

# CONSUMPTION DYNAMICS, HOUSING COLLATERAL AND STABILISATION POLICIES: A WAY FORWARD FOR POLICY CO-ORDINATION?

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NIESR Discussion Paper No. 486

Date: 1 February 2018

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This paper was first published in February 2018

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## *Abstract*

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JEL Classification: E31; E40; E51.

Keywords: Heterogeneous households, Credit constraints, Housing collateral, Asset prices, Bank lending, Macro-prudential tools, Fiscal and monetary policy.

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# CONSUMPTION DYNAMICS, HOUSING COLLATERAL AND STABILISATION POLICIES: A WAY FORWARD FOR POLICY CO-ORDINATION?

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December 2017

## Abstract

We decompose aggregate consumption of heterogeneous consumers by modelling both savers and their links to collateral constrained borrowers through a bank which prices credit risk. Savers own both firms and the commercial bank while borrowers require loans from the commercial bank to effect their consumption plans. The bank lends at a premium over the interest rate on central bank money in proportion to the riskiness of loans, the demand for loans, the asset price and the quantity of housing collateral. We show that even though house price do not represent wealth, aggregate consumption is closely related to movements in house prices. We consider the case for jointly determined macro-prudential, fiscal and monetary policies in order to minimise losses for a representative household. We consider the implications of loan default for our main results.

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# 1 Introduction

The interplay between household consumption and housing wealth seems to have become a dominant force in driving recent business cycle fluctuations but both the mechanism and its implications for policy remain rather opaque. We pursue a line of enquiry suggested by a number of recent studies that have employed collateral constrained models (see, among others, Almeida *et al.*, 2006 and Ortalo-Magné and Rady, 2006) to understand better this interplay. We explore the collateral channel of housing demand - a variant of the ‘financial accelerator’ model developed by Bernanke *et al.* (1999) and Kiyotaki and Moore (1997) - in order to disentangle the role of house prices in households’ consumption decisions within the framework of a micro-founded macroeconomic model (see Aoki *et al.*, 2004; Iacoviello, 2005; Lambertini *et al.*, 2013). We emphasize the role of financial intermediation in our model by having the links between savers and borrowers mediated by a bank that allows us to endogenise the External Finance Premium (EFP), defined as the difference between the cost of providing external funding by banks and the opportunity cost of internal funds. An advantage of our approach is that, as well as standard questions of monetary policy, we can then also consider the scope for using macro-prudential instruments to help stabilize welfare in our economy.

In this paper, we unbundle the representative agent assumption and consider two household types, savers and borrowers. Saver households maximize their lifetime expected utility, but are otherwise asset rich as they own the housing stock, financial intermediaries, firms and government bonds and behave as standard intertemporal optimizing consumers. Borrowers obtain loans from banks up to the expected loan to collateral value of houses. Banks intermediate between savers’ deposits and loans to borrowers on the basis of house prices, for which we derive an explicit demand function. We are thus able to analyze the interaction between both types of households, banks and assess the role of various policies in maximizing household welfare.

This work thus fills another gap in the literature. It has been forcefully argued that micro-founded macroeconomic models do not adequately model monetary imbalances or financial frictions. Standard models used for policy analysis have, by construction, no banks, borrowing constraints or any risk of default and so the risk free short-term interest rate suffices to model the monetary side of the economy (on this point, see Chadha *et al.*, 2014). As a consequence, money or credit aggregates and asset prices play no substantive role in explaining economic fluctuations. In comparison we stress that a link between credit and house prices may arise via collateral effects on credit and via the repercussions arising from credit supply fluctuations on house prices. We emphasize that even in a life-cycle model of household consumption, changes in house prices lead to changes in household spending and borrowing when homeowners try to smooth consumption over the life cycle.

