F. and Von Ungern-Sternberg, T. (1981) Liquidity and Inflation Effects on Consumer Expenditure in A. Deaton (ed.) *Essays in the Theory and Measurement of Consumer Expenditure*.

Holmans, A.E. (1991), Estimates of housing equity withdrawal by owner occupiers in the United Kingdom; 1970 to 1990, *Government Economic Service Working Paper*, No. 116.

Holmans, A.E. (1986), Flows of funds associated with house purchase for owner occupation in the United Kingdom; 1977-1984, *Government Economic Service Working Paper*, No. 92.

Hurd, M.D. (1989), Mortality risk and bequests, *Econometrica* Vol. 57, pp.779-814.

Mankiw, N.G. (1982), Hall's consumption hypothesis and durable goods, *Journal of Monetary Economics*, Vol. 10, pp.417-425.

Miles, D. (1992), Housing and the wider economy, *National Institute Economic Review*, February.

Modigliani, F. and Brumberg R. (1954), Utility analysis and aggregate consumption functions: an attempt at integration (mimeo.)

Muellbauer J. and Murphy, A. (1989), Why has UK personal saving collapsed? Credit Suisse First Boston.

Patterson, K. (1984) Some Properties of Consumption Functions with Integral Correction Mechanisms, *The Manchester School*

Poterba, J. (1984) Tax subsidies to owner-occupied housing: an asset market approach, *Quarterly Journal of Economics*, November, pp.729-752.

Russell, T. (1977), Rate of growth effects on aggregate savings: further analysis, *Review of Economic Studies* Vol. 44, pp.153-168.

Scott, A. (1990), Optimal consumption when capital markets are imperfect, LBS Centre for Economic Forecasting Discussion Paper No.-90, April

Tobin, J. and Dolde W.C. (1971), Wealth, liquidity and consumption, in Consumer Spending and Monetary Policy: the Linkages, Federal Reserve Bank of Boston Conference Series No. 5.

Topel, R. and Rosen, S. (1988), Housing investment in the United States, *Journal of Political Economy*, Vol. 96, pp.718-740.

Tsoukis, C. and Westaway, P.F. (1992), A forward-looking model of housing construction in the UK, *National Institute Discussion Paper*, New series No. 13.

Westaway, P.F. (1989), Modelling consumers expenditure in the UK: A disposable funds approach, paper presented at the Macroeconomic modelling bureau seminar on consumer spending, May.

Westaway, P.F. (1992), An algorithm for solving the consumer's utility maximisation problem in the presence of borrowing constraints, mimeo National Institute of Economic and Social Research.

Zeldes, S.P. (1989), Consumption and liquidity constraints: An empirical investigation, *Journal of Political Economy*, Vol. 97, pp.305-346.

Figure 2.0 : Labour income (grows at 2% per annum for 40 years, then halves and is flat thereafter)

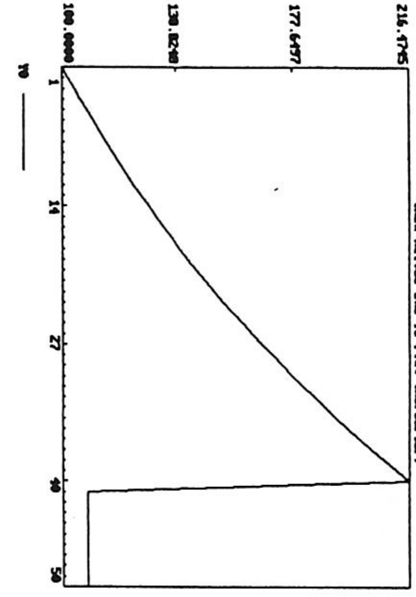

Figure 2.1(a): Consumption : Borrowing constraint b=infinity (u), 0 and 1.5 (p)

