AN OVERLAPPING GENERATIONS COMPUTABLE GENERAL EQUILIBRIUM (OLG-CGE) MODEL WITH AGE-VARIABLE RATE OF TIME PREFERENCE

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Abstract

It is generally accepted that people prefer to receive reward earlier rather than later. A positive rate of time preference is routinely used in economic models of intertemporal choice, for example OLG-CGE models. Calibrating an OLG-CGE model is challenging because age-profile data is usually not available. For example, researchers typically have no data on consumption by age group. The conventional way to proceed is to impose a constant rate of time preference, which implies smooth age profile for consumption.

The alternative methodology that we propose in this paper is to impose directly the bell-shaped consumption age profile from the National Transfer Accounts (NTA) and to introduce an age-variable rate of time preference. Compared with the conventional approach this variation leads to the lower level of savings and capital along the transition to a steady state. Our first results show that using the constant rate of time preference and smooth age consumption profile underestimates the negative effect of population ageing on the economy.

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1. Introduction

It is generally accepted that people prefer to receive reward earlier rather than later. This phenomenon is studied in economics, psychology and behavioural ecology. The terms used to describe this behaviour are "discounting", "time preference" and "impatience". In this paper we will be using the term "time preference" to describe preference for earlier reward and "rate of time preference" for the strength of this preference.

A positive rate of time preference is routinely used in economic models of intertemporal choice, for example in dynamic forward-looking computable general equilibrium (CGE) models. One type of such models, which we discuss in this paper, is overlapping generations CGE model (OLG-CGE), following the tradition of Auerbach and Kotlikoff (1987). The OLG-CGE model has been typically applied to studies related to demographic change and public policy (Börsch-Supan et al., 2006; Fehr et al, 2004, 2013; Georges et al, 2013; Sánchez-Romero et al., 2013).

Calibrating an OLG-CGE model is more challenging than calibrating a standard age-aggregated representative agent CGE model. The calibration itself requires a simulation because age-profile data is typically not available. For example, although aggregate consumption can easily be obtained from the System of National Accounts (SNA), researchers usually have no data on consumption by age group[1]. Consequently, such data needs to be generated at the calibration stage to provide the model with plausible initial paths. The conventional way to proceed is to assume a steady state with a stable population, and to use optimisation theory based on the Euler equation to derive the evolution of consumption over the lifecycle of an individual as a function of a) the inter-temporal elasticity of substitution, b) the market interest rate, and c) the rate of time preference. A typical assumption is to impose a constant rate of time preference, which implies smooth age profile for consumption (usually upward sloping as it is conventionally assumed that the rate of time preference should be higher than the interest rate).

Numerous empirical studies have demonstrated that private consumption age profile is bell-shaped. The most extensive study that aims to disaggregate national accounts by age is National

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[1] Speaking about consumption we mean private consumption which excludes spending on health and education. In our model health and education spending are modelled separately and also depend on age. See section 3.5.

Transfer Accounts project (United Nations, 2013). The project includes teams from over 40 countries and has a large collection of age disaggregated data from participating countries.

The OLG CGE modellers for a long time are making attempts to bring modelled consumption age profile closer to empirically observed shape. In more advanced OLG models with realistic demography, researchers include survival probabilities in the utility function to discount future consumption and allow adults to make consumption decisions on behalf of their children (Kotlikoff et al., 2007, Sánchez-Romero et al., 2012; Abio et al., 2015). This approach allows achieving hump-shaped age consumption profile, however it still does not guarantee that it will fit the data.

The alternative methodology that we propose in this paper is to impose directly the bell-shaped consumption age profile from the National Transfer Accounts (NTA). In order to do this, we need to introduce an age-variable rate of time preference. Theoretically, there are several studies which have discussed the possibility that the rate of time preference should vary with age (Ramsey, 1928; Rogers, 1994; Posner, 1995; Trostel and Taylor, 2001; Bishai, 2004). We compare the results from the model with age-variable rate of time preference (fitted consumption) with two specifications with constant rate of time preference: 1) model with perfect annuity market (smooth consumption) and 2) model with unintentional bequests and family composition (bell-shaped consumption). We demonstrate that introducing age-variable rate of time preference influences capital accumulation under the standard demographic shock and that using constant rate of time preference in both analysed specifications leads to underestimation of the negative effects of population ageing on the economy.

The rest of the paper is organised as follows. Section 2 presents a survey of the literature on time preference and its relationship with age. Section 3 describes three versions of the OLG-CGE model including the demographic structure, the producer and household problems, the investment, the public sector and market clearing conditions. Section 4 presents the calibration of the model and the difference between the three model specifications. Section 5 discusses the difference that introducing age-variable rate of time preference makes to simulation results. Section 6 concludes.

2. Time Preference

This section discusses the theoretical and empirical evidence from several studies regarding the age-variable time preference. We first explain why a constant rate of time preference is normally applied in the usual case for an individual's lifetime utility, and then discuss at which point we can apply the age-variable rate of time preference at the generational level.