There has been increasing interest in introducing a banking sector within micro-founded macroeconomic models in order to analyze economies where different financial assets are available to agents (see, for example, Canzoneri *et al.*, 2008 and Goodhart and Hofmann, 2008). We have framed a banking sector where the risk of households’ default on residential loans is explicitly modelled. This element of uncertainty might explain why the anticipation of potential defaults

leads to contractions of credit and deleveraging, even without the necessity of formal default events. Therefore, in our model, the accelerator effect from increasing asset prices operates through the ‘collateral’ channel of housing, and an attenuator operates via the lending rate which reflects the probability of default on residential loans. Our work confirms that a strong shocks amplification and propagation mechanism originates from the EFP (Goodfriend and McCallum, 2007) and from fluctuations in asset (housing) prices, which determines what we might wish to term a collateral channel, for propagating real and monetary shocks.

The paper contributes on three dimensions. The model captures the salient features of aggregate consumption dynamics and their apparent relationship to house prices, as it delivers strongly procyclical house prices with no wealth effects (Attanasio *at al.*, 2009). We show that house prices, which are forward-looking, are closely linked to the path of borrowers’ consumption, loan to value ratios, inflation and the lending rate. Secondly, consumption dynamics are shown to follow a higher order process when there are two types of households. Saver households have considerable volatility injected into their consumption titling plans by movements in real deposit rates. Borrower households need to generate sufficient collateral to allow credit to flow to them in the form of loans and these loans then suppress consumption in future periods and lead to a cycle in aggregate consumption. There are also spillovers in this economy from one type of consumer to the other, as changes in the expected price of durable goods affect borrower consumption via bank lending; the complementarity between consumption of the two types originates from the policy rate which, in our model, effects the deposit rate and savers’ consumption decision. Finally, we also consider the appropriate role of macro-prudential policies in stabilizing this economy. Our motivation is twofold. First, countercyclical macroprudential policy is linked to other policies that moderate cyclical fluctuations – above all monetary policy, which also affects macroprudential variables as asset prices and credit. Since macroprudential policy has direct or indirect effects on these variables, it is likely to influence the transmission mechanism of monetary policy. Finally, we study how a macroprudential rule for the loan-to-value ratio (LTV) interacts with a government feedback rule for fiscal policy.<sup>1</sup> We compute the optimal parameters of these rules both when fiscal and macroprudential policies act in a coordinated and in a non-coordinated way. We find that both policies acting together unambiguously improves the stability of the system in terms of welfare losses especially in presence of parameter uncertainty.

The paper is organized as follows. Section 2 describes how the model fits to the existing literature. Section 3 presents the model comprising a household sector with two types of agents - savers and borrowers - a banking sector, real and monetary sectors and both monetary and fiscal policy. Section 4 describes the steady-state of the model alongside the solution method employed. Section 5 illustrates the response of key variables in our model to real and financial shocks, reports the main results, and considers the appropriate role of stabilization policy in this class of model, noting that in a traditional representative agent framework active interest rates tend to be sufficient to obtain a welfare allocation close to optimal levels under commitment. Section 6 concludes and

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<sup>1</sup>There is a revival of interest in fiscal policy in macroeconomics. As in McKay and Reis (2016) we devote attention to taxes as a stabiliser for the business cycle, instead of focussing on government spending policies.

offers a tentative normative conclusion.

## 2 Background

The role of collateral constraints has been mainly assessed in a closed economy setting, where agents are constrained in the amount of funds they can borrow by the value of collateral they can pledge as a guarantee to the lenders. For example, in the presence of durable goods, Kiyotaki and Moore (1997) consider the case of collateral constraints with heterogeneous agents. Their analysis shows that the collateral constraint plays an important role in transmitting the effects of various shocks to other sectors through the ‘financial accelerator mechanism’. The benchmark model linking the macroeconomy to financial markets is Bernanke *et al.*’s model (1999) which Bernanke and Gertler (2001) exploit to analyze the supply-side effects of asset-price fluctuations and assess the implications of an explicit monetary-policy response to stock prices.<sup>2</sup>