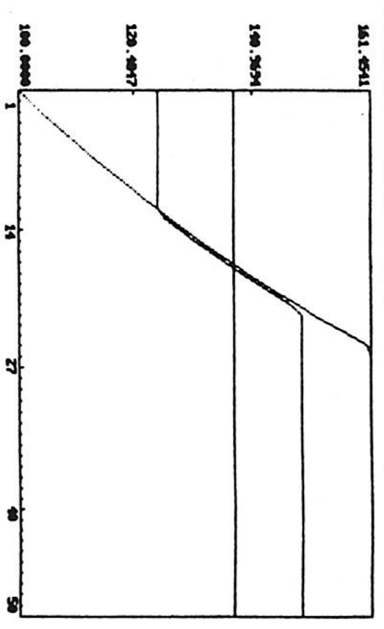

Figure 2.1(b): Wealth : Borrowing constraint b = infinity (u) , 0 and 1.5 (p)

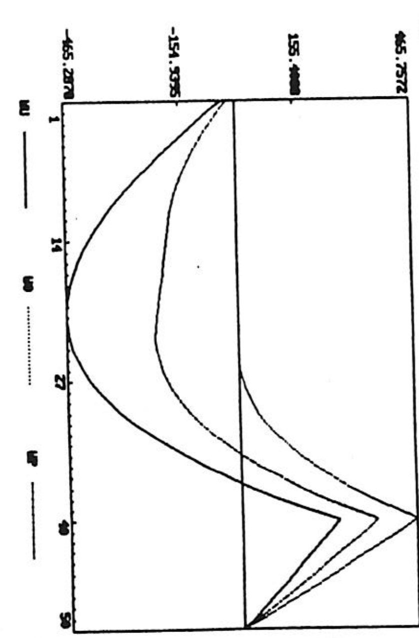

Figure 2.3(a): Consumption, high bequest; constrained (b=3) v unconstrained

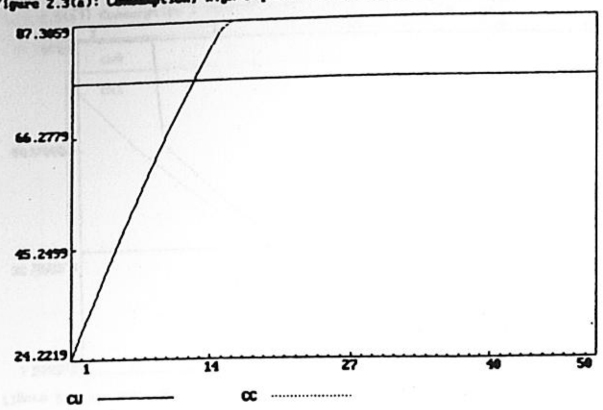

Figure 2.3(c): Total wealth

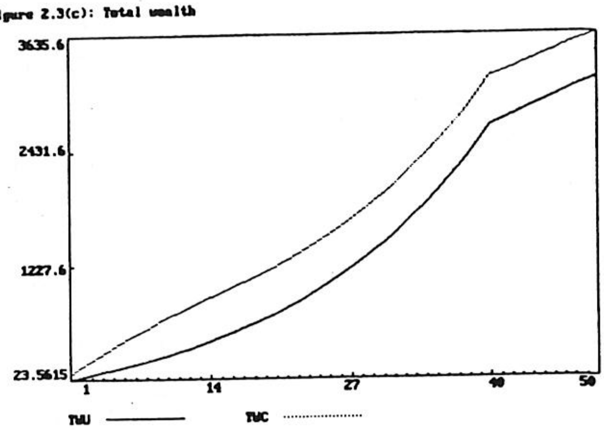

Figure 2.3(b): Housing

