Targetting Inflation with Nominal Interest Rates

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1 Introduction

While the UK government was committed to membership of the ERM for the exchange rate, interest rate policy was dedicated to the sole task of keeping sterling within its pre-assigned bands. By constraining the UK exchange rate to move in line with that of a low inflation country like Germany, the focus of monetary policy was on the intermediate target, the nominal exchange rate, rather than on the stated objective of policy, price inflation. Such a commitment to an intermediate target may be the most effective means of controlling the final objective, in particular, if that commitment is more credible. Indeed, as the commitment becomes more credible, so the monetary policy rule becomes closer and closer to one which sets interest rates equal to German rates. Once this stage is reached, the question of how to set domestic interest rates becomes trivial. The policy problem moves on to the design of monetary policy rules for the currency union as a whole.

As is well known, in September 1992, the UK authorities were forced to suspend Sterling's membership of the ERM. Since then, despite unchanged final objectives, the announced policy framework has altered operationally. Quoting the most recent Medium Term Financial Strategy (MTFS):

> 'It is the role of monetary policy to deliver low inflation. The aim is to keep underlying inflation ... in the range 1–4%. However, monetary policy influences inflation with a lag. Interest rate decisions are therefore based on an assessment of the prospects for underlying inflation in one to two years time. That assessment is based on a range of monetary indicators ... in particular, the growth of narrow and broad money and movements in the exchange rate and asset prices ... the extent of spare capacity in the economy and the overall stance of fiscal policy are also taken into account.'

MTFS, November 1993, paras. 2.04–2.05.

One of the final objectives of macroeconomic policy, price inflation, now appears as a direct target with a range of desired values to be achieved. From a policy optimisation perspective, it seems eminently sensible to pay direct attention to the final objectives of policy. This obviously contrasts with the ERM regime where the target for price inflation was implicitly given by the inflation rate of the anchor country, or in the earliest vintages of the MTFS where the target values for inflation had to be inferred from the target growth rates of the different monetary aggregates.

This paper is concerned with the design of interest rate policy rules in this policy environment, both in theory and in practice. Of course, as the MTFS acknowledges, any policy for interest rates must take into account the potentially complicated lagged response from interest rates on to inflation. We would argue that this is best done by using a macroeconomic model which takes into account all channels of influence of interest rates.

The MTFS does not make explicit how different monetary indicators should be interpreted in their effect on future inflation. We would argue that these monetary variables, if important, should already be incorporated in the inflation transmission mechanism embedded in the macroeconomic model. Moreover, as we seek to illustrate more clearly below, a stated policy which attempts to anticipate future inflation may well be consistent with an interest rate rule which reacts to the current values of a range of such indicators.

The MTFS also remains unclear on the exact criteria by which interest rate policy should be designed to control inflation. Since the real objective of policy is to achieve higher living standards (or perhaps more properly maximise discounted future consumption), should the government be indifferent to any output losses, albeit temporary ones, that may be incurred in bringing inflation down through tight monetary policy? We clarify this issue in our analysis below (for an earlier critique of the role of the short term output-inflation trade off in the MTFS, see Westaway and Wren-Lewis, 1993).

If macroeconomic models are to be used for the design of monetary policy rules, it is vital to understand clearly how they determine price inflation. Section 2 addresses this question, by focusing on the logically prior question of how the price level itself is determined. We argue that for the type of macroeconomic models used for policy analysis in the UK (for example, by HM Treasury), sensible price level behaviour requires a monetary policy rule other than fixed interest rates, nominal or real. This suggests that previous model simulation work, particularly that based on rational expectations models, solved under the assumption of fixed nominal interest rates, has been misleading.

In section 3, we examine the monetary policy design problem by analysing a stylised four equation macromodel incorporating the most important features of the large scale empirically based model. This simple framework allows a number of basic issues in the design of monetary policy to be illustrated. We show how possible problems of price level indeterminacy are resolved by a simple proportional rule on the price level, while the level of inflation itself can be controlled by an integral feedback (which relates the change in interest rates to the inflation error). Under the inflation targetting regime, we show how the amount of price level slippage in the face of shocks to inflation will depend inversely on the interest rate feedback coefficient but will be exaggerated further if policymakers attempt to take output deviations into account. We also illustrate how a simple inflationary bias will emerge with such rules if the desired or target level of output is above the natural rate, just as in the Barro-Gordon analysis. We illustrate the important role of price inertia in determining the appropriate interest rate policy; inflation control can be made very precise when price setters and foreign exchange markets are forward-looking simply by announcing a sufficiently vigorous feedback rule on interest rates. This is contrasted with a situation where price setting is not forward-looking, in which case inflation control may become unstable if too strong an interest rate rule is anticipated.

Section 4 uses the stylised model to analyse optimal interest rate policy. We show how the fully optimal policy can be expressed as an integral feedback rule on all the states of the model (including the error in inflation) plus a time-varying term in the lagged states, this latter term accounting for the inherent time-inconsistency of this policy.[1] The coefficients for this rule are compared with those on the time-consistent policy rule which feeds back on the current predetermined states only. The results from these optimal policies are compared with optimal simple rules, derived by restricting certain feedback parameters to be zero.

[1] For a similar derivation of these results, see Currie and Levine (1985).

Section 5 compares the performance of different *simple* rules on the full NIESR model. For price level shocks, we illustrate how interest rate rules designed to change the level of M0 and money GDP perform very similarly to rules which directly target the price level itself. More interestingly, we illustrate the effects of an announced policy to reduce the rate of inflation by 1%. As with the stylised model, we show how the assumption of full credibility of the announcement can imply a counter-intuitive fall in interest rates and an implausibly quick improvement in inflation. We show how this effect can be modified by assuming that the private sector learns more gradually about the monetary authorities' true policy intentions.

2 Monetary policy and prices

Two separate arguments have been advanced to imply that the assumption of fixed nominal interest rates have implications for the price level which make such a policy infeasible. The first is the familiar Wicksell argument that such a policy is dynamically unstable. Any positive shock to inflation will almost always cause real interest rates to fall thus stimulating demand. This will, in turn, further boost inflation. With fixed nominal interest rates, prices rise without limit.

The second argument is associated with the indeterminacy of the price level. If a macroeconomic model can be expressed entirely in real terms, that is by dividing all nominal variables by the price level, then the model will usually be consistent with a unique real equilibrium, but one which can be associated with any price level. In a model where there is nominal inertia and where expectations are backward-looking, then the price level will be tied down by its own lagged values. However, as Sargent and Wallace (1975) noted, if the price level is infinitely flexible or if it is forward-looking (or one of its determinants is forward-looking), then the price level will be indeterminate in the sense that there is no unique rational solution and that real equilibrium is consistent with an infinity of nominal equilibria.[2] This argument was established for a simple analytic rational expectations model by Sargent and Wallace (1975). A voluminous subsequent literature sought to disprove this assertion for more general models but as

[2] McCallum (1986) has drawn a distinction between solutions which have a unique real equilibrium for any price level and one where only the long run real solution is unique but the transient path will be affected by the choice of price level.

will be illustrated below, the result remains relevant for a wide range of empirically relevant models.

Price level behaviour with fixed nominal interest rates in large scale macromodels has been investigated by Gagnon and Henderson (1990). The price equations in these models were typically characterised by expectations-augmented Phillips curves. For those models without forward expectations in the price equations, they conclude that there may be instability in some models but not others. The *absence* of instability is attributed to either model inadequacy (for example due to a non-vertical Phillips curve) or the presence of wealth effects. These have a stabilising influence in that assets denominated in nominal terms are diminished by inflation which in turn reduces expenditure.

For the one model they consider with rational expectations, they are unable to find a solution, and conclude that there is price level indeterminacy. This accords with McKibbin (1993) who believes that the MSG2 model is similarly indeterminate. Henderson and McKibbin (1993a, b) argue that the price level may be determinate but that the conditions under which it is are rather obscure. Given that the policy instrument of the major central banks is mostly the nominal interest rate then any simulation is at best incomplete in the absence of a monetary policy rule and possibly invalid.

These issues should be familiar to macromodellers in the UK, although typically the problem has not been seen as one of price level indeterminacy but rather one of how to solve the exchange rate under the assumption of forward-looking model consistent expectations. As we argue below, these issues are very closely related. To show this, it is useful to write down a stylised macroeconomic model which shares the most important characteristics of the large macromodels used in the UK. The equations are as follows;

$$y_{t+1} = \beta_1 y_t - \beta_2(r_t - \Delta p^e_{t+1}) + \beta_3(e_t - p_t) \tag{1}$$

$$\Delta p^e_{t+1} = \alpha_a \Delta p_t + (1-\alpha_a)\Delta e_t + \mu(e_t - p_t + \tau y_t) \tag{2a}$$

$$p_t = \alpha_b(p_{t-1} + p_{t+1}) + (1 - 2\alpha_b)(e_t + \tau y_t) \tag{2b}$$

$$e^e_{t+1} = e_t + r_t \tag{3}$$

where y is the log of output, r the nominal interest rate, p the log of the price level and e the log nominal exchange rate.[3]

Equation (1) is a dynamic IS curve with output depending on its lag, real interest rates and competitiveness (the real exchange rate). Equations ($2a$) and ($2b$) are two different versions of the price equation. Both of these are typical of the type of price equation typically employed in the main UK macromodels, but differ in an important respect from those analysed in Gagnon and Henderson (1990). Rather than taking the form of conventional Phillips curves, the equations for wages and prices are based on the imperfectly competitive framework popularised by Layard and Nickell (1985). The resulting long run expression for the price level can be written as a function of overseas prices (here simplified to be the exchange rate) with a unit elasticity and domestic output relative to capacity. In reduced form, the deviation of prices from its long run level then modifies the usual Phillips curve relationship. We consider two different versions of this equation; equation ($2a$) where price expectations are formed adaptively, and equation ($2b$) where they are forward-looking.[4]

The uncovered interest parity condition (3) which determines the nominal exchange rate completes the model. For now, we are treating nominal interest rates as exogenous. In analysing the behaviour of these stylised models, the parameter settings are $\beta_1 = 0{\cdot}75$, $\beta_2 = 0{\cdot}25$, $\beta_3 = 0{\cdot}2$, $\alpha_a = 0{\cdot}75$, $\alpha_b = 0{\cdot}45$, $\mu = 0{\cdot}2$ and $\tau = 1$. The α parameters in the two price equations are not strictly comparable but are chosen so that the speed of response of prices in response to an exogenous shock to the exchange rate are roughly equal.

The model has a number of notable features. It is a natural rate model, and output cannot be permanently moved from equilibrium. The real interest rate and real exchange rate are similarly constrained. Both static and dynamic homogeneity in prices ensure this. This allows either model to be rewritten to eliminate the price level. For example, using equation ($2a$), we obtain

$$y_{t+1} = \beta_1 y_t - \beta_2 i_t + \beta_3 \rho_t \tag{1'}$$

[3] In common with the international literature on exchange rates e is the domestic price of foreign exchange, *i.e.* a rise in e represents a devaluation. Foreign prices and interest rates are normalised to zero.