Empirical work has also focussed on the relationship between consumption and house price. Attanasio *et al.* (2009) stress that over the past 25 years, house price and consumption growth have been highly correlated. Three main hypotheses for this have been proposed: increases in house prices raise household wealth and so their consumption; house price growth relaxes borrowing constraints by increasing the collateral available to households; and house prices and consumption are together influenced by common factors. Using microeconomic data from the Family Expenditure Survey (FES) for UK, they find that the relationship between house prices and consumption is stronger for younger -typically borrowing constrained- than older households, contradicting the wealth channel. Using data from the British Panel Household Survey (BHPS) for the years 1997-2008 Table 1 shows that there is a positive correlation between consumption<sup>3</sup> and house prices<sup>4</sup> for borrowers<sup>5</sup> validating the house price effect hypothesis on consumption for credit constrained agents. Changes in house prices, in fact, affect consumption by changing the degree to

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<sup>2</sup>The financial accelerator mechanism of Bernanke *et al.* (1999) has only recently been reconsidered in standard DSGE models (see, among others, Gilchrist *et al.*, 2009).

<sup>3</sup>Total consumption from BHPS (1997-2008) is defined as the sum of durable and non durable consumption. DURABLE CONSUMPTION includes the following items: Cost of satellite bought in past year (da G a R), Cost of cable TV bought in past year (da G a R), Cost of landline phone bought in past year (da G a R), Cost of mobile phone bought in past year (da P a R), Cost of TV bought in past year (da G a R), Cost of VCR bought in past year (da G a R), Cost of deep freeze bought in past year (da G a R), Cost of washer bought in past year (da G a R), Cost of tumble drier bought in past year (da G a R), Cost of dish washer bought in past year (da G a R), Cost of microwave bought in past year (da G a R), Cost of computer bought in past year (da G a R), Cost of CD player bought in past year (da G a R), Extra loan for other consumer goods (da A a R). NON DURABLE CONSUMPTION includes the following items for services and food: Amount spent on gas (W)XPGASY, Amount spent on electricity (W)XPLECY, Amount spent on oil (W)XPOILY, Amount spent on coal/other (W)XPSFLY, Total weekly food and grocery bill (W)XPFOOD. In Table 1 we define  $C(\text{NONDURABLE})=C(\text{FOOD})+C(\text{SERVICES})$  and  $C\_TOT=C(\text{DURABLE})+C(\text{NONDURABLE})$ .

<sup>4</sup>We consider the real house value defined as the ratio between the house value and the consumer price index (base year 2005).

<sup>5</sup>In line with the borrowing constraint hypothesised in our model borrowers are homeowners facing a mortgage or loan repayment against a house purchase.

which credit constraints are binding.<sup>6</sup> Savers,<sup>7</sup> instead, display a negative or zero correlation between house prices and consumption. Therefore, an increase in house prices which raises household wealth does not affect consumption, contradicting the wealth channel hypothesis. Hence, homeowners who are not facing credit constraints seem to be more hedged against fluctuations in house prices; these fluctuations have no effect on their real wealth and do not affect their consumption choices.

Our work relates to different strands of literature. First, it is strictly related to some recent DSGE models with heterogeneous agents<sup>8</sup> and durables (housing). Such as Iacoviello's study (2005), where he introduces a borrowing constraint tied to housing values both for impatient households and for entrepreneurs; in this framework a rise in asset prices increases the borrowing capacity of the debtors (both households and firms), allowing them to consume and invest more. Hence collateral effects can significantly strengthen the response of the real economy to demand shocks, including those hitting house prices.