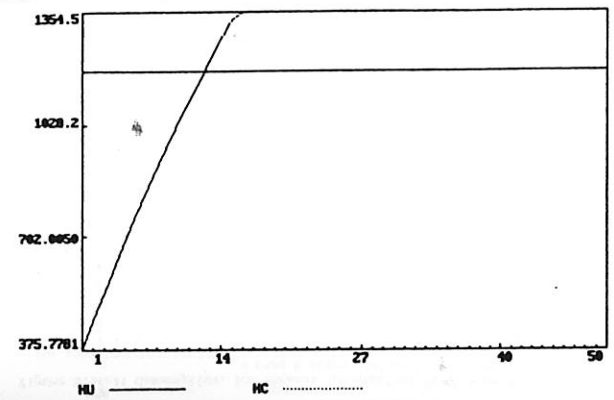

Figure 2.3(d): Financial wealth

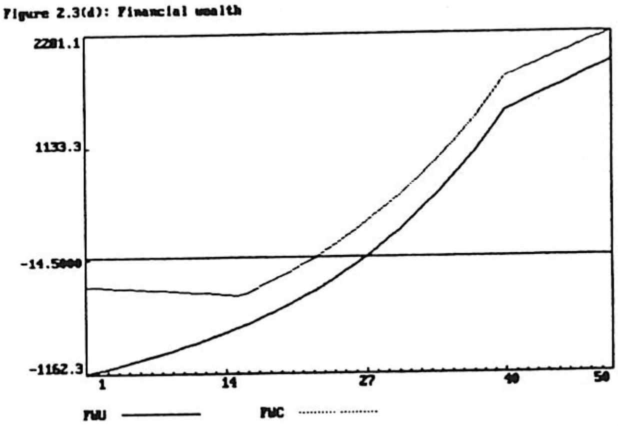

Figure 2.2(a): Consumption (borrowing unconstrained)
High bequest (c20), low bequest (c1)

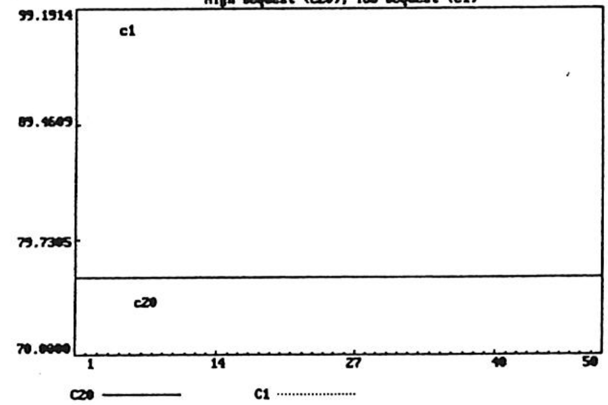

Figure 2.2(c): Total wealth

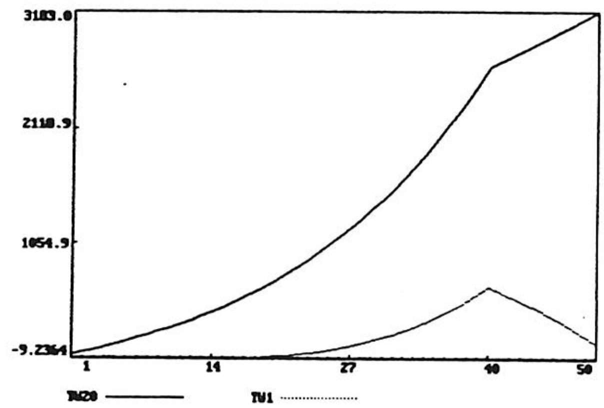

Figure 2.2(b): Housing (high and low bequests)