[4] For a full description of the determination of prices in the NIESR model, see Soteri and Westaway (1993).

$$\Delta p^e_{t+1} = \Delta p_t + (1-\alpha)\Delta\rho_t + \mu(\rho_t + \tau y_t) \tag{2a'}$$

$$\rho^e_{t+1} = \rho_t + i_t \tag{3'}$$

with

$$i_t = r_t - \Delta p^e_{t+1} \tag{4}$$

$$\rho_t = e_t - p_t \tag{5}$$

where ρ is the real exchange rate and i is the real interest rate.[5]

It is useful to write the model in matrix form to facilitate the analysis of its stability properties under different policy assumptions. For example, equations ($1'$), ($2a'$) and ($3'$) can be written in matrix form as

$$\begin{bmatrix} z_{t+1} \\ \rho^e_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_t \\ \rho_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} r_t \tag{6}$$

where

$$z_t = [T_t \quad r_{t-1} \quad y_t \quad i_{t-1} \quad \rho_{t-1} \quad \Delta p_t]'$$

and A and B are appropriately partitioned matrices. T_t is the target variable, included as a state with equation $T_{t+1} = T_t$. This facilitates inflation targets different from zero. As we require the *nominal* interest rate to be retained as the monetary instrument the real interest rate becomes a state variable. The nominal interest rate is also included as an artificial state. This allows changes to be used in the cost function, as detailed below.

Of course, it should be immediately clear from the fact that the model can be specified entirely in real terms that the exchange rate is indeterminate since, with exogenous nominal interest rates, equation (3) or ($3'$) has a pure unit root. Since the exchange rate is a forward-looking jump variable, the jump in the exchange rate in the face of any shock will be arbitrary. Some mechanism is required to remove this indeterminacy, or technically, to move the root associated with the exchange rate outside the unit circle so that the saddlepath stability condition is satisfied.

[5] In this form the model has no trending variables for an arbitrary non-zero inflation rate.

2.1 An aside on solving large models with fixed interest rates

For some time, many macromodellers in the UK have argued that this unit root in the exchange rate equation can be removed by the appropriate inclusion of a risk premium term. The NIESR model has incorporated a term in the ratio of net overseas assets to GDP (see NIESR, 1994). The new Strathclyde model has adopted the same specification. The Warwick Macroeconomic Modelling Bureau have argued, mainly on econometric grounds, for a term in the current balance to GDP ratio. By including such terms, it has been argued that the model can be solved with an endogenous exchange rate but with fixed nominal interest rates. Indeed, the Warwick Macromodelling Bureau run comparative simulations across models on precisely this basis (see Church *et al.*, 1993, for example). The question of whether these terms should be included should ultimately be based on statistical grounds, but if it is to be argued that their presence removes the unit root in the exchange rate equation, it must be argued that the level of the nominal exchange rate affects the risk premium in the long run. If not, then the unit root is preserved and the model will not solve.[6]

We would argue that if the full model is coherently specified, then it is unlikely that the level of the exchange rate and prices will influence the outcome for real variables. Putting this another way, we would expect that a full model simulation which exogenously changes the level of the nominal exchange rate should, in the long run, return to the same equilibrium (see Westaway, 1992a, for an illustration of this feature on the NIESR UK model, and Barrell *et al.*, 1992, for an exposition of this argument in the context of a world model). The crux of this argument is as follows; if each domestic sector of the model embodies a well-defined stock-flow equilibrium (guaranteed by the consumption-income-wealth relationship for the personal sector and by some form of solvency condition for the government sector) then by identity, the overseas sector current balance or asset stock must similarly return to the same equilibrium position, regardless of the underlying price level. A number of counter-arguments are sometimes made to this in an attempt to justify the use of exogenous nominal interest rates with forward-looking exchange rates. It is suggested that the presence of nominal rigidities,

[6] In Fisher *et al.* (1992) no long run effect from the exchange rate on the basic balance term is found in the Treasury model. They interpret this as a problem.

for example the existence of non-interest bearing money, can tie the price level down. However, this is ruled out if behavioural relationships which target sectoral wealth to income ratios are enforced. Of course, this may not be entirely realistic, but the postulated existence of such non-rational behaviour is a rather unsatisfactory basis on which to determine the exchange rate in model simulations.

In practice, too, it is often the case that the finite nature of the time horizon over which forward-looking macromodels are solved can often deliver a spurious solution. For example, if the current balance to GDP ratio does not return to base for a purely nominal shock simply because the simulation period is too short then this will prompt a determinate jump in the exchange rate which in theory may be quite unjustified. Model inadequacies which may show up in long simulation bases will have a similar effect. In either case, again, this is a very shaky basis on which to base endogenous exchange rate simulation results.

3 Closing the model with interest rate rules

Having argued that the stylised model is indeterminate in prices when the model is solved 'open-loop', that is with interest rates exogenous, we now turn to analysis of the 'closed-loop' model, that is when interest rates are manipulated according to some policy reaction function. Since the model is linear, we can solve the model in this section using the solution technique developed for rational expectations models by Blanchard and Kahn (1980).

One policy rule commonly adopted in model simulations is to adjust nominal interest rates so that the real interest rate, however defined, stays constant. In fact, this policy rule is also problematic for a number of reasons. First, an attempt may be made to fix the real interest rate at a level inconsistent with the equilibrium of the model. Of course, it can be argued that in open economy models with internationally mobile capital, the real interest rate is determined by the world level so fixing real interest rates will be acceptable in simulation mode. However, this policy rule does not solve the problem of price level indeterminacy since constant real interest rates will be consistent with any level of price inflation. It is easy to show this using our stylised model. We would assert that this result also holds in a wide range of larger models, even when full model dynamic homogeneity may not hold.

We now turn to consideration of simple feedback rules for interest rates. What properties must the rule have to tie down the price level? Does it matter that interest rates must be increased to curb inflation but in the long run will themselves be lower? Does this have implications for the appropriate sign of the feedback coefficients?

For simplicity, we focus on simple proportional and integral rules which feedback on prices or inflation and on a range of other indicators.[7] As we shall illustrate below, these simple rules are closely related to the type of rules which emerge from a full optimisation exercise.

3.1 Feedback control on prices

The obvious form of control rule to adopt in order to solve the problem of price level indeterminacy is to incorporate a price level target explicitly in the interest rate reaction function, as follows;

$$r_t = \theta(p_t - p_t^*)$$

This may be interpreted as a proportional control rule on the price level. For the parameters set as in equations (1) to (3) above, and with θ set to unity, the saddlepath stability condition is met for both models (the unstable roots are 1·5854 for the model with the backward-looking price equation and a complex pair with modulus 1·4944 for the model with the forward-looking version).

Charts 1(a) and (b) show the response of interest rates, prices and output for each model. Both show nominal interest rates rising to drive the price level to its new target level 1% below base. With backward-looking expectations in prices, the interest rate rise is 1%, approximately twice as much as when price expectations are forward-looking. The resulting fall in output of 0·75% is correspondingly around twice as large in the backward-looking case. As with all real variables, this movement in output away from equilibrium is temporary for both models although the response is much more oscillatory when expectations are adaptive.

It is interesting to examine the sensitivity of these responses to changes in the parameter in the feedback rule. In the backward-looking case, as the strength of the

[7] Phillips (1954) represented the first attempt to apply this form of control theory to an economic model. More recent examples include Westaway (1986), Weale *et al.* (1989).

feedback coefficient on prices is increased, so price level control is at first improved as the magnitude of the initial interest rate hike is increased but gradually this occurs at the expense of an increasingly oscillatory response in the price level around its required target level until eventually the response becomes dynamically unstable; chart 2(a) illustrates for θ=8.0 when the unstable root is 2·8. On the other hand, when the price level acts as a forward-looking jump variable like the exchange rate, then as the gain on the feedback rule is increased so the initial movement in interest rates increases but it results in much closer control of the price level about its target without any oscillatory response. The higher is θ, the faster this control becomes; chart 2(b) illustrates for $\theta = 1000$ (when the unstable roots are 13·1). In both cases, as the dynamic response of prices is quickened (for example, by lowering the coefficient α_b in equation (2*b*), so the required movement in interest rates becomes smaller and price level control becomes more accurate.

It is also interesting to examine the response when the feedback parameter becomes arbitrarily small. This feedback rule has sometimes been used as a benchmark model for optimal control exercises which illustrates how the model behaves when the monetary policy instrument is 'almost' exogenous; see Currie and Levine (1985), for example. Charts 3(a) and (b) illustrate for $\theta = 0·001$. For both models, since interest rates hardly move at all, the exchange rate jumps immediately to its required long run level 1% below base. With forward-looking prices, this is associated with a slow monotonic convergence of the price level to its target. With the backward-looking price equation, however, prices display a long cycle bordering on instability which shows little sign of settling down.

3.2 Feedback control on inflation

Although it is quite common to simulate macromodels by artificially adjusting the target price level, it is much more realistic, as supported by the earlier quote from the MTFS, to use interest rates to control inflation. The most simple rule would appear to be a simple extension of the proportional rule, replacing the price level target by the inflation target, *i.e.*

$$r_t = \theta(\Delta p_t - \Delta p_t^*).$$

However, as with the closely related fixed real interest rate rule, it does not determine the price level. This is obvious once this interest rate rule is incorporated in the model specified in real terms since the price level is still completely absent. As usual, this indeterminacy is confirmed by calculating the roots of the model.

One obvious way to target inflation is simply to use the proportional control rule on the price level and alter the rate of change of the price level target. In fact, it is easy to show that this is equivalent to an integral control rule of interest rates on inflation, *i.e.*

$$\Delta r_t = \theta(\Delta p_t - \Delta p_t^*).$$

Clearly, this is the first difference of the proportional rule on the price level. By setting $\theta = 1$ in this interest rate rule, the price level indeterminacy is indeed removed and the saddlepath stability condition is satisfied for both models with exactly the same unstable roots as when the equivalent proportional rule was used on the price level. It should be noted that, despite the fact that the long run movement in interest rates is always in the opposite direction to that initially required to bring inflation down, the appropriate integral control coefficient is still positively signed.

Charts 4(a) and (b) show the results for interest rates, inflation and output. Perhaps the most striking aspect of these results is the contrast between the interest rate responses. With backward-looking prices, interest rates initially rise by 1%, the magnitude of the feedback parameter, causing inflation to fall a very small part of the way towards its new target level. On the other hand, when prices behave as a 'jump' variable, inflation actually overshoots its target level causing interest rates to

fall immediately, although real interest rates do of course rise.[8] As with the price level simulations, the adverse temporary output response is much larger when expectations are adaptive.

This difference between the models can be emphasised still further if we again analyse how inflation control is affected as the strength of the announced feedback control coefficient is increased. In the forward-looking case, as θ is increased, so the change in the inflation rate and the corresponding change in the interest rate become self-fulfilling. The quicker are the price dynamics, the easier it is to bootstrap the inflation rate to its target level. In the limit, with a very large feedback coefficient, the inflation rate can change instantaneously to its target level with no change in any other real variables. Observationally, this simulation will appear as a 1 point cut in interest rates bringing about a one point fall in inflation, but importantly, this is not the same as announcing a 1% cut in the level of interest rates since that policy would not yield a determinate solution.[9] Chart 5(a) illustrates for $\theta = 1000$. By contrast, with more inertia in the backward-looking price equation, it is possible to speed up the initial control of inflation but if the feedback is too strong, the response becomes increasingly oscillatory until eventually interest rates become unstable. Chart 5(b) illustrates for $\theta = 8$.

This is an important difference between backward and forward-looking models which seems to show the inherent stability advantages of the forward-looking version which is robust to feedback rules which are either very weak or very strong. Of course, all of the above simulations are run on the assumption that the policy rule is completely believed and understood. Later, we will relax this assumption in our simulations carried out on the full NIESR model.