Our paper also relates to the literature on optimal policy with heterogeneous consumers and collateral constraints. It relates to the general literature of adding financial frictions to the New Keynesian Model (NKM), among the few papers which adopts a similar approach we refer to Gertler and Karadi's paper (2013). The role of macroprudential policy has been analysed in several papers.<sup>9</sup> Antipa *et al.* (2010) show that macroprudential policies can smooth credit cycles and smooth recessions. Borio and Shim (2007) and N'Diaye (2009) focus on the interaction between countercyclical prudential regulation and monetary policy. Rubio and Carrasco-Gallego (2014) find that a countercyclical LTV rule that reacts to credit growth can moderate lending booms, and be welfare enhancing because it delivers a more stable system, in terms of output, inflation and financial stabilization. Recently, Kannan *et al.* (2012) examine the potential role of monetary policy in mitigating the effects of asset price booms and study the role of macroprudential instruments (based on a LTV rule) in a NKM with a banking sector and financial accelerator effects. The main feature of this model is the presence of financial intermediaries. In fact, the analysis assumes that savers cannot lend to borrowers directly, whereas banks take deposits from savers and lend them to borrowers, charging a spread that depends on the net worth of borrower. They find that having monetary policy which responds to credit conditions or introducing a loan-to-value rule for borrowers helps to reduce the volatility of the output gap and credit aggregates when the economy is hit by financial or housing demand shocks; however, here the functional form for the

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<sup>6</sup> This is also in line with the findings by Aoki *et al.* (2004) who pointed out that a rise in house prices increases the collateral available to homeowners encouraging them to borrow more and to finance higher consumption. More recent evidence shows that large effects of house prices changes on household durable spending and non-durable consumption (for example, Mian *et al.* (2013); Kaplan *et al.* (2016)).

<sup>7</sup>Savers are defined as those who own their house outright without mortgage or loans repayment.

<sup>8</sup>Iacoviello (2005) assumes that the heterogeneity among agents is in the discount rates. Aoki *et al.* (2004) assume instead that a certain fraction of households have accumulated enough wealth so that their consumption decisions are well approximated by the permanent income hypothesis. The other households do not have enough wealth to smooth consumption and they face borrowing constraints.

<sup>9</sup>For discussions of the objectives of macroprudential policy and the concept of systemic risk see Brunnermeier *et al.* (2009).

determination of the spread is assumed rather than derived from a profit maximization problem. Whereas in our model, savers and borrowers face not only different degrees of impatience but also different interest rates; in fact the wedge between the deposit rate and lending rate generates sources for banks' profits and credit frictions. Moreover, we assume that the interest rate wedge is not constant, but varies with expected durable goods prices (i.e., the collateral value), and the amount of granted loans. Since durable goods are secured for loans, changes in the expected price of durable goods will affect lending rate, borrowers' credit availability and consumption. Ordinarily, however, previous work in the field does not consider strategic interactions between macroprudential and fiscal policy as in our model which is the first to address it. Since there is an extensive and established consensus that the origin of the last crisis is related to real estate booms and busts, we have focused on the effects of a fiscal tool that pricks the house price growth and we analyse its interaction with countercyclical macroprudential tools.

### 3 The Model

It is therefore natural to consider heterogeneous agents who are either saver or borrowers (collateral constrained) and the link with the loan to value and the lending rate. As shown in Figure 1, the consumption of savers and of borrowers will be negatively correlated and this will act to reduce overall business cycle variance of aggregate consumption. At the unconstrained equilibrium, the external finance premium, the wedge between borrower and saver interest rates, is driven to zero and consumption is maximised at  $C^*$  for borrowers. When we add in an external finance premium, the level of consumption is lower for borrowers and higher for savers, as the latter save less. Indeed, as we move to the left of  $C^*$ , the consumption of borrowers falls and that of savers increases at time  $t$ , and thus the consumption of these two types of households may tend to be negatively correlated. The market determined external finance premium,  $efp_t$ , reflects the sensitivity of borrower household consumption and  $C^{efp}$  is one possible equilibrium where consumption by borrower households is constrained. Since creditor-borrower dynamics exacerbate intra and intertemporal volatility it may be appropriate to place a 'tax' on supply and this will tend to reduce further the consumption of borrowers. The tax can be any policy that reduces the supply of savings at every given interest rate and may include, for example fiscal intervention that taxes the housing collateral, or simply macroprudential policies that limit supply in lending. The basic result would be to further limit the consumption of borrower-households to the point  $C^{mpi}$ , at which the existing magnitude of the external finance premium is augmented by the  $MPI_t$ . The lower level of consumption here by borrowers is designed to reduce the build-up in financial risks over the business cycle and can be modified separately from the policy rate thus offering policymakers an extra degree of freedom (Chadha, 2014).