In economics, the objective of a time preference is to evaluate the (relative) value of a reward at an earlier time compared to the same reward in the future (Fetter, 1927). The subject of time preference was initiated in the theory of capital by various researchers, such as Rae (1905), Marshall and Marshall (1920), Ramsey (1928), Fisher (1930), Pigou (1932), Harrod (1948), and Böhm-Bawerk (1959).

Ramsey (1928) raises the issue of dynamic inconsistency, which states that without a constant discount rate, an individual will change his decision if he is at a different time period. The decision which he makes today may not be the same tomorrow. Therefore, to avoid this inconsistency, he states that a single person has to apply a constant discount rate for both the present utility and the future utility.

In the dynamic models with perfect foresight (like the OLG CGE model that we are discussing here) age-variable rate of time preference does not lead to dynamic inconsistency. Because of the perfect foresight agents know that when they get older they will have different rate of time preference and take it into account when making their initial optimising decisions. This way they don't have to change their decision in the future.

Several empirical studies show that individuals have the time/age-variable rate of time preference. Bishai (2004) constructs an empirical model with endogenous rate of time preference, and concludes that the rate of time preference estimated by using the National Labour Survey of Youth declines from 27 to 30 years old, but starts to increase quickly and consistently as individuals grow older. His findings also suggest that schooling is another factor that causes changes in rate of time preference.

There are several researchers who try to explain why people change their discount rate over

their lifetime. Rogers (1994) explains that the marginal rate of time preference should vary with age, with young adults discounting the future at a higher rate than seniors. Posner (1995) believes that there are several different selves in an individual's life. The present and the future selves are different and they make separate decisions. People weigh their present consumption more heavily than their future consumption.

Trostel and Taylor (2001) propose that time preference changes with age because expected utility from consumption (eventually) falls as people's mental and physical abilities (eventually) decline with age. To prove this point, they construct a utility function varying with age, and estimate the Euler equation with independent variables, such as interest rate, age, survival probability, permanent income, and education using the data from the Panel Study of Income Dynamics. The key finding is that the rate of time preference increases over the life cycle. A "surprising" result is that the estimated coefficient on survival probability is not statistically significant. The explanation is that the planned bequests and annuities can eliminate the impact from survival probability. The same conclusion was arrived at by Yaari (1965), whose study demonstrates that the impact of survival uncertainty can be eliminated by annuities and planned bequests. Other researchers, such as Kotlikoff et al. (1982), conclude that private pensions, social security, and familial transfers can also act as imperfect annuities. Trostel and Taylor (2001) propose that it is aging that causes the time preference, not the probability of dying. Our calibration also shows that the rate of time preference increases with age (s

3. Model Description

In this paper we compare three models. Fore simplicity of presentation we call them 1) the model with "fitted consumption" – this is the model with time variable rate of time preference that allows to perfectly fit age consumption profile; 2) the model with "smooth consumption" – this is the model with constant rate of time preference and perfect annuity market which results in smooth upward-sloping age consumption profile; 3) the model with "bell-shaped consumption" – this is the model with constant rate of time preferences and unintentional bequests and family composition, which generated bell-shaped age consumption profile. They have the same structure except for the

household problem. Below we describe the demographic structure of the model and outline the main features of the production, household and government sectors.

3.1 Demographic Structure

The population is divided into 19 generations or age groups (i.e., 0-4, 5-9, 10-14, 15-19, …, 90-94). Demographic variables, fertility, mortality and net-migration rates are assumed to be exogenous.[2] This is a simplifying assumption given that such variables are likely endogenous and affected by, for example, changes in economic growth. Every cohort is described by two indices. The first is *t*, which denotes time. The second is *g*, which denotes a specific generation or age group.

The size of the cohort belonging to generation *g+k* in any period *t* is given by the following two laws of motion:

$$Pop_{t,g+k} = \begin{cases} Pop_{t-1,g+k+5}\, fr_{t-1} & for \quad k = 0 \\ Pop_{t-1,g+k-1}\left(sr_{t-1,g+k-1} + mr_{t-1,g+k-1}\right) & for \quad k \in [1,18] \end{cases} \tag{4}$$

The first equation simply implies that the number of children born at time *t* (age group *g+k = g*, i.e. age group 0-4) is equal to the size of the first adult age group (*g+k+5=g+5*, i.e. age group 20-24) at time *t-1* multiplied by the "fertility rate", *fr*, in that period. If every couple has two children on average, the fertility rate is approximately equal to 1 and the size of the youngest generation *g* at time *t* is approximately equal to the size of the first adult generation *g+5* one year before. A period in the model corresponds to five years and a unit increment in the index *k* represents both the next period, *t+k,* and, for an individual, and a shift to the next age group, *g+k*.

The second law of motion gives the size at time *t* of any age group, *g+k,* beyond the first generation, as the size of this generation a year ago times the sum of the age specific conditional survival rate, *sr,* and the net migration rate, *mr,* at time *t-1.* In this model the fertility rates vary across time, while the survival and net migration rates vary across time and age. For the final generation (i.e., the age group

90-94 (*k=18*)), the conditional survival rate is zero. This means that everyone belonging to the oldest age group in any period dies with certainty at the end of the period.