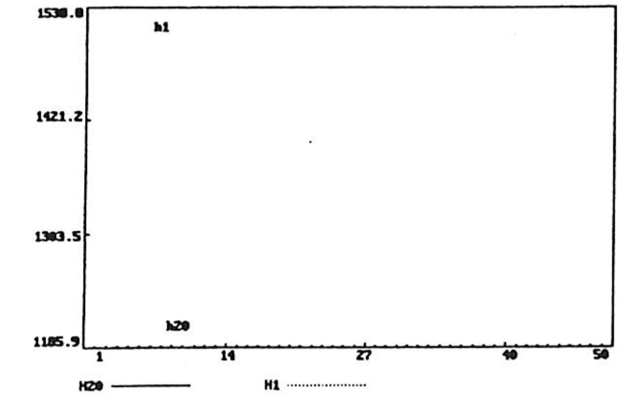

Figure 2.2(d): Financial wealth

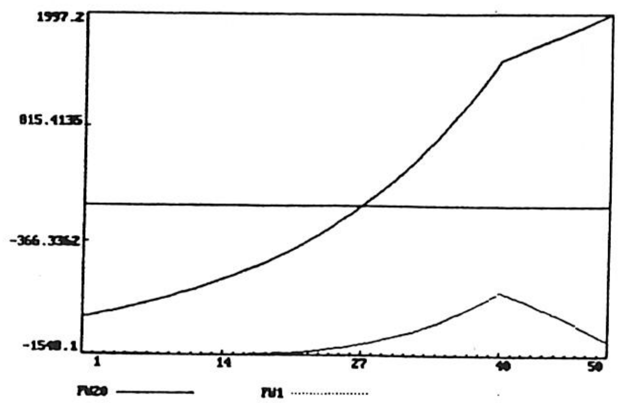

Figure 2.5(a): Consumption : Unconstrained, phg=0 (CU0), phg=.01 (CU1)
and constrained ,phg=.01 (CC1)