A number of additional points of interest emerge from this simple analysis of the integral control rule of interest rates on inflation. Because this control rule is equivalent to a proportional rule on the price level, then there will be a steady state offset between the price level and its target level whenever the target level is growing relative to base

[8] This overshooting response of the inflation rate is reminiscent of the overshooting result in the Dornbusch exchange rate model, although it is achieved by a slightly different mechanism.

[9] It has sometimes been wrongly claimed that an open loop permanent cut in interest rates would bring about an infinite fall in interest rates and have implemented temporary changes instead. Rather, we would argue that any open loop simulation is undefined.

(see Salmon, 1982). In fact, it is easy to calculate the amount of price level slippage for any change in the inflation target.

Given the feedback rule,

$$\Delta r_t = \theta(\Delta p_t - \Delta p_t^*)$$

suppose we are driving the inflation rate down by 1%, then the interest rate must end up 1% lower. Since the price level slippage will be given by the sum of the inflation errors, then this slippage multiplied by the feedback parameter must equal the sum of the interest rate changes. Therefore, the slippage will amount to $1/\theta$. Interestingly, this slippage will still occur, even if the authorities are explicitly targetting the price level with a proportional feedback rule. To achieve a price level target, integral control on the level of prices will be required.[10] The issue of how whether to choose a level or rate of change for any nominal anchor, whether it is the price level itself or the money supply, is an interesting question in itself which we do not address here.[11]

3.3 Feedback control on additional indicators

So far, since we have been focusing on the issue of price level determinacy, we have only focussed on rules which feed back on the level or rate of change of prices. However, it is quite possible to implement different interest rate feedback rules which achieve other objectives without compromising the stability condition required for the model to solve.

(a) Feedback on nominal variables;

It is easy to show that the price level will be determinate so long as interest rates feedback on the level of some nominal variable. For example, interest rate policy is often defined in terms of control of some monetary aggregate on the grounds that money causes inflation. Obviously, this causal role does not apply in our stylised model since prices are ultimately driven by overseas prices and the exchange rate. Yet, it is is

[10] Again, see Salmon (1982) for a technical explanation.

[11] An interesting exchange on this question between Sir Samuel Brittan and Stanley Fisher recently appeared in the columns of the Financial Times; see 'Beware Ivy League Central Bankers', June 13, for example.

quite possible to incorporate an equation for the money supply into this model where money is purely passive, for example,

$$m_t = \lambda m_{t-1} + (1 - \lambda)(p_t + y_t).$$

In the long run this money stock will grow in line with nominal income but has no influence on the model at all. In fact, this type of equation for a monetary aggregate is quite similar to the way such equations are often specified in many large macromodels; for example, the equations for M0 and M4 in the NIESR model are both passively endogenous in the sense that they are almost entirely post-recursive ('output variables'). Nevertheless, it is quite possible to specify an interest rate rule which drives such a money stock to a particular level which will thereby tie down the price level. This will be illustrated in section 5 with a simulation on the NIESR model.

(b) Feedback on the exchange rate

This type of feedback rule will serve to determine the exchange rate in the same way that the above rules have been used to tie down the price level. This in turn will determine the price level. However, since the equations for the interest rate and exchange rate can be solved independently of the rest of the model, then there is no reason for either variable to move in simulations once the exchange rate has achieved its desired level. This type of rule can be used to justify running simulations under the fixed interest rate fixed exchange rate assumption.

(c) Feedback on output

So far, all of the rules considered have assumed that interest rate policy is assigned to one target only. This may be an accurate characterisation of how exchange rate policy was announced during the UK's period of ERM membership, or how inflation targetting is described in the quote from the MTFS at the beginning of this paper. In practice, however, policymakers are likely to be concerned with more than one policy objective. In particular, they are unlikely to be indifferent to the output consequences of any particular policy rule, even if they know that in the long run, output must return to its natural rate (see Westaway and Wren-Lewis, 1993).

In the next section, we will consider more deeply the appropriate specification of policy in such an environment of multiple objectives. For now, we will mention briefly the implications of modifying the rules considered so far to take into account movements in output away from equilibrium.

First, consider the implications of modifying an inflation targetting rule by incorporating an effect from output disequilibrium, that is, interest rates rise by less for a given upward shock to inflation so as to mitigate the adverse consequences for output, *i.e.*

$$\Delta r_t = \theta_1(\Delta p_t - \Delta p_t^*) + \theta_2(y_t - y^*)$$

where y^* is the equilibrium level of output (which will be the base level in our simulations). In general, of course, there will be limitations imposed by the stability of the model on the maximum size of the coefficient on y. Compared to our earlier analysis, the most obvious effect of including this output effect is to increase the amount of price level slippage in the face of a unit shock to inflation by $(\theta_2/\theta_1) \times \sum_0^\infty (y_t - y^*)$. This also implies that for a given inflation target, an integral control rule on inflation will succeed in keeping the price *level* at its implicit target unless the inflation rule is augmented by some other indicator variable such as output. Interestingly, too, it is easy to show how this type of rule can impart an inflationary bias to policy whenever policymakers attempt to target output around a level which is above its natural rate, say at $\hat{y}_t$. Suppose that the policy rule is specified as

$$\Delta r_t = \theta_1(\Delta p_t - \Delta p_t^*) + \theta_2(y_t - \hat{y}_t)$$

then, in equilibrium, inflation will be

$$\Delta p_\infty = \Delta p_\infty^* + \frac{\theta_2}{\theta_1} \times (\hat{y}_\infty - y^*).$$

This inflationary bias emerges for much the same reasons as the familiar Barro-Gordon analysis, although importantly, our simple result does not depend on the existence of forward-looking expectations. Ammer and Freeman (1994) discuss the issues involved and attempt to assess the success of anti-inflationary policies in the UK and elsewhere. Interest rate rules which react to different combinations of inflation and output have been analysed extensively in the context of world models in a number of papers in

Bryant *et al.* (1993) where the circumstances under which this type of policy might be desirable are examined. This rather informal type of analysis rather begs the question that if policymakers do have a well defined objective function, what is the optimal strategy for interest rates and how well can such an optimal strategy be mimicked by these more simple rules? We turn to these issues in the next section.

4 Optimal Control

For ease of comparison with the stated objectives of the MTFS in the UK, we analyse policy under the stylised assumption that policymakers are only concerned with inflation relative to some target level and output relative to its equilibrium path. In this section, we choose to focus only on the backward-looking version of the price equation (*i.e.* equation (2a)). For analytical simplicity we use a quadratic objective function of the form

$$V = \sum_{t=0}^{\infty} \delta^t \left((1-\kappa)(\Delta p_t - T_t)^2 + \kappa y_t^2 + \frac{1-\kappa}{\gamma} \Delta r_t^2 \right) \quad (7)$$

with $0 \geq \kappa \geq 0{\cdot}5$. We penalise *changes* in interest rate movements and not deviations from base. This is because otherwise it will never be optimal to achieve a non-zero inflation rate target. This is a reflection of the desirable property of such cost functions that they introduce an integral action on the targets, so that the target is hit in steady state (Salmon, 1982, Kwakernaak and Sivan, 1972).

The choice of κ determines how much the policy maker cares about inflation relative to output, and the weighting on interest rate changes retains the same proportionate costs between inflation and interest rate movements regardless of the value of κ. This formulation implies that for non-zero κ output should be returned to its natural rate whilst inflation can be targetted at an arbitrary level.

In order to calculate optimal policies we augment the matrix form of the model (6) with an equation which defines variables which we are interested in but which do not appear as states of the model, *i.e.*

$$\begin{bmatrix} \Delta p_t - T_t \\ y_t \\ \Delta r_t \end{bmatrix} = C \begin{bmatrix} z_t \\ \rho_t \end{bmatrix} + D r_t \quad (8)$$

where C and D are appropriately defined matrices. If we further define

$$W = \begin{bmatrix} 1-\kappa & 0 & 0 \\ 0 & \kappa & 0 \\ 0 & 0 & \frac{1-\kappa}{\gamma} \end{bmatrix}$$

then (7) can be written in matrix form as

$$V = \sum_{t=0}^{\infty} \delta^t \left([\, z_t' \quad \rho_t \,] Q \begin{bmatrix} z_t \\ \rho_t \end{bmatrix} + 2 [\, z_t' \quad \rho_t \,] U r_t + r_t R r_t \right) \tag{9}$$

where $Q = C'WC$, $U = C'WD$ and $R = D'WD$.

The first order conditions for a minimum of (6) and (9) yield a model under under control given by

$$\begin{bmatrix} I & \delta B R^{-1} B' \\ 0 & \delta(A' - UR^{-1}B') \end{bmatrix} \begin{bmatrix} \begin{pmatrix} z_{t+1} \\ \rho^e_{t+1} \end{pmatrix} \\ \begin{pmatrix} \xi^1_{t+1} \\ \xi^2_{t+1} \end{pmatrix} \end{bmatrix} = \begin{bmatrix} A' - U'R^{-1}B & 0 \\ -(Q - UR^{-1}U') & I \end{bmatrix} \begin{bmatrix} \begin{pmatrix} z_t \\ \rho_t \end{pmatrix} \\ \begin{pmatrix} \xi^1_t \\ \xi^2_t \end{pmatrix} \end{bmatrix} \tag{10}$$

where $(\,\xi^{1\prime}_t \quad \xi^2_t\,)'$ is a partitioned vector of costate variables (Lagrange multipliers) associated with the predetermined and free states respectively.

A standard control problem (without rational expectations) would be solved by finding a constant matrix S such that $\xi_t = S z_t$ which puts (10) on the saddlepath[12] with a feedback rule for the instruments of $r_t = -F \begin{bmatrix} z_t \\ \rho_t \end{bmatrix}$ with F given by

$$F = (R + \delta B'SB)^{-1}(U' + \delta B'SA). \tag{11}$$

The state variables would then evolve with transition matrix $A - BF$, *i.e.*

$$\begin{bmatrix} z_{t+1} \\ \rho^e_{t+1} \end{bmatrix} = (A - BF) \begin{bmatrix} z_t \\ \rho_t \end{bmatrix}. \tag{12}$$

With some states non-predetermined this does not give sufficient conditions for an optimum. This can be shown to require $\xi^2_0 = 0$ (Calvo, 1978). ξ^2 is then treated as an additional predetermined variable. Partition S so that $\xi^2_t = S_{21} z_t + S_{22} \rho_t$, so $\rho_t = -S_{22}^{-1} S_{21} z_t + S_{22}^{-1} \xi^2_t$. Substituting into (12) for ρ_t and ρ^e_{t+1}, it is easy to establish that the predetermined variables evolve according to

$$\begin{bmatrix} z_{t+1} \\ \xi^2_{t+1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ S_{21} & S_{22} \end{bmatrix} (A - BF) \begin{bmatrix} I & 0 \\ -S_{22}^{-1} S_{21} & S_{22}^{-1} \end{bmatrix} \begin{bmatrix} z_t \\ \xi^2_t \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} z_t \\ \xi^2_t \end{bmatrix}. \tag{13}$$

[12] This is done this by an eigenvalue-eigenvector approach which was the inspiration for the Blanchard and Kahn (1980) solution method.