Time variable fertility and time/age-variable net migration and conditional survival rates are calibrated based on exogenous population projections. This permits a precise modelling of the demographic scenarios of any configuration within the model.

3.2 Production

At any time *t*, a representative firm hires labour and rents physical capital to produce a single good using a Cobb-Douglas technology. The production function thus reads:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} \tag{5}$$

where Y denotes output, K is physical capital L denotes effective units of labour, A is a scaling factor and α represents the share of physical capital in output. The market in which the representative firm operates is assumed to be perfectly competitive. Factor demands thus follow from the solution to the recursive profit maximization problem:

$$re_t = \alpha A \left(\frac{K_t}{L_t} \right)^{\alpha - 1} \tag{6}$$

$$w_t = (1 - \alpha) A \left(\frac{K_t}{L_t} \right)^{\alpha} \tag{7}$$

where re and w denote, respectively, the rental rate of capital and the wage rate.

3.3 Household sector

Household behaviour is captured by 19 representative households that interact in an Allais-Samuelson overlapping generations structure representing each of the age groups. Individuals enter the labour market at the age of 20, retire at age 65, and die at the latest by age 94. Younger

generations (i.e. 0-4, 5-9, 10-14 and 15-19) are fully dependent on their parents and play no active role in the model. However, they do influence the public expenditure. An exogenous age/time-variable survival rate determines life expectancy.

Adult generations (i.e. age groups 20-24, 25-29, …, 90-94) optimise their consumption-saving patterns over time. The household's optimization problem consists of choosing a profile of consumption over the life cycle that maximizes a CES type inter-temporal utility function, subject to the lifetime budget constraint.

In the model specifications without family composition the inter-temporal preferences of an individual born at time *t* are given by:

$$U = \frac{1}{1-\theta} \sum_{k=4}^{20} \left\{ \Pi_{l=4}^{k} \left[\frac{1}{1+\rho_{g+l}} \right] \Pi_{m=0}^{k} sr_{t+m,g+m} \left((C_{t+k,g+k})^{1-\theta} \right) \right\} \tag{8a}$$

where *C* denotes consumption and θ represents the inverse of the constant inter-temporal elasticity of substitution. Parameter ρ is the pure rate of time preference, and depending on the version of the model can be constant or age-variable. In case if rate of time preference is constant the discount factor collapses to a conventional expression $\left[\frac{1}{1+\rho} \right]^{k-4}$. Future consumption is also discounted at the unconditional survival rate, $\prod_k sr_{t+k,g+k}$, which is the probability of survival up to the age *g+k* and period *t+k*. It is the product of the age/time-variable conditional survival rate, $sr_{t+k,g+k}$, between periods *t+k* and *t+k+1* and ages *g+k* and *g+k+1*.

In case we take family composition into account the inter-temporal preferences of an individual born at time *t* are given by:

$$U = \frac{1}{1-\theta} \sum_{k=4}^{20} \left[\frac{1}{1+\rho} \right]^{k-4} \prod_{m=0}^{k} p_{t+m,g+m} \lambda_{t+k,g+k} (C_{t+k,g+k})^{1-\theta} \tag{8b}$$

where λ_g is a number of equivalent adult consumers in a household whose head is *g* years old. We discuss how it is calculated in the calibration section.

The household is not altruistic, i.e. it does not leave intentional bequests to children. In different specifications of the model households can either leaves unintentional bequests or insure their future via a perfect annuity market, as described theoretically by Yaari (1965, case C) and implemented in an OLG context by Borsch-Supan et al (2006).

If we assume unintentional bequests, the household's dynamic budget constraint takes the following form:

$$HA_{t+1,g+1} = Y_{t,g}^L \left(1 - \tau_t^L - Ctr_t\right) + Pens_{t,g} + TRF_{t,g} + \left(1 + r_t\left[1 - \tau^K\right]\right)HA_{t,g} + Beq_t - (1 + \tau^c)C_{t,g} \quad (9a)$$

where *HA* is the level of household assets, *r* is the rate of return on physical assets, τ^K is the effective tax rate on capital, τ^L the effective tax rate on labour, τ^C the effective tax rate on consumption, *Ctr* is the contribution rate to the public pension system, Y^L is the labour income, *Pens* is the level of pension benefits, *TRF* is public transfers other than pensions and *Beq* are unintentionally bequests distributed equally across generations.

If we assume a perfect annuity market, the household's dynamic budget constraint takes the following form:

$$HA_{t+1,g+1} = \frac{1}{sr_{t,g}} \times \left[Y_{t,g}^L\left(1 - \tau_t^L - Ctr_t\right) + Pens_{t,g} + TRF_{t,g} + \left(1 + r_t\left[1 - \tau^K\right]\right)HA_{t,g} - \left[1 + \tau^C\right]C_{t,g}\right] \quad (9b)$$

The intuition behind the term *1/sr* is that the assets of those who die during period *t* are distributed equally between their surviving peers. Since the survival rate at time *t* in age group *g* is less than one, therefore at time *t+1* everyone in their group has more assets. This is the mathematical description of the perfect annuity market.