In this section we illustrate the main features of our model summarized in Figure 2. The economy operates over an infinite-time horizon and comprises a continuum of households in the interval  $\mathbb{R} \in [0, 1]$ . Households who consume, work and demand housing and financial assets, are

divided into two groups, which we refer to as saver (creditor) households and borrower households.<sup>10</sup> Saver households maximize their lifetime expected utility facing a CIA constraint, while borrower (leveraged) households, whose ability of borrowing is endogenously linked to the market value of their housing wealth, face both liquidity and collateral (borrowing) constraints. The latter ‘doubly-constrained’ collateralize their debt repayment in order to borrow from financial intermediaries and use these additional credit lines and money to finance their current consumption. The dichotomy between savers, who are essentially standard optimizing consumers and borrowers who are liquidity constrained is key to this paper.

The banking sector collects money in the form of time deposits from savers and lends against housing equity to households who are borrowing constrained. Saver households purchase a positive amount of bonds and time deposits and do not borrow from banks, while leveraged households borrow a positive amount of money from banks and have no deposits. We assume that the savers are also the owners of monopolistic firms in the production sector and of financial intermediaries in the banking sector. Saver households derive utility from consumption of non-durable goods (consumption goods) while leveraged households derive utility from consumption of both non-durable goods and durable goods (housing services). Leveraged households supply labor to firms. Entrepreneurs produce differentiated intermediate goods using leveraged households’ labor. They sell the differentiated goods at a price which includes a markup over the purchasing cost and is subject to adjustment costs. Finally, the monetary authorities set the policy interest rate endogenously, in response to inflation and output, and fiscal policy can be set passively or actively to foster stabilization.

### 3.1 Households’ Utility Maximization

#### 3.1.1 Saver Households

The preferences for this type of household can be expressed as:

$$\max U = E_0 \sum_{t=0}^{\infty} \beta^t (\log c_t + \chi_B \log b_t) \quad (1)$$

where  $\beta \in (0, 1)$  is the discount or time-preference factor that measures how patient people are,  $c_t$  denotes household’s real consumption of non-durable goods and  $b_t$  real government bond holdings with a weight coefficient  $\chi_B > 0$ .

The representative saver household maximizes the above utility function subject to a sequence of constraints expressed in real terms (i.e., in units of consumption):

(i) Resource constraint:

$$c_t + b_t + d_t \leq \Delta q_t H_t + \frac{R_{t-1}^B b_{t-1}}{\pi_t} + \frac{R_{t-1}^D d_{t-1}}{\pi_t} + \Theta_t + \Pi_t - \tau_y Y_t \quad (2)$$

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<sup>10</sup>We use saver and creditor, as well as borrower and leveraged as interchangeable terms.