The system under control can be solved as we have enough initial conditions. The trajectories of the instruments can then be determined from

$$\begin{aligned} r_t &= -F_1 z_t - F_2 \rho_t \\ &= -(F_1 - F_2 S_{22}^{-1} S_{21}) z_t - F_2 S_{22}^{-1} \xi^2_t \\ &= -\tilde{F}_1 z_t - \tilde{F}_2 \xi^2_t \end{aligned} \tag{14}$$

where writing the feedback rule as a function of ξ^2 emphasises the time inconsistent nature of the optimal solution. (14) can be solved recursively for ξ^2 (remembering that $\xi^2_0 = 0$) gives

$$\xi^2_t = \sum_{i=0}^{t-1} \Gamma^i_{22} \Gamma_{21} z_{t-1-i} \tag{15}$$

so that using (15) in (14)

$$r_t = -\tilde{F}_1 z_t - \sum_{i=0}^{t-1} \tilde{F}_2 \Gamma^i_{22} \Gamma_{21} z_{t-1-i} \tag{16}$$

gives the optimal policy as a function of current and past discounted states, depending on the period in which the policy is implemented. Re-implementing the policy at any time past that initial period necessarily involves different values for current and future dated instruments.

To avoid such problems, it might be better to only consider time-consistent policy rules. The widely used solution method of Oudiz and Sachs (1985) restricts the feedback rule to the predetermined states alone. This can be likened to a dynamic game equilibrium, where a fixed point is sought between two sets of competing agents, the policy maker and a private sector. In equilibrium the pair $r_t = -F z_t$ and $\rho_t = -N z_t$ for fixed F and N is a time consistent (subgame perfect) equilibrium. See Blake (1992) for details and exact solution algorithm used here.

5 Optimal policy rules

For there to be such a thing as an optimal policy it must be with respect to a particular loss function. The fixed parameters in (7) are assumed to be $\delta = 0{\cdot}9875$ and $\gamma = 100$, with $\kappa \in [0, 0{\cdot}5]$. We carried out a number of exercises on the model, both to calculate fully optimal time-inconsistent and time consistent policies, and optimal (in a sense defined below) simple policies. They can be usefully contrasted.

5.1 Optimal time inconsistent and time consistent policies

Using the algorithms outlined in the previous section we calculated optimal policies for various values of κ at increments of 0·05. In Table 1 we give the welfare losses for two deterministic displacements to the state. The first is an announced reduction in the target for inflation of 1%, the second a disturbance to output at t_0 of unity.

Table 1. Welfare losses for optimal control.

	Welfare losses			
	Time inconsistent		Time consistent	
κ	Inflation	Output	Inflation	Output
0·00	0·5200	0·0010	0·5227	0·0012
0·05	0·5532	0·0398	0·5590	0·0401
0·10	0·5808	0·0766	0·5926	0·0772
0·15	0·6032	0·1116	0·6231	0·1124
0·20	0·6206	0·1449	0·6504	0·1461
0·25	0·6334	0·1767	0·6740	0·1781
0·30	0·6416	0·2071	0·6937	0·2088
0·35	0·6454	0·2361	0·7094	0·2381
0·40	0·6448	0·2640	0·7207	0·2661
0·45	0·6398	0·2906	0·7273	0·2989
0·50	0·6303	0·3161	0·7288	0·3185

It is apparent that as might be expected an increase in κ raises the relative advantage[13] of the time inconsistent policy to the time consistent policy in targetting inflation. Surprisingly, there is no improvement in comparing the responses to the output disturbance.

Turning to the coefficients of the optimal rules, Tables 2 and 3 give the values of $-F$ for both types. Both sets of rules always have in common a zero coefficient on the lagged real interest rate (more on this below) and a sign switch on the output variable as soon as output is included in the loss function. For the time consistent control rules (Table 3) a high degree of constancy is observed in all the other parameters for the various loss functions. This is less true of the time inconsistent policies (Table 2) as

[13] Given by the ratio of the welfare losses.

Table 2. Policy rules for optimal time inconsistent control.

	Feedback coefficients						
	State variables						Costate
κ	T_t	r_{t-1}	y_t	i_{t-1}	ρ_{t-1}	Δp_t	ξ_t
0·00	−0·0529	0·3132	−0·0091	0·0000	−0·1849	0·7397	−1·9767
0·05	−0·0151	0·3004	0·0431	0·0000	−0·1787	0·7147	−1·9393
0·10	0·0188	0·2879	0·0933	0·0000	−0·1733	0·6933	−1·9073
0·15	0·0494	0·2755	0·1419	0·0000	−0·1688	0·6751	−1·8802
0·20	0·0768	0·2634	0·1892	0·0000	−0·1650	0·6598	−1·8576
0·25	0·1015	0·2513	0·2356	0·0000	−0·1618	0·6473	−1·8394
0·30	0·1236	0·2391	0·2814	0·0000	−0·1593	0·6373	−1·8252
0·35	0·1433	0·2269	0·3269	0·0000	−0·1575	0·6298	−1·8150
0·40	0·1607	0·2146	0·3724	0·0000	−0·1562	0·6247	−1·8088
0·45	0·1759	0·2020	0·4183	0·0000	−0·1555	0·6221	−1·8066
0·50	0·1890	0·1891	0·4649	0·0000	−0·1555	0·6219	−1·8085

Table 3. Policy rules for optimal time consistent control.

	Feedback coefficients					
	State variables					
κ	T_t	r_{t-1}	y_t	i_{t-1}	ρ_{t-1}	Δp_t
0·00	−0·1093	0·2293	−0·0183	0·0000	−0·2200	0·8800
0·05	−0·1113	0·2234	0·0232	0·0000	−0·2220	0·8879
0·10	−0·1123	0·2171	0·0650	0·0000	−0·2238	0·8952
0·15	−0·1125	0·2103	0·1069	0·0000	−0·2256	0·9022
0·20	−0·1120	0·2031	0·1491	0·0000	−0·2272	0·9089
0·25	−0·1110	0·1956	0·1915	0·0000	−0·2288	0·9154
0·30	−0·1095	0·1877	0·2344	0·0000	−0·2304	0·9217
0·35	−0·1075	0·1795	0·2778	0·0000	−0·2320	0·9280
0·40	−0·1052	0·1709	0·3219	0·0000	−0·2336	0·9343
0·45	−0·1025	0·1619	0·3670	0·0000	−0·2352	0·9407
0·50	−0·0995	0·1524	0·4134	0·0000	−0·2368	0·9471

there is a sign switch in the coefficient on the target for inflation (again we return to this below).

Graphing the responses under control show what might be expected from the welfare losses. In Charts 6(a) and (b) both the optimal time consistent and time inconsistent policies are shown for two values of κ, 0 and 0·5 which are the extreme values considered. For $\kappa = 0$, the two sets of rules give almost the same result in targetting inflation, both showing good control and a marked effect on output. For $\kappa = 0{\cdot}5$ they are more dissimilar with the time inconsistent solution trading off a larger output loss for quicker control of inflation, as well as engineering an immediate fall in interest rates.

It should be no real surprise that the time inconsistent policy for $\kappa = 0$ is not very time inconsistent from the parameters of the rule (16). Note this is the only practical way that the policy rule could actually be implemented. The values in the various matrices are $S_{21} = [-0{\cdot}695 \quad 0{\cdot}067 \quad 0{\cdot}169 \quad 0{\cdot}000 \quad -0{\cdot}157 \quad 0{\cdot}628]$, $S_{22} = 0{\cdot}340$, $\Gamma_{21} = [0{\cdot}032 \quad 0{\cdot}042 \quad 1{\cdot}008 \quad 0{\cdot}000 \quad 0{\cdot}019 \quad -0{\cdot}074]$ and $\Gamma_{22} = 0{\cdot}351$. As Γ_{22} is a small number the effects of lagged states die away fairly quickly. For $\kappa = 0{\cdot}5$ $\Gamma_{22} = 0{\cdot}431$ so the effects persist much longer, although not excessively so.

These policies are often looked upon as a form of benchmark that simple rules should be able to mimic if they can be considered as serious policy options. It is with this in mind that we reinterpret the coefficients of the policy rules and design optimal simple rules.

5.2 Optimal simple policy rules

The presence of the change term for interest rates in the cost function rather than a levels term is designed to ensure that the inflation target is met in the long run. If this is the case, it might be that the optimal policy can be written including an integral feedback on the target. This does turn out to be the case. The coefficients of both the optimal time consistent and time inconsistent policies can be expressed as

$$\Delta r_t = \theta_1(\Delta p_t - T_t) + \theta_2 i_{t-1} + \theta_3 y_t + \theta_4 \rho_{t-1} + \theta_5 \xi_t^2 \qquad (17)$$

where θ_5 is zero for time consistent policies. The real interest rate, output and real exchange rate all return to zero in equilibrium, so this formulation ensures that the inflation target is met. The information in Tables 2 and 3 is reinterpreted in this way in Table 4, where there is now a non-zero feedback on the lagged real interest rate that is implicit in the simple state feedback equations.

Written this way it is possible to compare the optimal or time consistent with several types of simple rule by directly comparing the re-written coefficients and the associated losses.[14] In particular, we can search for simple rules which dominate the time consistent policy. It may seem an obvious point, but optimal time consistent policies are optimal only subject to the constraint of being time consistent. In general some shock specific rule will dominate it, and not necessarily one that feeds back on all the states.

A problem with optimal simple rules is that they are not usually certainty equivalent. In other words, they are state dependent. We have chosen to optimise subject to an announced shock of a 1% reduction in the target for inflation. This contrasts with the approach adopted by McKibbin (1993) who uses the covariances of the expected shocks in the loss function. That approach will yield a simple rule which is superior on average. Ours gives the rule which is the best for the exercises described above.

What classes of simple rule should we consider? Obviously, θ_5 is always zero for any simple rule. We assume that θ_1 is always retained as a non-zero coefficient as only then will the inflation target be achieved. Subject to this, then there are eight possible shock-specific optimal rules, one with all of θ_2–θ_4 non-zero, three with any two non-zero, three with only one non-zero and one with all three zero. The strategy we followed was to find the optimal rule[15] for each of the eight regimes for $\kappa = 0$ and use these as starting values for $\kappa = 0{\cdot}05$, and so on. If the algorithm failed no further search strategy was tried.

The best feedback rule on the entire state for the case of $\kappa = 0$ turned out to be

$$\Delta r_t = 0{\cdot}0536(\Delta p_t - T_t) - 0{\cdot}7443 i_{t-1} - 0{\cdot}0197 y_t - 0{\cdot}2424 \rho_{t-1}. \qquad (18)$$

This gave almost the same loss as the time inconsistent policy, and a very similar profile for interest rates. In Table 4 we give all the optimal policy rules that we were able to calculate, including the optimal time consistent and time inconsistent policies rewritten

[14] This is in marked contrast to Currie and Levine (1985) who find fully optimal rules that are 'exceedingly complicated and opaque: ... (where it is) not possible to provide any intuitive explanation of the implied feedback responses' (p. 66, *n*1).

[15] Numerical optimisation was carried out using the BFGS variable metric method (for details see, for example, Press *et al.*, 1992). Each function evaluation (*i.e.* calculation of the welfare loss) was performed by solving the appropriate Lyapunov equation (Kwakernaak and Sivan, 1972, 471–472). For this size of model this can be done efficiently by vectorisation.

Table 4. Optimal time inconsistent, time consistent and simple rules.