Labour income is defined as:

$$Y_{t,g}^L = w_t EP_g LS_g \quad (10)$$

where *LS* is the exogenously given supply of labour. It is assumed that labour income depends on the individual's age-specific productivity. In turn, it is assumed that these age-specific productivity differences are captured in age-earnings profiles. These productivity profiles are quadratic functions of age:

$$EP = \gamma + (\lambda_t)g - (\psi)g^2, \quad \gamma, \lambda, \psi \geq 0 \tag{11}$$

with parametric values estimated from micro-data (as discussed in the calibration section). Differentiating different versions of household utility function, subject to different versions of lifetime budget constraint, with respect to consumption yields the following three different first-order conditions for consumption, commonly known as Euler equation:

1) model with smooth consumption

$$\frac{C_{t+1,g+1}}{C_{t,g}} = \left(\frac{\left[1 + r_t(1 - \tau^K)\right]}{1 + \rho} \right)^{1/\theta} \tag{12a}$$

2) model with bell-shaped consumption

$$\frac{C_{t+1,g+1}}{C_{t,g}} = \left(\frac{\lambda_{t+1,g+1}}{\lambda_{t,g}} \frac{\left[1 + r_t(1 - \tau^K)\right]sr_{t,g}}{1 + \rho} \right)^{1/\theta} \tag{12b}$$

3) model with fitted consumption

$$\frac{C_{t+1,g+1}}{C_{t,g}} = \left(\frac{\left[1 + r_t(1 - \tau^K)\right]}{1 + \rho_g} \right)^{1/\theta} \tag{12c}$$

Please note that depending on the version of the model, rate of time preference either constant, ρ, or age-variable, ρ_g.

3.4 Investment and Asset Returns

The law of motion for the capital stock, *Kstock*, takes into account:

$$Kstock_{t+1} = Inv_t + (1-\delta)Kstock_t \qquad (13)$$

where *Inv* represents investment, δ is the depreciation rate of capital.

Capital markets are assumed to be fully integrated. This implies that financial capital is undifferentiated from physical capital, so that the interest rate parity holds:

$$1 + r_t = re_t + (1-\delta) \qquad (14)$$

where *r* and *re* denote the net and gross rates of return to physical capital, respectively.

3.5 Government Sector

The Government's budget constraint reads:

$$\sum_g Pop_{t,g}\left\{\left(\tau_t^L + Ctr_t\right)w_t EP_{t,g} LS_{t,g} + \tau^C C_{t,g} + \tau^K r_t HA_{t,g}\right\}$$
$$= Gov_t + \sum_g Pop_{t,g}\left(TRF_{t,g} + Pens_{t,g}\right) \qquad (15)$$

where Gov is public consumption. The left-hand side of the constraint contains the government revenues. The right-hand side of the equation represents different categories of government expenditure including transfers to households and pension benefits. Note that the pension program is a part of the overall government budget.

Public expenditures per person, *GEPC,* are fixed per-person and hence total expenditure, *Gov,* depends only on the size of the total population, *TPop*.

$$Gov_t = TPop_t GEPC \qquad (16)$$

In the simulations presented in this paper we use the wage tax rate, τ_t^L, as the only endogenous

policy variable that adjusts in every period to achieve a balanced government budget. Wage tax does not generate efficiency distortions, given the absence of an endogenous labour-leisure decision.

3.6 Market and Aggregation Equilibrium Conditions

Perfect competition is assumed in all markets. The equilibrium condition in the goods market requires that the UK's output be equal to aggregate absorption, which is the sum of aggregate consumption, investment and government spending:

$$Y_t = \sum_g Pop_{t,g} C_{t,g} + Inv_t + Gov_t + GovH_t + GovE_t \tag{17}$$

Labour market clearing requires that the demand for labour be equal to the supply:

$$L_t = \sum_g Pop_{t,g} LS_g EP_g \tag{18}$$

Similarly, the units of capital accumulated up to period t must equal the units of capital demanded by the representative firm in that period:

$$Kstock_t = K_t \tag{19}$$

In the same vein, equilibrium in the financial market requires total stock of private wealth accumulated at the end of period t to be equal to the value of the total stock of capital accumulated at the end of period t:

$$\sum_g Pop_g HA_{t,g} = Kstock_t \tag{20}$$

4. Calibration

The model is calibrated using 2010 data for the UK where available. The data for the demographic baseline shock is taken from the 2010-based principal population projections produced by the Office for National Statistics (ONS). Population projections are used for calibration of the fertility, survival and net migration rates used in the model. They are calculated according to the formulae used in the model to replicate the demographic process (section 3.1).

The data on public finances and GDP components are taken from the ONS and HM Treasury. The effective labour income, consumption and capital tax rates are calculated from the corresponding government revenue categories and calibrated tax bases, namely total labour income (compensation of employees), aggregate private consumption and total capital income (gross operating surplus). Data on total amount of pensions are taken from the Government Actuarial Department (GAD). Based on this information, the effective pension contribution rate and the average size of pension benefits are calculated. The average pension per person is obtained by dividing the total amount of pension benefits by the total number of people of pension age. For simplicity, it is assumed that both males and females start receiving pension benefits at the age of 65.