Households enter each period  $t$  with real saving deposits at the bank,  $d_t$ . Saving deposits pay a gross nominal interest rate,  $R_t^D$ , at the end of the period, while short-term government bonds,  $b_t$ , pay a (gross) nominal interest rate  $R_t^B$  and  $\pi_t \equiv \frac{P_t}{P_{t-1}}$  is the (gross) inflation rate. In our model, saving deposits (or saving accounts) like a risk-free financial assets provide a store of value and no liquidity services to households, since we assume that they cannot be withdrawn from the bank before the beginning of the next period. Following Canzoneri *et al.* (2008) we put government bonds into household utility to reflect their value in providing liquidity services in addition to being store of value; the liquidity services are represented as direct contributions to household's utility and therefore they command a liquidity premium, for this reason the bond rate will be lower than the saving deposit rate. We also assume that savers enters each period with an endowment of a fixed credit good, therefore it does not enter savers' preferences. We might think of the credit good as housing services,  $H_t$ , which savers sell at time  $t$  to the borrowers who demand and consume housing services in the same period. Hence, the term  $\Delta q_t H_t = q_t H_t - \frac{q_{t-1} H_{t-1}}{\pi_t}$  stands for net real housing holdings with  $q_t$  denoting the relative price of residential goods expressed in term of non-durables,  $q_t \equiv \frac{Q_t}{P_t}$  where  $Q_t$  is the nominal house price. The terms  $\Theta_t$  and  $\Pi_t$  denote real profits (dividends)<sup>11</sup> respectively from firms and banks owned by saver households; and the term  $\tau_y Y_t$  is taxation. In accordance with our time convention,  $d_t$  and  $b_t$  are respectively bank saving accounts and short-term government bonds accumulated in period  $t$  and carried over into period  $t + 1$ .

By differentiating (1) subject to (2) with respect  $c_t$  and  $b_t$ , the efficiency conditions for saver household are:<sup>12</sup>

- *Consumption:*

$$\frac{1}{c_t} - \lambda_t^s = 0 \quad (3)$$

- *Bonds:*

$$\frac{\chi_B}{b_t} - \lambda_t^s + \beta \frac{\lambda_{t+1}^s R_t^B}{\pi_{t+1}} = 0 \quad (4)$$

Replacing  $\lambda_t^s$  we get:

$$\frac{\chi_B}{b_t} = \left( \frac{1}{c_t} - \frac{\beta R_t^B}{\pi_{t+1} c_{t+1}} \right) \quad (5)$$

- *Deposits:*

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<sup>11</sup>Profits rebated to saver households by the real and banking sectors are respectively  $\Theta_t = \int_0^1 \Theta_{z,t} dz$  and  $\Pi_t = \int_0^1 \Pi_{j,t} dj$ .

<sup>12</sup>For the sake of simplicity we will omit the expectations operator.

$$-\lambda_t^s + \beta \frac{\lambda_{t+1}^s R_t^D}{\pi_{t+1}} = 0 \quad (6)$$

and replacing  $\lambda_t^s$

$$\frac{1}{c_t} = \beta \frac{R_t^D}{c_{t+1} \pi_{t+1}} \quad (7)$$

Equation (7) is the relation between the marginal utility of current period consumption, next period consumption and the real interest rate. With respect to the standard consumption Euler condition, in (7) there is an additional cost defined as the opportunity cost of holding positive money balances, which is given by the foregone one-period deposit interest rate,  $R_t^D$ . Whilst (4) defines the demand for bonds,  $B_t$ , which depends positively on the bond rate,  $R_t^B$ , and inversely on the the difference of marginal utility of consumption in two consecutive periods.

### 3.1.2 Borrower Households

Each household is allowed to acquire housing services by owning a house. Owning a house in our model serves a dual purpose; it provides the household housing services, and also allows household to own equity. Housing enters in this model both as a good but also as an asset which can be used as collateral to get loans in the credit market. This group of households is facing an additional borrowing constraint that limits the amount they can borrow to the expected market value of their housing holdings; this sort of home equity release scheme allows households to access their housing wealth for financing consumption and housing.