κ	Coefficients on: $(\Delta p_t - T_t)$	i_{t-1}	y_t	ρ_{t-1}	ξ_t^2	Loss
0·00	0·0529	−0·6868	−0·0091	−0·1849	−1·9767	0·5200
	0·1093	−0·7707	−0·0183	−0·2200		0·5227
	0·0536	−0·7443	−0·0197	−0·2424		0·5200
	0·1071	−0·1592	0·2355			0·5215
	0·0555	−0·6991		−0·2246		0·5200
	0·1202		0·3056	0·0654		0·5221
	0·3009	−0·5696				0·5619
	0·5616			0·5930		0·7187
	0·1274		0·3441			0·5399
	1·3533					1·3788
0·10	−0·0188	−0·7121	0·0933	−0·1733	−1·9073	0·5808
	0·1123	−0·7829	0·0650	−0·2238		0·5926
	0·0403	−0·2022	0·2111			0·5821
	0·0596		0·2879	0·0843		0·5829
	0·2374	−0·6455				0·6182
	0·4900			0·6377		0·7464
	0·1047		0·3557			0·6170
	1·3890					1·6954
0·20	−0·0768	−0·7366	0·1892	−0·1650	−1·8576	0·6206
	0·1120	−0·7969	0·1491	−0·2272		0·6504
	0·0173		0·2681	0·1001		0·6228
	0·1868	−0·7220				0·6544
	0·4314			0·6675		0·7561
	0·0971		0·3724			0·6756
	1·4234					2·0108
0·30	−0·1236	−0·7609	0·2814	−0·1593	−1·8252	0·6416
	0·1095	−0·8123	0·2344	−0·2304		0·6937
	0·0000		0·2523	0·1178		0·6442
	0·1454	−0·8029				0·6722
	0·3792			0·6875		0·7504
	0·0959		0·3942			0·7154
	1·4568					2·3252

in the form (18). The policy rule above is the third rule reported, and the welfare loss identical to four significant figures to the fully optimal rule.

Examining the case where $\kappa = 0$, many of the policy rules are almost as effective

Table 4. Continued...

κ	Coefficients on: $(\Delta p_t - T_t)$	i_{t-1}	y_t	ρ_{t-1}	ξ_t^2	Loss
0·40	−0·1607	−0·7854	0·3724	−0·1562	−1·8088	0·6448
	0·1052	−0·8291	0·3219	−0·2336		0·7207
	0·1106	−0·8925				0·6725
	0·3302			0·6995		0·7304
	0·0977		0·4218			0·7352
	1·4895					2·6385
0·50	−0·1890	−0·8109	0·4649	−0·1555	−1·8085	0·6303
	0·0995	−0·8476	0·4134	−0·2368		0·7288
	0·0802	−0·9962				0·6551
	0·2820			0·7036		0·6957
	0·1010		0·4570			0·7332
	1·5219					2·9509

as the fully optimal rule, with three that do not feedback on the entire state better that the time consistent policy. A notable feature is that the information content of each variable differs depending on which others are included. For example there is a sign switch in the coefficient on output when any other variable is dropped, and in the lagged real exchange rate where the lagged real interest rate is dropped.

As κ was increased the optimal full state feedback rule became unstable in the parameter search. This is because if we consider the feedback on the deviations of inflation from target the integral control for the fully optimal rule becomes perversely signed for the time inconsistent policy although it remains correctly signed for the time consistent policy. In trying to get as close to optimality the search algorithm fails.

If κ exceeds 0·1 the coefficient on output is always positive, but the sign change in the lagged real exchange rate term persists. From the welfare losses it is clear that the simple integral feedback on inflation does not perform very well relative to the other policy rules, and the simple expedient of including output works quite well.

This is confirmed by Chart 7, where we show the two extreme cases again with the fully optimal and the two simplest rules, a feedback on inflation and a feedback including output. The latter performs rather well, but feeding back on inflation only means that there must be an increase in interest rates in the first period of the optimal coefficient. Including output as an indicator variable changes this completely.

6 Interest rate rules on the NIESR model

In the last three sections, by employing a stylised model which shares certain important features of large macromodels, we have attempted to clarify a number of key issues which are relevant for the practical implementation of interest rate policy. It is instructive to illustrate these points using the full NIESR model. Apart from the greater degree of complication which pertains in a large macromodel, the main distinction between the full model simulation results and the small model results is that it is necessary to solve the full non-linear model using numerical methods. In principle, this does not present a problem *per se* but in practice, the solution of models with forward-looking expectations can be problematic especially when an unstable root of the model is close to the unit circle. Obviously, a more vigorous feedback rule can solve that problem but this will often be at the expense of the stability of the model solution as a whole. We do not discuss these issues further here but acknowledge that this aspect of the problem requires further analysis.

First, we illustrate how interest rate policy can be used to target different nominal anchors. In all model simulations in this section, we adopt the pure open arbitrage version of the exchange rate equation. Previous versions of the model incorporated a risk premium in the net overseas assets ratio to GDP. All simulations described here are run with a fiscal policy rule which fixes the PSBR to GDP ratio at base levels.

We conduct three separate simulations where the price level (RPI excluding interest payments), money GDP and M0 are respectively driven down by 1% using a proportional feedback rule with a feedback parameter of 0·25.

Charts 8(a)–(b) show the outcomes for the price level and M0 target. The results are similar in many respects to the analogous exercise performed on the stylised model. The targetted nominal variable falls to its required level after about three years with interest rates rising by 0·25% on impact. With a money GDP target, prices fall in a very similar trajectory to the price targetting case. When M0 is the target, prices are also driven down at a similar speed but because there is a long run effect from

cumulated interest rates in the expression for financial liberalisation used in the M0 equation (see NIESR, 1994), then there is a velocity shift which implies that prices fall by less than 1% in the long run.

We now turn to consideration of the motivating topic of this paper, that is the use of interest rate rules to control inflation. Charts 9(a)–(b) show the outcome of an integral feedback rule on inflation (with a feedback coefficient of 0·25) which is simulated to respond to a 1% cut in the target rate of inflation; see chart 9(a). Almost immediately, interest rates fall in anticipation of the lower rate of inflation in the future. This result is very reminiscent of the interest rate response in the stylised model with forward-looking prices. The price level falls on impact such that annualised quarterly inflation overshoots its target level and nominal interest rates therefore fall on impact. Recall, that in the long run, the price level must slip by 4% (1/0·25) relative to the inflation trajectory starting from its previous starting point. The exchange rate jumps by even more resulting in a 6% initial appreciation of the real exchange rate which combines with the also temporary rise in real interest rates to cause a temporary fall in output which peaks at 0·65% after 2 years. Inflation broadly settles down at its new lower level after about 5 years when the nominal exchange rate is also depreciating at its new lower rate. Interestingly, output in the long run is higher with lower inflation because of a lack of full model dynamic homogeneity with respect to prices, despite the fact that the wage-price system itself does embody this property. The inflation effects mainly arise from the cost of finance due to non-neutralities in the tax system. These effects from inflation on the real equilibrium are an interesting topic in their own right, see Pemberton (1989) for example, but we do not discuss this issue further in this paper.[16]

It might be argued that the interest rate response in this simulation is rather implausible since it is unlikely that interest rates would really immediately fall on announcement of such a policy. If wage and price expectations are formed in a backward-looking autoregressive manner, then, just as in the stylised model with backward-looking prices, so the initial movement in interest rates in the the full model might be

[16] A term in the nominal interest rate in the consumption function, justified on the grounds that it captures borrowing constraints mainly through front-end loading effects, can exaggerate the inflation non-neutrality still further. This effect has been switched out for the purposes of these simulations.

upwards (although even in the stylised model, the initial increase in interest rates is very small when the feedback parameter is as small as 0·25). The well-known disadvantage to this modelling strategy is that the lagged response of prices actually mixes up effects which arise from structural inertia with those arising from expectational errors.

Alternatively, it may be that agents do have forward-looking expectations but that they no not actually believe that policymakers will see through the policy because of the adverse output consequences of the policy tightening measure. For example, suppose that private sector agents initially only attached a probability of 0·1 to the announcement that the inflation target was to be 1% lower but that as they continually observed that policymakers were actually moving interest rates consistent with their announced lower target, so they gradually adjusted their perceived probability by 0·1 each quarter, until after ten quarters, they fully believed the governments policy intentions. It is possible to illustrate this type of learning mechanism by performing a series of stacked simulations, where period by period, private sector agents gradually revise their expectation of the inflation target. For a similar exercise in the context of learning about devaluation within the ERM, see Westaway (1992a, b).[17] This is illustrated in Chart 9(b) as the 'fast learning' simulation. The initial movement in interest rates is now positive and inflation takes slightly longer to adjust to its new target level compared to the case when credibility is immediate. The output costs are correspondingly larger as interest rates needed to be higher before inflationary expectations responded. It is possible to simulate an even larger initial increase in interest rates by assuming that expectations take even longer to adapt to the new policy announcement. Chart 9(b) also shows the results of an alternative 'slow learning' simulation where the private sector does not adjust its perceived level of the inflation target at all for five periods, whereupon it adjusts fully to the correct value in the next five periods. Since inflation takes even longer to adjust, interest rates are higher still in the first year, before eventually adjusting to the same long run level 1% below base.

[17] This type of learning assumes that agents understand how the economy operates but are unsure of the policymakers intentions. For an approach which assumes that agents use Kalman filtering to learn about the policy reaction function and the model, see Hall (1990).

7 Conclusion

In this paper, we have attempted to show how a highly stylised version of the output, price, interest rate and exchange rate equations of a large macroeconomic model can be used to highlight a number of important issues which should face policymakers when they attempt to use interest rates to control inflation. We have then illustrated some of these issues using the NIESR model which has been solved both assuming that expectations are formed rationally and alternatively that agents learn about any change in policy regime gradually.

The main points worthy of emphasis are as follows;

- The primary role of interest rate policy is to determine the price level.
- If policymakers are interested in maximising some measure of welfare related to an inflation target and a natural rate of output, then the optimal policy rule will ideally pay attention to all currently dated variables.
- No explicit account of forward-looking information needs to be taken into account beyond the transmission mechanism of the model which will influence the parameters of the feedback rule.
- Simple feedback rules may work almost as well as their fully optimal counterparts. The fully optimal rules, whether time consistent or time inconsistent, will be optimal in all circumstances whereas the choice of simple rule will be shock specific.
- If an integral feedback rule for interest rates on inflation is adopted, then there will be no price level slippage unless either the inflation target is changed or policymakers attempt to take output deviations into account. If policymakers attempt to target output above its natural rate then there will be inflation slippage.

It is also possible to draw a number of more speculative conclusions. First, as the exercise on the simple model has shown, the precise size and sign of the coefficients on the indicator variables such as output can vary according to the form of the policy rule chosen and the preferences of policymakers. The most robust rule would seem to be one where inflation simply reacts to the deviation of inflation from its target level although typically this will not perform as well as one where other indicators are taken into account. Nevertheless, the simpler rules also have advantages if agents are attempting to learn about a change in policy regime. Of course, even this simple rule can lead to instability in some models (usually when expectations are adaptive) but this suggests that robustness checks should be carried out across different versions of the same model and also across different macroeconomic models.

Of course, there are a many aspects to the monetary policy design problem which we have glossed over in this analysis. Two issues are particularly important.

First, the models in this paper have assumed that money supply plays a purely passive role. A more complete structural model of the financial sector might have a more active role for financial effects where the exchange rate moves to clear the relative demand and supply of sterling assets. Previous attempts to model the exchange rate in this way in a macromodelling context have been carried out by Keating (1985) and Pain and Westaway (1994). It is important to know whether the policy conclusions which emerge from this type of approach is substantively different to the standard approach which assumes perfect capital mobility.