The source of the labour market data is the Labour Force Survey (LFS, Q2 2008- Q1 2013). Two labour market characteristics are derived from the data: age-specific employment rates and age-specific productivity profiles. The latter are estimated via the use of age-earning regressions of the Mincerian type (Mincer, 1958).

To arrive at a number of equivalent adult consumers in a representative household we need coefficients of equivalence between consumption of children of various ages and adults of various ages and family composition. The first item is calculated from the NTA consumption profile. For example, 5-9 y.o. children consume about 50% as much as 25-29 y.o. adults. Than we use age-specific fertility rates (ASFR) to allocate children to adults. By using ASFRs we essentially assume that couples belong to the same age group.

The (5-year) inter-temporal elasticity of substitution ($1/\theta$) is set to 1.5 and (5-year) interest

rate to 27.6% (5% a year).

During the calibration stage, model parameters are solved endogenously under the assumption that the observed base year data represent steady state equilibrium. Thus, it is a reverse of the simulation stage in which the model solves for the future states of the economy, given parameter values, exogenous values and base year data. Our calibration strategy is to calibrate the following parameters:

- the scaling parameter in the production function (A) and the capital income share of output (α) – using the data on output, capital and labour demands;

- aggregate level of capital and capital depreciation rate – using the assumption of the steady state equilibrium and base year level of investment and capital market equilibrium condition;

- age-specific per capita consumption and the rate of time preference – using data on aggregate level of consumption and household optimisation problem.

In the conventional approach the rate of time preference is constant and is used to generate plausible consumption and capital ownership profiles for each age group, for which data are usually not available. In the model specification with smooth consumption the rate of time preference, the interest rate and the inter-temporal elasticity of substitution together determine the slope of the consumption profile across the age groups. If the rate of time preference is smaller than the interest rate – the representative household is more patient than the financial market and the consumption profile is upward sloping (conventional assumption). In the model specification with bell-shaped consumption survival rates also plays a role and age consumption profile has two humps: one because of children's consumption and the other one because of declining consumption in old age. In the specification with fitted consumption we introduce the age-variable rate of time preference, which is calibrated to replicate the exogenously given consumption age profile from the NTA.

Figure 2 compares exogenously set (5-year) interest rate and three rates of time preference – two constant from specification with smooth and bell-shaped consumption, and one time-variable calibrated using NTA consumption age profile data. The age-variable rate of time preference

increases gradually from young to old, and then declines after the age of 75. Our hypothesis is that this decline is connected with the paucity of data no which consumption profile for older age groups is based. The age-variable rate of time preference is lower than the interest rate for the young, which reflects that those generations are more patient than the general financial market. On the other hand, the older age groups have a rate of time preference higher than the interest rate, which implies that they are more impatient than the overall financial market. Both constant rates of time preference that satisfy the same model parameters and generate the same level of aggregate consumption are lower than the interest rate.

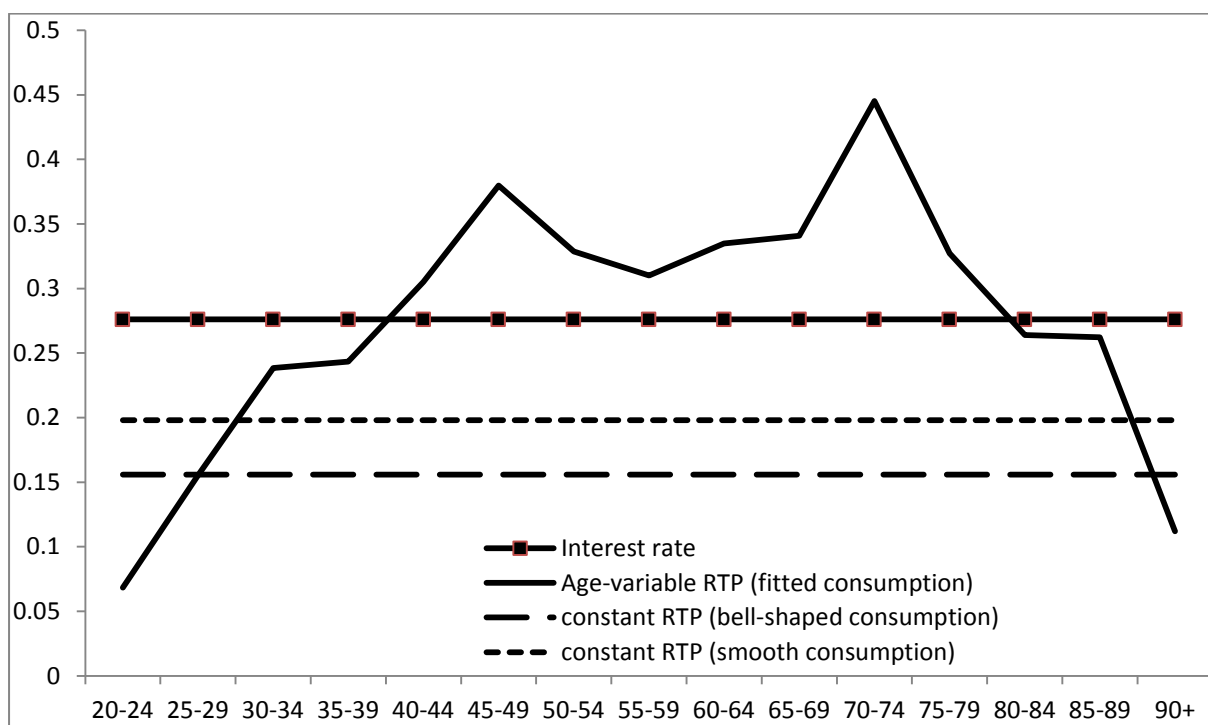**Figure 2. Interest rate and calibrated rates of time preference (5-year)**