The representative borrower household's maximization problem then reads as:

$$\max U^b = E_0 \sum_{t=0}^{\infty} \check{\beta}^t \left( \log c_t^b + \chi_H \log H_t - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) \quad (8)$$

where the discount factor is  $\check{\beta} \in (0, 1)$  and  $\check{\beta} < \beta$  indicating that borrowers are also more impatient than savers.  $H$  denotes services from the fixed stock of residential goods (housing services), with a weight coefficient  $\chi > 0$ ;  $N$  denotes labour supplied by leveraged households to the goods sector, and  $\varsigma > 0$  is the labour disutility parameter and it is equal to the inverse of the (Frisch) elasticity of labour supply with respect to the real wage.<sup>13</sup> The superscript  $b$  denotes borrower (leveraged) consumers who are subject to both liquidity and borrowing constraints.

This household maximizes the above utility function subject to the following constraints expressed in real terms (i.e. in units of consumption):

(i) Resource constraint:

$$c_t^b + \Delta q_t H_t + \frac{R_{t-1}^L l_{t-1}}{\pi_t} \leq l_t + w_t N_t - \tau_h q_t H_t \quad (9)$$

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<sup>13</sup>The Frisch elasticity of labour supply is defined as the elasticity of the labour supply with respect to wages holding the marginal utility of consumption constant. Empirical estimates on the Frisch elasticity of labour supply are numerous. Prescott (2005) estimates that the Frisch elasticity of labour supply is 3 in the United States, so  $\varsigma = 0.3$ . With  $\varsigma$  approaching 1, the utility function becomes linear in leisure.

(ii) Borrowing constraint:

$$l_t \leq \frac{\kappa_t q_{t+1} \pi_{t+1} H_t}{R_t^L} \quad (10)$$

In (9) among the resources there are wage earnings  $w_t N_t$  from supplying labor to the goods sector with  $w_t$  denoting real wage and loans from the banking sector expressed in real terms,  $l_t$ , with  $R_{t-1}^L$  denoting the nominal interest rate on previous period borrowing. Finally, the last term on the right hand side of the resource constraint denotes tax payments to the government in the form of a property tax,  $\tau_h q_t H_t$ , levied on the value of the housing assets where  $\tau_h$  is the house-tax rate.

Following the ‘collateral’ channel of housing, our work aims at disentangling the important role of housing wealth in the households’ decisions of consumption over the life-cycle. According to (10) in each period  $t$  borrower households cannot borrow from banks more than a fraction,  $\kappa_t$ , of the expected value of today’s stock of housing which in real terms is equal to  $q_{t+1} \pi_{t+1} H_t$ . The term  $\kappa_t = \kappa (Y_t)^{-f_k} \xi_{k,t}$  depends on the Loan-to-Value (LTV) parameter  $\kappa$  and on the output gap,  $Y_t$ . We assume for the moment that  $f_k = 0$  hence the LTV is a fixed parameter subject to a stochastic shock.

This approach is a variant of the ‘financial accelerator’ model developed by Bernanke *et al.* (1999) where borrowing is procyclical with respect to the underlying business cycles which affect asset prices and therefore the value of the collateral. The collateral channel can work either by relaxing a liquidity constraint directly, by rising the loan to value ratio, or by providing equity that can be extracted at some point in the future, affecting individuals’ consumption decisions. Among other things, the collateral channel can also amplify the effects of monetary policy in the economy (see Goodfriend and McCallum, 2007; Chadha *et al.*, 2014; Aoki *et al.*, 2004). As house prices affect the collateral value of houses, then real house price fluctuations have a considerable role in determining the access to credit lines (10).

We then differentiate (8) subject to (9) and (10) with respect to  $c_t$ ,  $l_t$ ,  $W_t$  and  $N_t$ . The efficiency conditions for leveraged consumer are:

• *Consumption:*

$$\frac{1}{c_t^b} - \lambda_t^b = 0 \quad (11)$$

• *Loans:*

$$\lambda_t^b - \frac{\check{\beta} R_t^L \lambda_{t+1}^b}{\pi_{t+1}} = \nu_t \quad (12)$$

and replacing  $\lambda_t^b$  :

$$\frac{1}{c_t^b} - \frac{\check{\beta} R_t^L}{c_{t+1}^b \pi_{t+1}} = \nu_t \quad (13)$$

where  $\nu_t$  is the Lagrange multiplier associated to the borrowing constraint. Equation (11) implies that the borrowing constraint is binding in a neighborhood of the steady state, that is  $\nu > 0$  if and only if  $(1 - \check{\beta}R^L) > 0$ . And since we log-linearize the model around the steady state and assume that uncertainty is low, then the borrowing constraint will always bind along the dynamics and equation (10) will hold with equality.