Second, the analysis of this paper has assumed a very simple information structure where all currently dated variables are assumed to be known with equal certainty. In reality, information on the latest movements of the monetary aggregates may be more timely than other current dated variables, for example GDP itself. In these circumstances, monetary indicators may play an important role in influencing the movement of interest rates. Formally, however, this role as a contemporaneous indicator (often attributed to M0 or retail sales) ought to be incorporated in the macroeconomic model itself and the preceding analysis of the appropriate design of interest rate rules would carry through as before.

In concluding, the main message which emerges from this paper is that even when it is assumed that the model of the economy is known with certainty, the monetary policy design problem is extremely complicated. It seems obviously important to at least understand how policy ought to be carried out in these circumstances so that some policy framework can be set down for the more awkward situation of massive uncertainty which actually faces policymakers and other agents in the economy who must respond to the government's actions. In practice, the UK government's response

to this problem has been to remain vague, perhaps deliberately, about how interest rates will respond in any particular circumstance. This may allow UK policymakers to keep themselves some freedom of manoeuvre for any eventuality but this may only be gained at the expense of expectational uncertainty. Perhaps a simple rule of the form analysed in this paper, that is, to increase interest rates by θ whenever current inflation is 1% off target, may anchor expectations more effectively than any poorly defined state contingent policy.

References

Ammer, John and Richard T. Freeman (1994) 'Inflation targeting in the 1990s: The experiences of New Zealand, Canada, and the United Kingdom', International Finance Discussion Paper No. 473, Board of Governors of the Federal Reserve System.

Barrell, Ray, Jan Willem In 't Veld and Andrew Gurney (1992) 'The real exchange rate, fiscal policy and the role of wealth: An analysis of equilibrium in a monetary union', *Journal of Forecasting*, 11, 361–388.

Blake, Andrew P. (1992) 'Time consistent mixed precommitment macropolicy', National Institute Discussion Paper No. 7, new series.

Blanchard, Olivier and Charles M. Kahn (1980) 'The solution of linear difference models under rational expectations', *Econometrica*, 48, 1305–1311.

Bryant, Ralph C., Peter Hooper and Catherine L. Mann (editors) (1993) *Evaluating Policy Regimes: New Research in Empirical Economics*, Washington: The Brookings Institution.

Buiter, Willem H. and Richard C. Marston (editors) (1985) *International Economic Policy Coordination*, Cambridge: Cambridge University Press.

Calvo, Guillermo A. (1978) 'On the time consistency of optimal policy in a monetary economy', *Econometrica*, 46, 1411–1428.

Church, Keith, Peter Mitchell, Peter Smith and Kenneth Wallis (1993) 'Comparative properties of models of the UK economy', *National Institute Economic Review*, 145, 87–107.

Currie, David, and Paul Levine (1985) 'Simple macropolicy rules for the open economy', *Economic Journal*, Supplement, 95, 60–70.

Fisher, Paul, David Turner and Kenneth Wallis (1992) 'Forward unit root exchange rate dynamics and the properties of large scale macroeconometric models' in Colin Hargreaves (editor) *Macroeconomic Modelling of the Long Run*, Aldershot: Edward Elgar.

Gagnon, Joseph E. and Dale W. Henderson (1990) 'Nominal interest rate pegging under alternative expectations hypotheses', in Hooper *et al.*, *op. cit.*.

Hall, Stephen G. (1990) 'Modelling the sterling effective exchange rate using expectations and learning', Bank of England Discussion Paper No. 33.

Henderson, Dale W. and Warwick J. McKibbin (1993a) 'An assessment of some basic monetary-policy regime pairs: Analytical and simulation results from simple multi-region macroeconomic models', in Bryant *et al.*, *op. cit.*.

Henderson, Dale W. and Warwick J. McKibbin (1993b) 'A comparison of some basic monetary policy regimes for open economies: Implications of different degrees of instrument adjustment and wage persistence', International Finance Discussion Paper No. 458, Board of Governors of the Federal Reserve System.

Hooper, Peter, Karen H. Johnson, Donald L. Kohn, David E. Lindsey, Richard D. Porter and Ralph Tryon (editors) (1990) *Financial Sectors in Open Economies: Empirical Analysis and Policy Issues*, Board of Governors of the Federal Reserve System.

Kwakernaak, Huibert and Raphael Sivan (1972) *Linear Optimal Control Systems*, Wiley-Interscience.

Keating, Giles (1985) 'The financial sector of the London Business School model', in David Currie (editor), *Advances in Monetary Economics*, London: Croom Helm.

Layard, Richard and Stephen J. Nickell (1985) 'The causes of British unemployment', *National Institute Economic Review*, 111, 62–85.

McCallum, Bennett T. (1986) 'Some issues concerning interest rate determinacy, price level determinacy and the real bills doctrine', *Journal of Monetary Economics*, 17, 135–160.

McKibbin, Warwick J. (1993) 'Stochastic simulations of alternative monetary regimes using the McKibbin-Sachs global (MSG2) model', in Bryant *et al.*, *op. cit.*.

NIDEM (1994) *National Institute Domestic Econometric Model*, May.

Oudiz, Gilles and Jeffrey Sachs (1985) 'International policy coordination in dynamic macro-economic models', in Buiter and Marston, *op. cit.*.

Pain, Nigel and Peter F. Westaway (1994) 'Towards a structural model of the UK exchange rate', *Greek Economic Review*, forthcoming.

Pemberton, James (1989) 'The macroeconomics of inflation non-neutrality', *Economic Journal*, 99, 62–82.

Phillips, A.W. (1954) 'Stabilisation policy in a closed economy', *Economic Journal*, 64, 290–323.

Press, William H., Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery (1992) *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, second edition, Cambridge: Cambridge University Press.

Salmon, Mark (1982) 'Error correction mechanisms', *Economic Journal*, 92, 615–629.

Sargent, Thomas J. and Neil Wallace (1975) ' "Rational" expectations, the optimal monetary instrument and the optimal money supply rule', *Journal of Political Economy*, 83, 241–254.

Soteri, Soterios and Peter F. Westaway (1993) 'Explaining price inflation in the UK', *National Institute Economic Review*, 144, 85–95.

Weale, Martin, Andrew Blake, Nicos Christodoulakis, James Meade and David Vines (1989) *Macroeconomic Policy: Inflation, Wealth and the Exchange Rate*, London: Unwin Hyman.

Westaway, Peter F. (1986) 'Some experiments with simple feedback rules on the Treasury model', *Journal of Economic Dynamics and Control*, 10, 239–246.

Westaway, Peter F. (1992a) 'To devalue or not to devalue: An analysis of UK exchange rate policy in the ERM', National Institute Discussion Paper No. 16, new series.

Westaway, Peter F. (1992b) 'A forward-looking approach to learning in macroeconomic models', *National Institute Economic Review*, 140, 86–97.

Westaway, Peter F. and Simon Wren-Lewis (1993) 'Is there a case for the MTFS?', *Manchester School*, 61, 227–247.

Chart 1(a): Price level targetting: Stylised model.
Backward-looking price equation; $\Theta_p = 1$

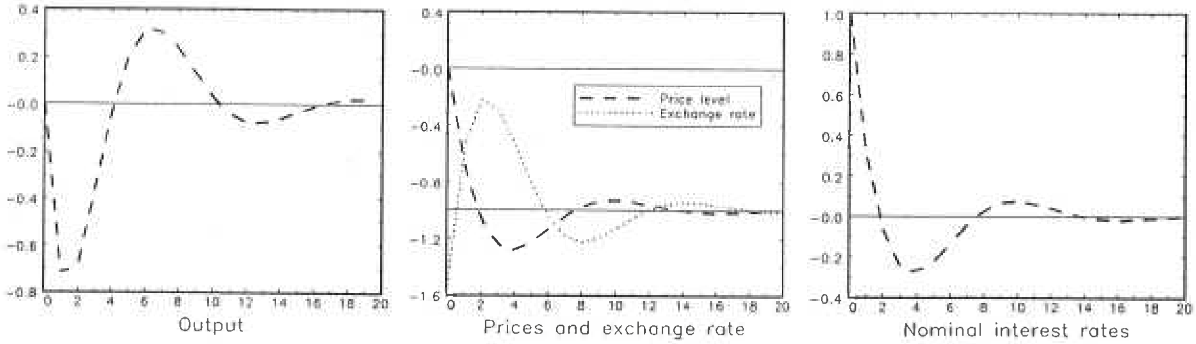


Chart 1(b): Price level targetting: Stylised model.
Forward-looking price equation; $\Theta_p = 1$

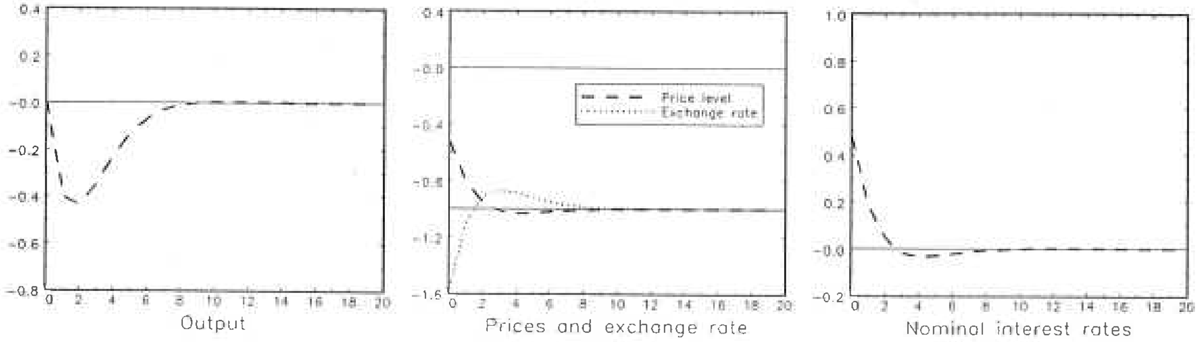


Chart 2(a): Price level targetting: Stylised model.
Backward-looking price equation; $\Theta_p = 8$

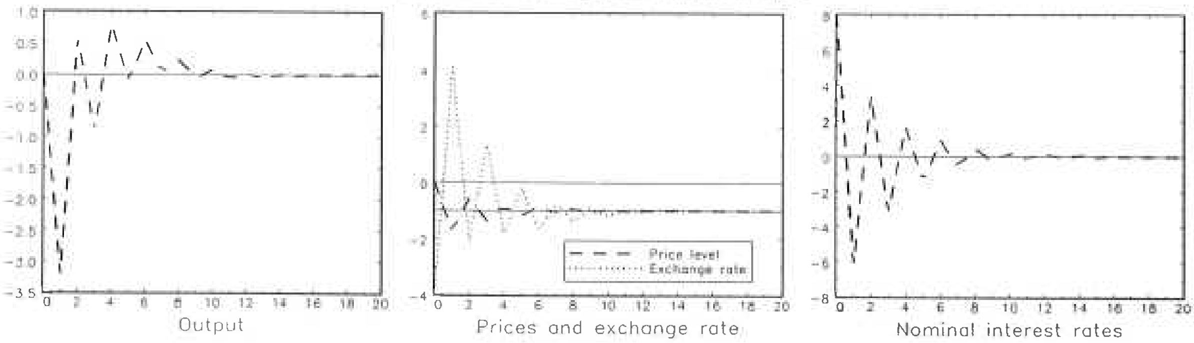


Chart 2(b): Price level targetting: Stylised model.
Forward-looking price equation; $\Theta_p = 1000$