Figure 3 presents three consumption age profiles: 1) smooth; 2) bell-shaped; and 3) fitted to the NTA data. The smooth consumption age profile underestimates level of consumption of the younger age groups and overestimates consumption of older age groups (60+). The profile with two humps also underestimates the level of consumption in the young age and overestimates after the age of 50.

**Figure 3. Normalised consumption age profiles[3]**
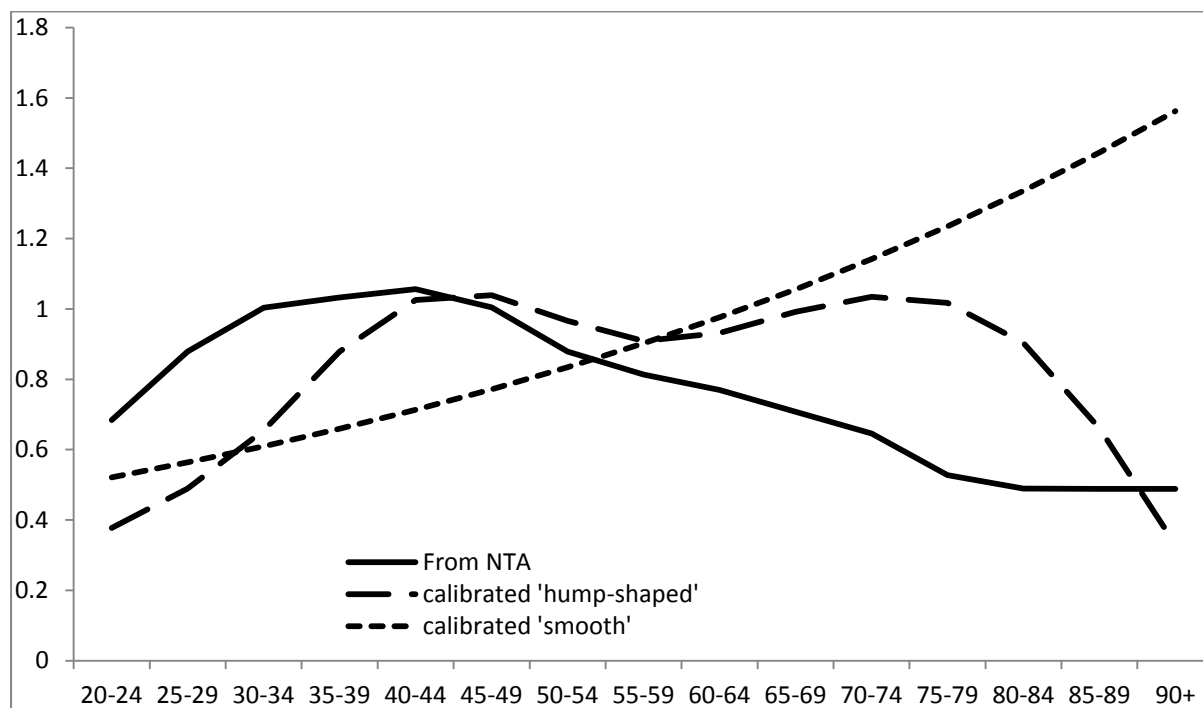
5. Simulation Results

We compare the performance of three versions of the model – with smooth, hump-shaped and fitted consumption profiles – under the standard demographic shock. As a demographic scenario we use the 2010-based ONS principal population projections.

The three versions of the model are calibrated on the same base year data and have the same parameters. But they have different rates of time preference. The difference in this parameter results in different consumption age profiles (see Figure 3). Combined with the savings led capital accumulation model (savings determine investments), this results in different level of savings and investment in the three versions of the model. The reason is that higher consumption in the old age predicted by the specifications with the constant rate of time preference requires a higher level of savings to fund this consumption and correspondingly a higher level of investment. In this model, the only motive for saving is provision for the old age.

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[3] The sum of consumption across the life cycle in the NTA profile is normalized to 1.