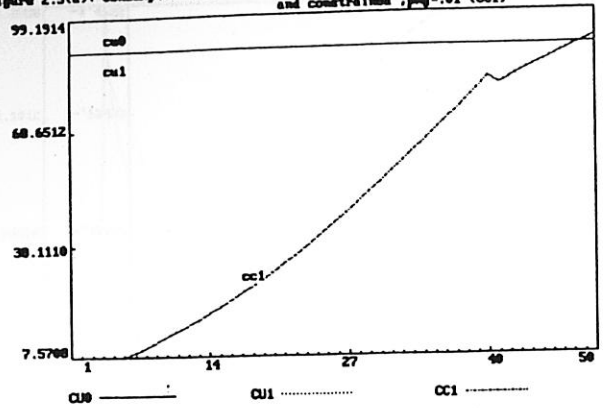

Figure 2.5(c): Total wealth

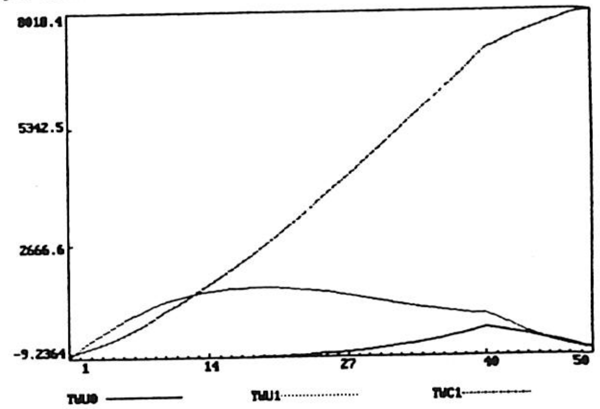

Figure 2.5(b): Housing

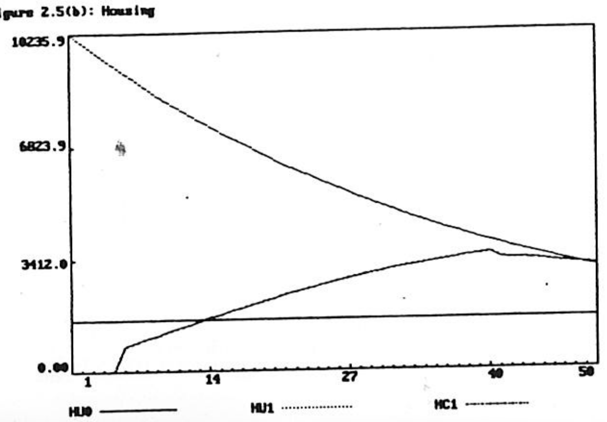

Figure 2.5(d): Financial wealth

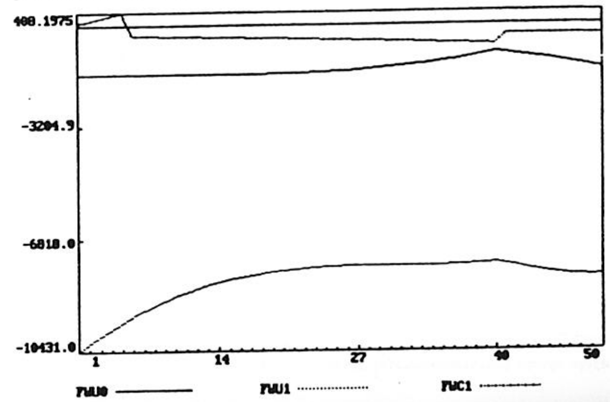

Figure 2.4(a): Consumption, low bequest (constrained (b=3) v unconstrained)
+ case H where minimum housing =750

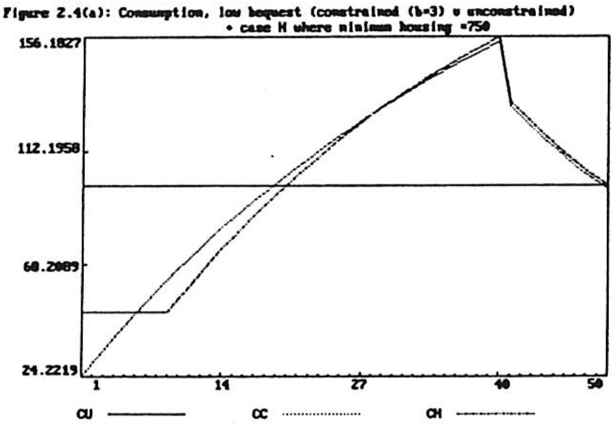

Figure 2.4(c): Total wealth

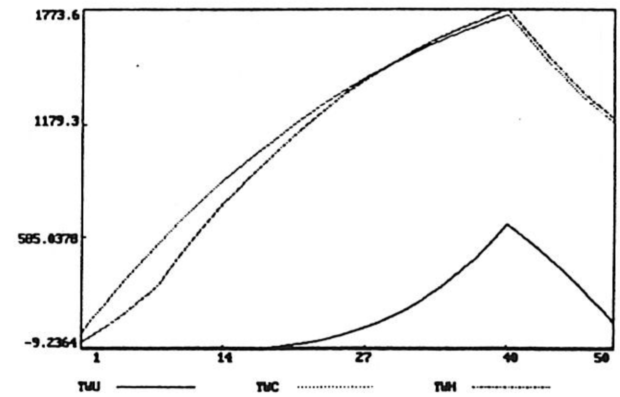

Figure 2.4(b): Housing (HH excludes rented component =750 for years 1-7)