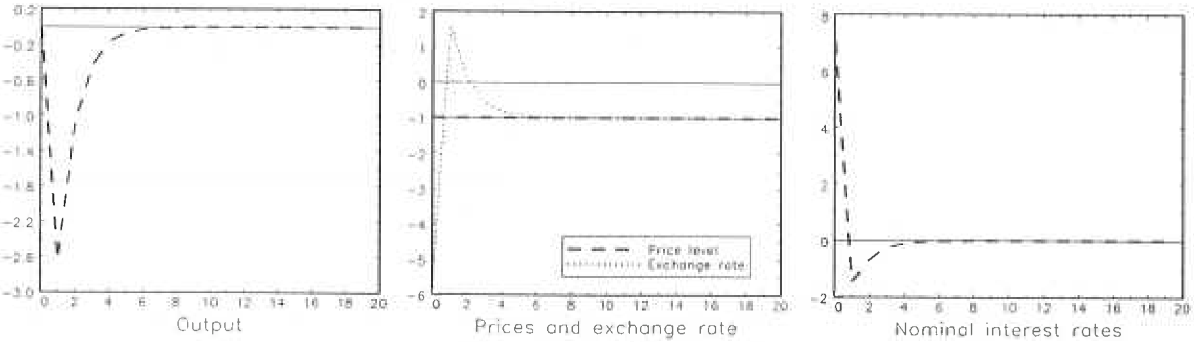


Chart 3(a): Price level targetting: Stylised model.
Backward-looking price equation: $\Theta_p=0{\cdot}001$

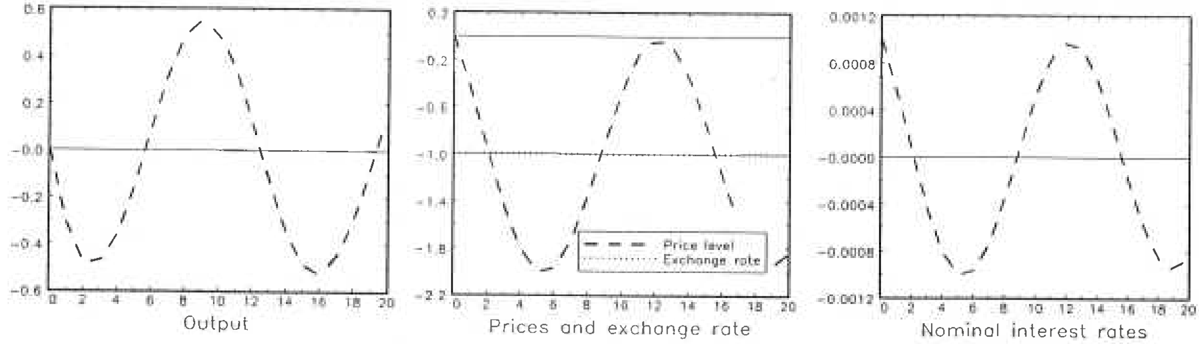


Chart 3(b): Price level targetting: Stylised model.
Forward-looking price equation: $\Theta_p=0{\cdot}01$

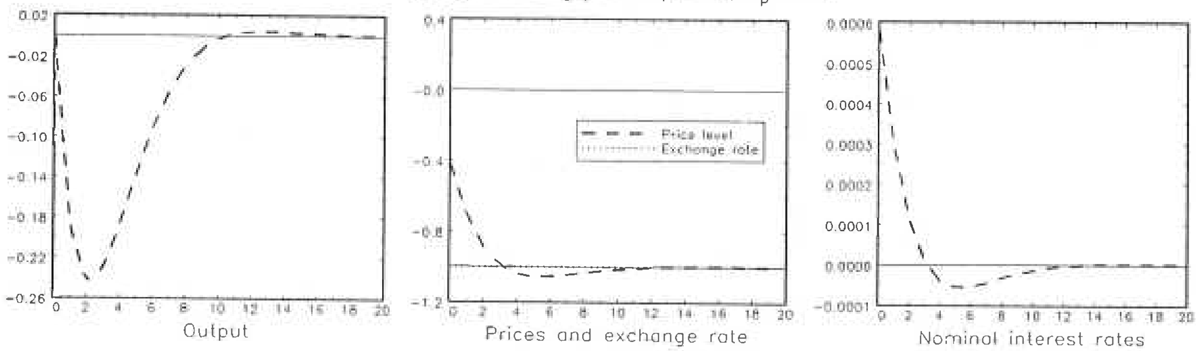


Chart 4(a): Inflation targetting: Stylised model.
Backward-looking price equation: $\Theta_i=1$

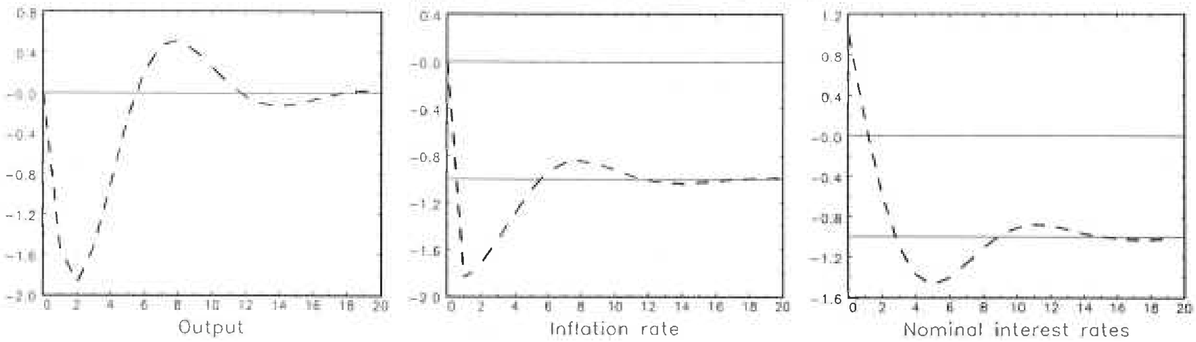


Chart 4(b): Inflation targetting: Stylised model.
Forward-looking price equation: $\Theta_i=1$

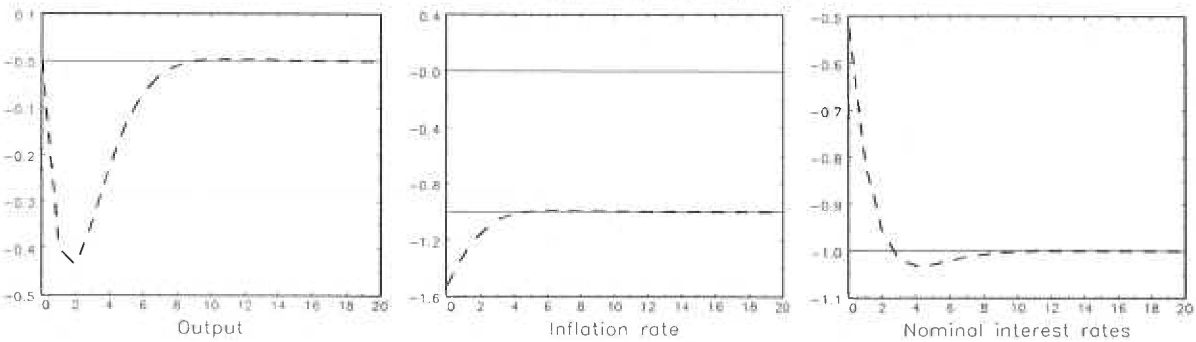


Chart 5(a): Inflation targetting: Stylised model.
Backward-looking price equation; $\Theta_i=8$

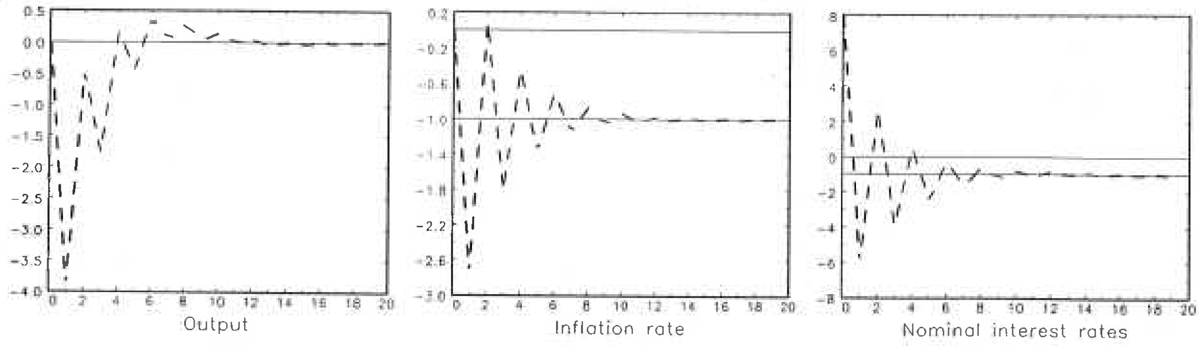


Chart 5(b): Inflation targetting: Stylised model.
Forward-looking price equation; $\Theta_i=1000$

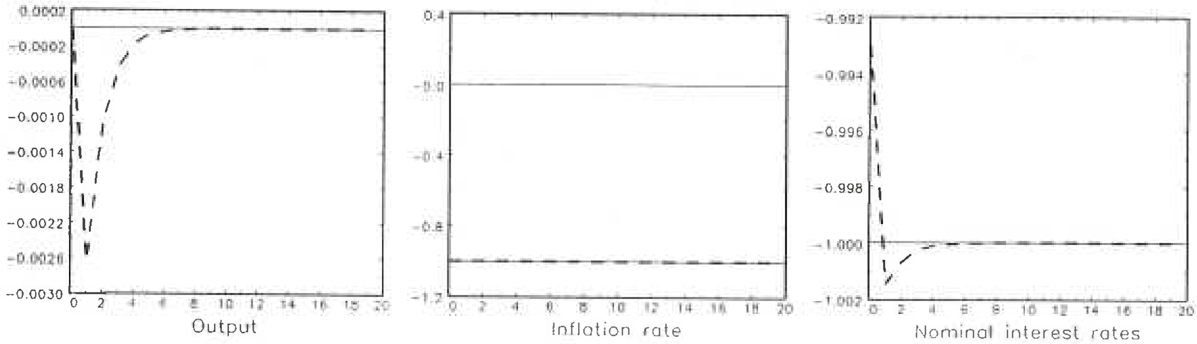


Chart 6(a): Inflation targetting: Stylised model.
Backward-looking price equation; Optimal control; $\kappa=0{\cdot}0$