Figure 4. Output, % difference
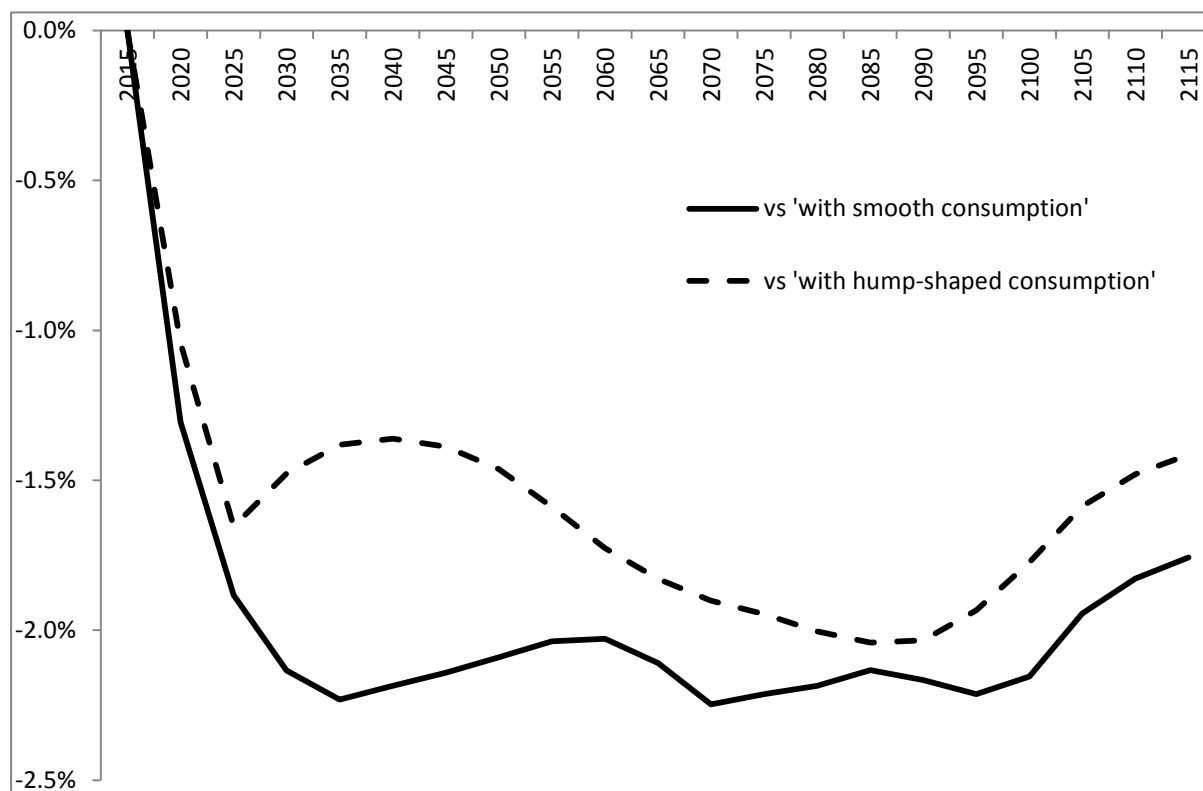
The results presented below show results for the model specification with the age-variable rate of time preference (fitted consumption profile) relative to the specifications with constant rate of time preference (smooth and hump-shaped consumption profiles). Figure 4 demonstrates differences in the level output – an age-variable rate of time preference leads to lower level of capital, which results in a lower level of output.