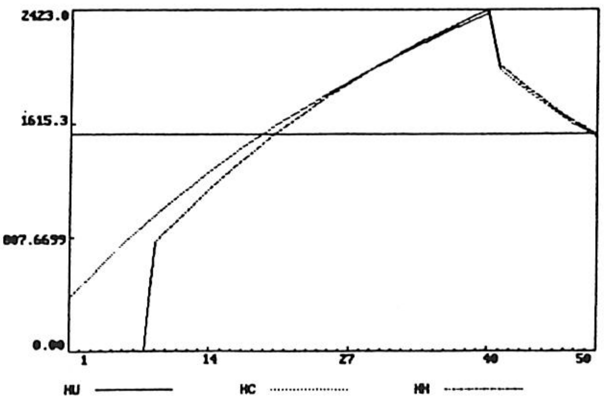

Figure 2.4(d): Financial wealth

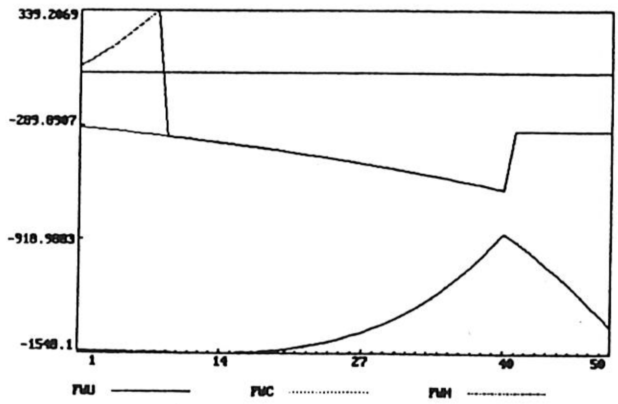

Figure 3.3(a): Consumption to income ratios; b changes from 0 to 100
Low bequest (cyr1) and high bequest (cyr20) cases

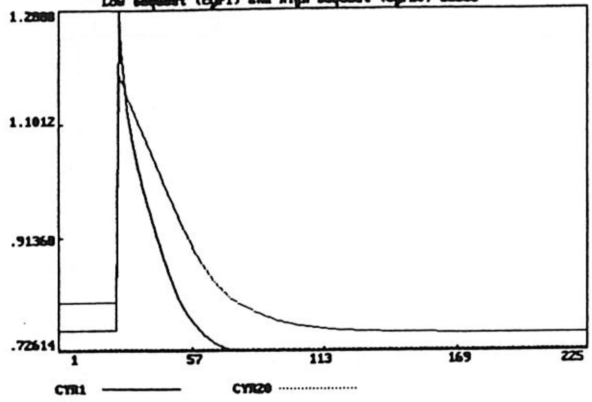

Figure 3.6(a): Consumption to income ratio; b changes from 2 to 5
A (no change in ph), B (ph jumps 3x) C (ph jumps 6x)

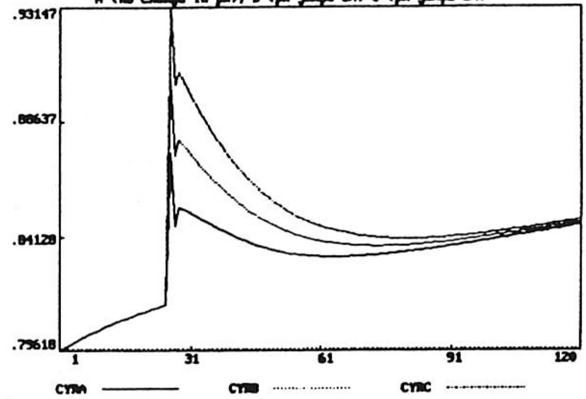

Figure 3.3(b): Total wealth to income ratios; as 3.3(a)