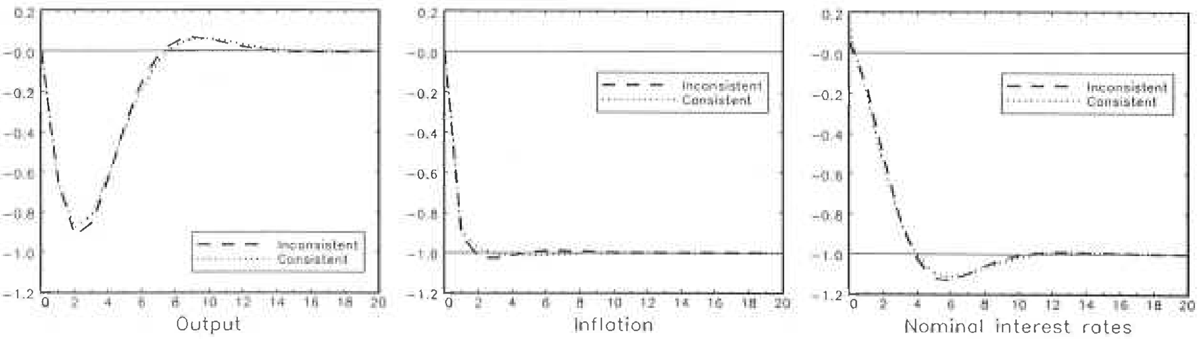


Chart 6(b): Inflation targetting: Stylised model.
Backward-looking price equation; Optimal control; $\kappa=0{\cdot}5$

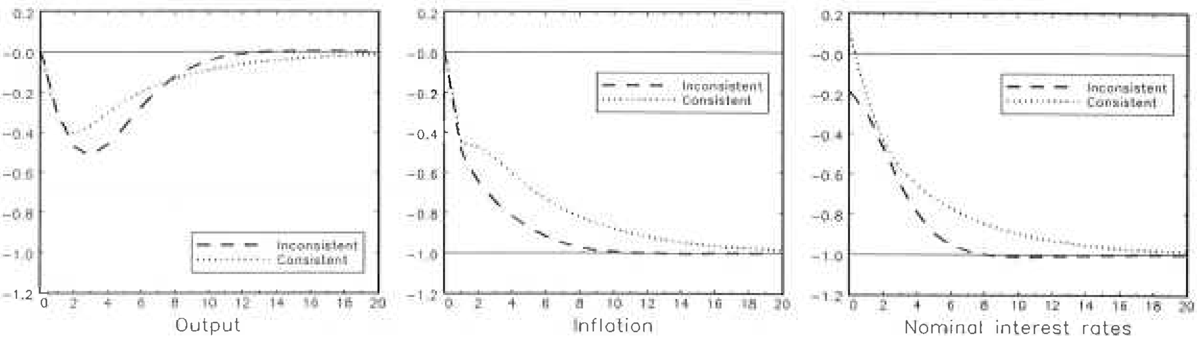


Chart 7(a): Inflation targetting: Stylised model.
Backward-looking price equation; Optimal and simple rules; $\kappa=0{\cdot}0$

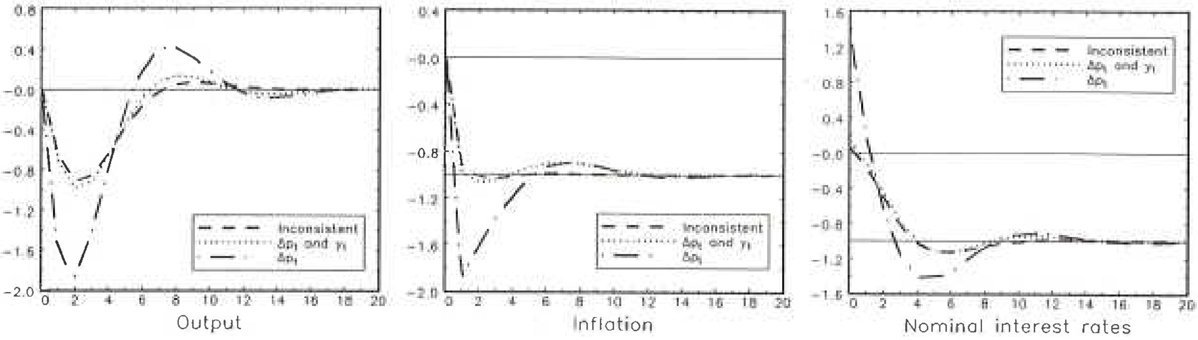


Chart 7(b): Inflation targetting: Stylised model.
Backward-looking price equation; Optimal and simple rules; $\kappa=0{\cdot}5$

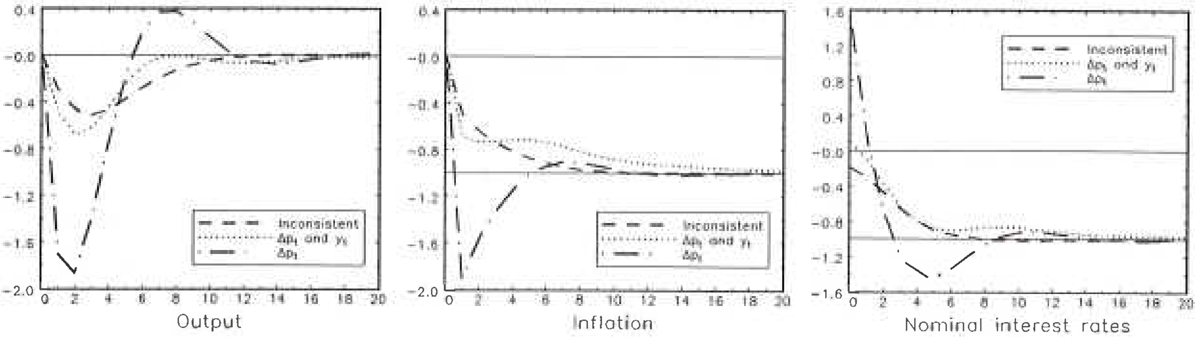


Chart 8(a): Price level targetting
NIESR model; $\Theta_i=0{\cdot}25$

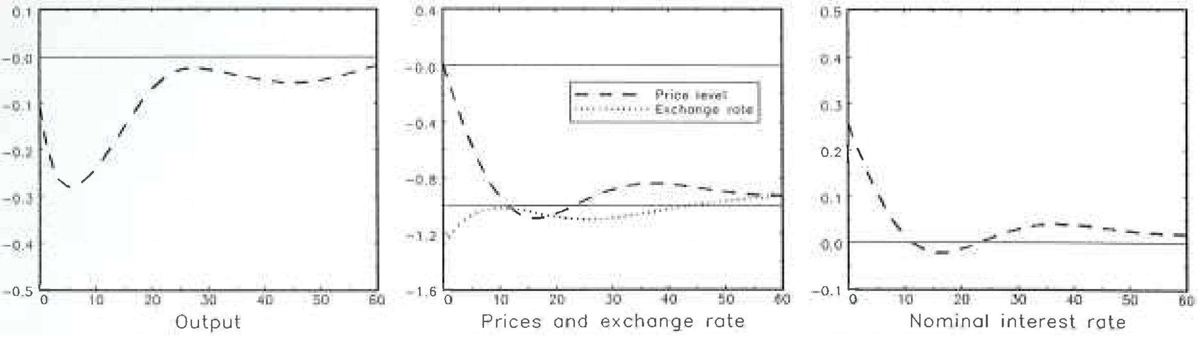
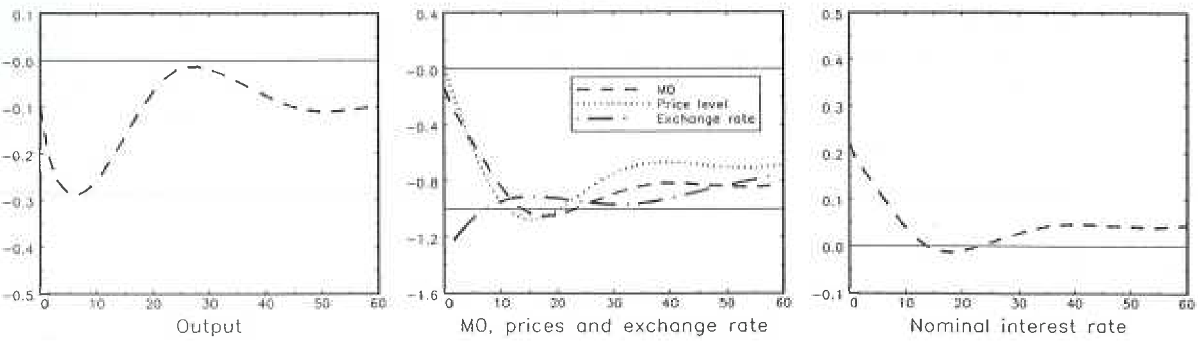
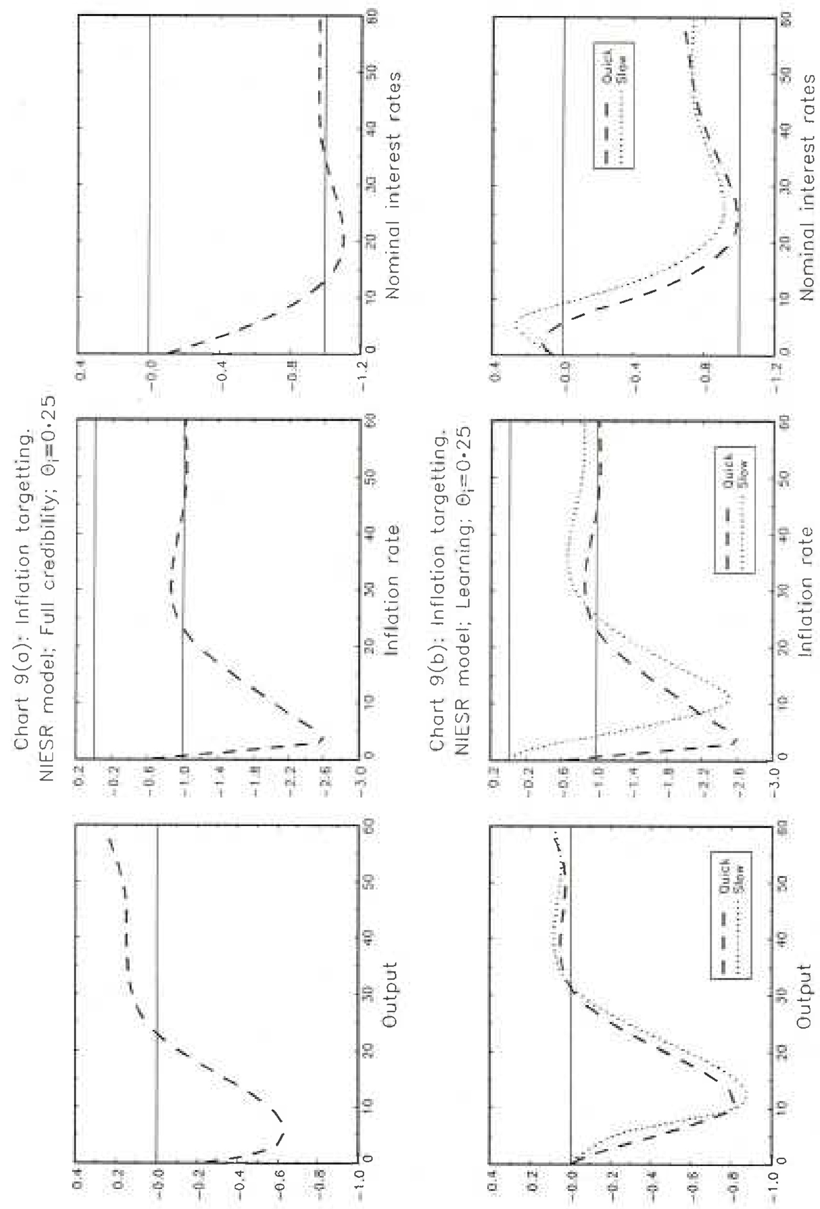


Chart 8(b): M0 targetting.
NIESR model; $\Theta_i=0{\cdot}25$





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