Since the level of labour supply and demand is the same in all versions of the model, this means that capital labour ratio is lower in the version with age-variable rate of time preference (Figure 5). This results in higher cost of capital and interest rate (Figure 6). Please note that the demographic shock in all versions of the model leads to higher capital labour ratio and lower interest rates, however the decline is smaller in the version with age-variable rate of time preference.
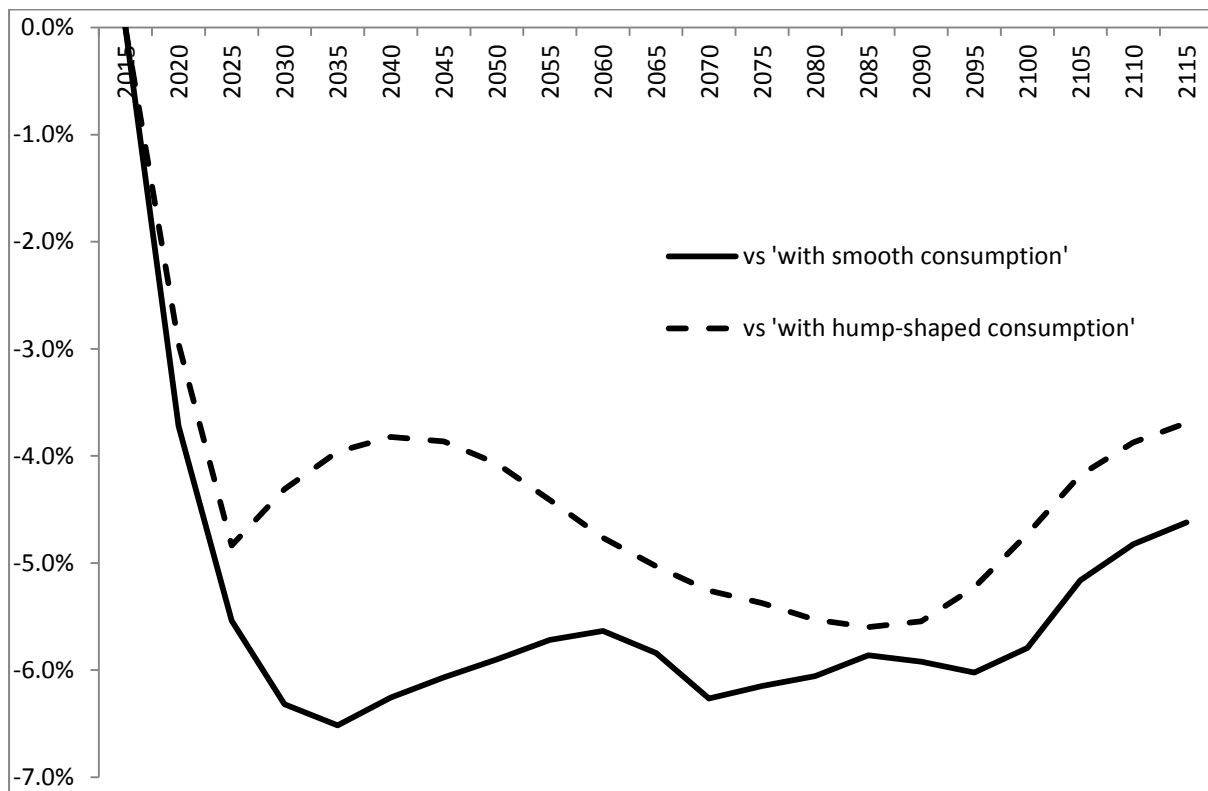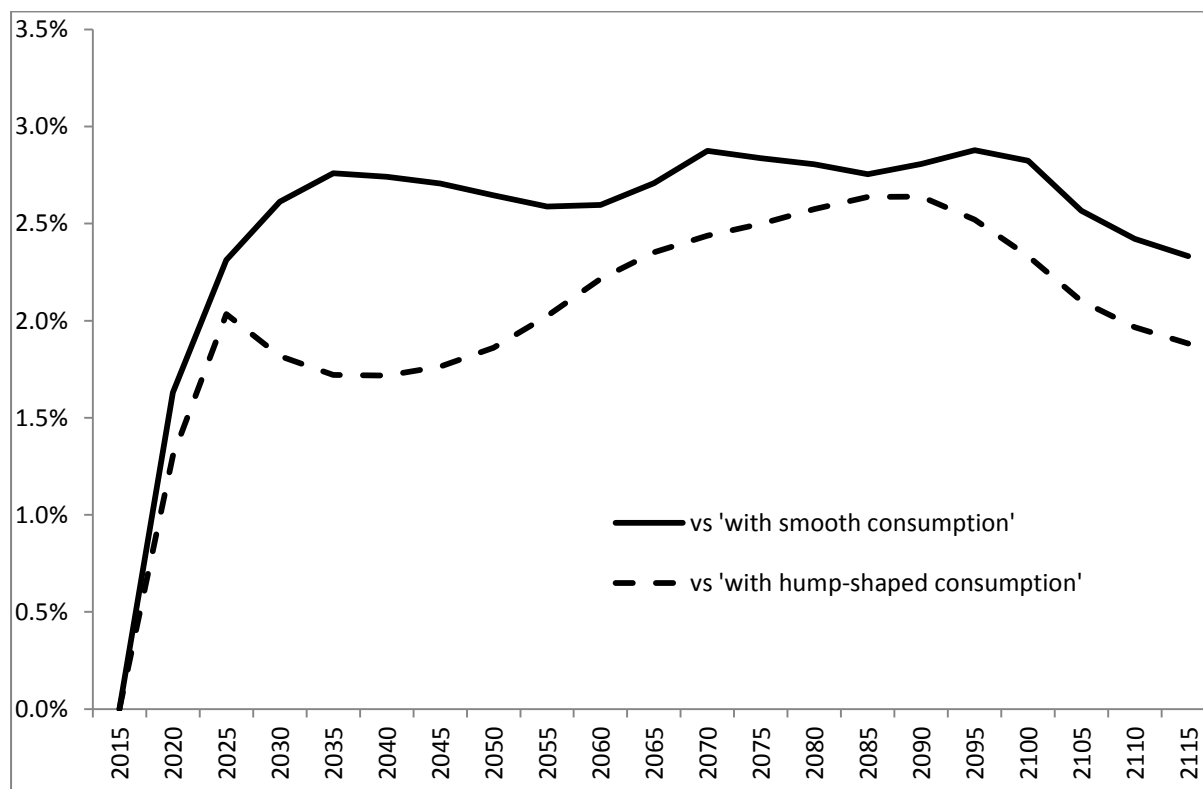
Figure 5. Capital labour ratio, pp difference

Figure 6. Interest rate, pp difference

6. Conclusion

This paper develops a methodology that introduces an age-variable rate of time preference to an OLG-CGE model calibrated on the per capita NTA consumption profile. Compared with the conventional models that use constant rate of time preference, this variation leads to the lower level of savings and capital along the transition to a steady state. The difference in the level of savings and capital is going to be larger the bigger is the difference between consumption age profiles in the older age groups. Our results show that using specification with constant rate of time preference underestimates the negative effect of population ageing on the economy.

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