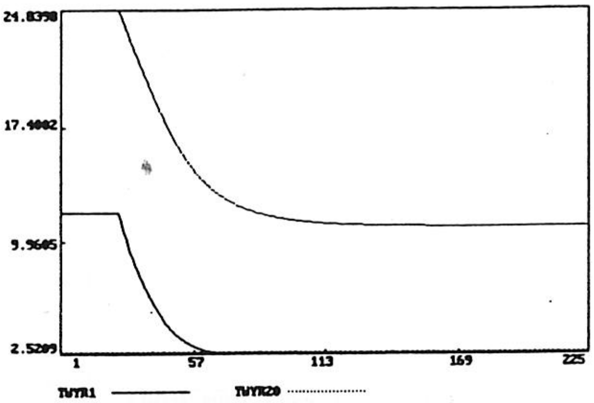

Figure 3.6(b): Housing stock

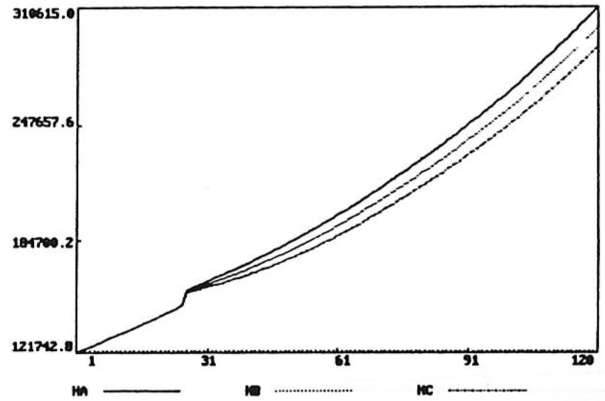

Figure 3.1(a): Simple model without durables or bequests
Consumption to income ratio; b changes from 0 to 100

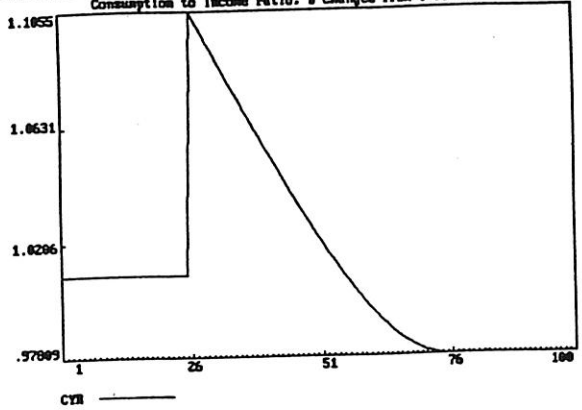

Figure 3.2(a): Convergence of c/y for alpha=20 (uss=unconstrained steady state
u=unconstrained, c= constrained with b=0)

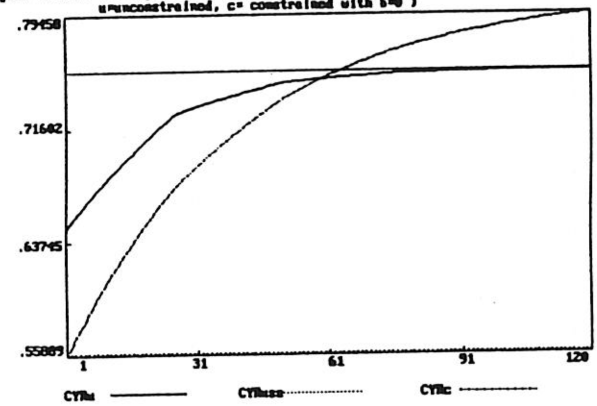

Figure 3.1(b): Wealth to income ratio (as 3.1(a))

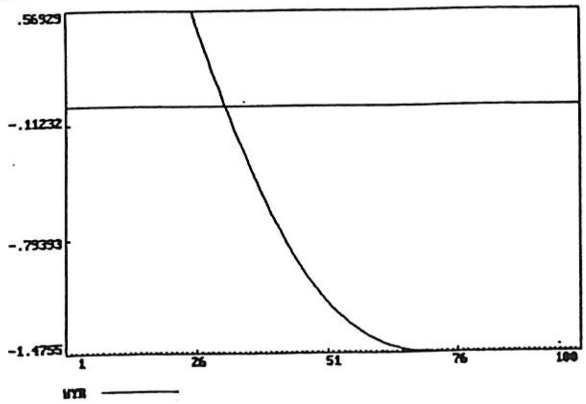

Figure 3.2(b) : Convergence of c/y with alpha = 1 (unconstrained (u)
and constrained (c))

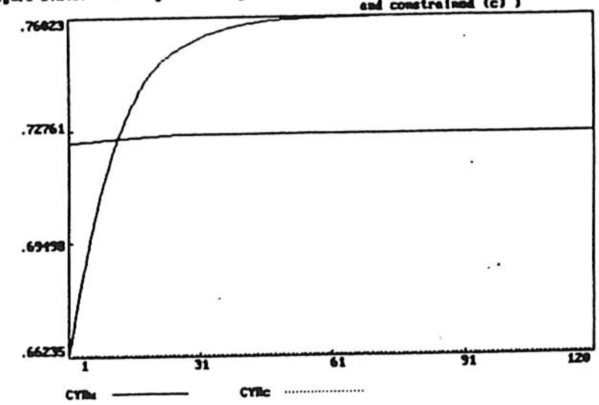

Figure 3.4(a): Consumption to income ratio: alpha =20
A (PHG=0, B=100) ,B (PHG=.01 B=100) C (PHG=.01 B=0)

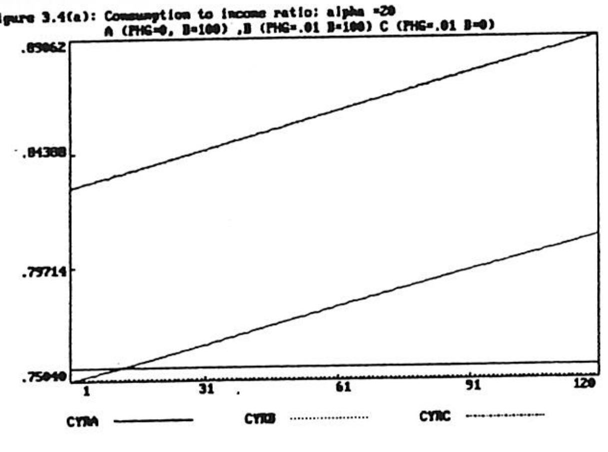

Figure 3.4(b): Housing to income ratio

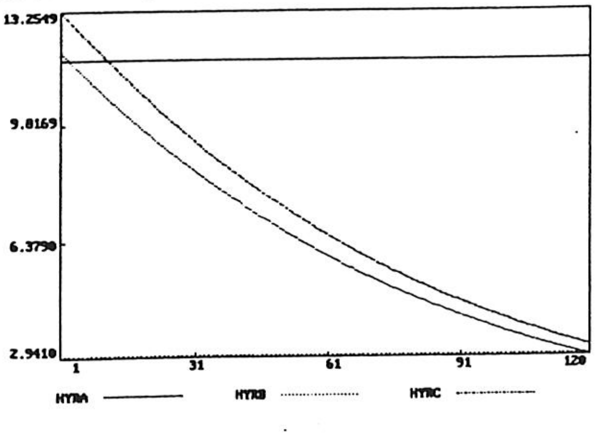

Figure 3.4(c): Total wealth

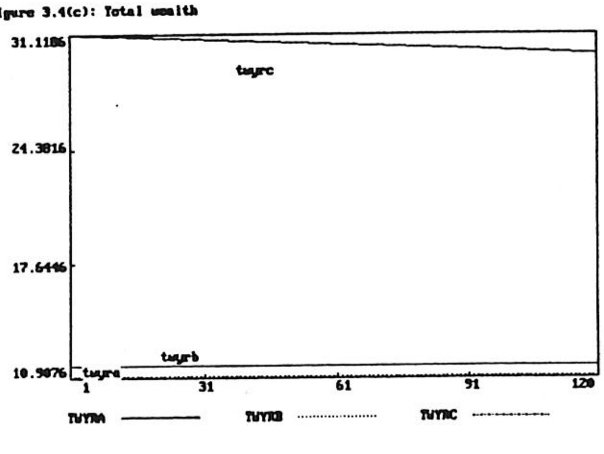

Figure 3.4(d): Financial wealth