A binding collateral constraint, implying  $\nu_t > 0$ , has two main effects on household's decisions: (i) it prevents a consumption smoothing behavior (11); (ii) increases the marginal value of housing as it is also used as collateral (see below (15)).

- *Residential Goods:*

$$\frac{\chi_H}{H_t} - \lambda_t^b q_t + \frac{\check{\beta} q_t \lambda_{t+1}^b}{\pi_{t+1}} - \tau_h q_t \lambda_t^b + \nu_t \frac{\kappa_t q_{t+1} \pi_{t+1}}{R_t^L} = 0 \quad (14)$$

Replacing  $\lambda_t^b$  and given (11) the above relationship can be also rewritten as follows:

$$\frac{\chi_H}{H_t} = q_t \left[ \frac{(1 + \tau_h)}{c_t^b} - \frac{\check{\beta}}{c_{t+1}^b \pi_{t+1}} \right] - \nu_t \frac{\kappa_t q_{t+1} \pi_{t+1}}{R_t^L} \quad (15)$$

- *Labor Supply:*

$$N_t^\zeta c_t^b = w_t \quad (16)$$

The usual Euler condition (11) states that the utility foregone in sacrificing a unit of current consumption is equal to the expected marginal benefit of future additional consumption appropriately discounted. But because of the CIA constraint, households who wish to consume more of nondurable goods face the opportunity cost given by foregoing returns on interest-bearing assets - as they must hold higher positive money balances which do not pay a return. In addition, the collateral constraint implies that because the borrowing capacity, and therefore the availability of loans, is strictly tied to the real value of housing holdings we are also expecting a higher demand of housing.

Equation (15) can be interpreted as a modified intertemporal Euler condition for residential goods. It states that the purchase of durable goods (housing services) is partly an investment. In fact, (15) shows that the path of housing consumption is optimal when the marginal cost of acquiring one unit of housing which comprises a housing tax,  $\tau_h$ , (the first term on the RHS) is equal to its marginal utility (the last term on the RHS). The latter depends on (i) the direct utility gain of each additional unit of real estate; (ii) on the value of housing used as collateral to borrow funds from the credit market; (iii) from the expected utility coming from next period net resale value of each unit of housing purchased in the previous period. Therefore, in (15) the last term on the right-hand side is linked to the shadow value of the collateral constraint (10) which depends on several model variables. The first is the loan-to-value ratio,  $\kappa_t$ , which is a measure of the flexibility of the credit market. The second variable is represented by the real expected house prices,  $q_{t+1}$ , which directly affects the ability of households to get loans by relaxing their collateral

constraint. Therefore, when house prices rise, especially in case of bubbles and overconfidence in future house prices, households can borrow and spend more. This implies that with a cheaper and easier access to home equity lines of credit, a rise in house prices will allow for additional borrowing to finance consumption. Another variable is the interest rate on loans,  $R_t^L$ , which is negatively related to housing demand, since the amount of debt to be repaid is increasing with the interest rate charged by banking intermediaries on the new borrowing against housing collateral. Whilst the term into the square brackets is the difference between the marginal utility of consumption of two consecutive periods can be interpreted as a modified version of a standard Euler equation for consumption as a measure of the collateral constraint effects. It states that there is the possibility of expanding consumption by means of purchasing a unit of housing and increasing borrowing via a relaxation of the collateral constraint. As recalled above, these results occur because the borrowing constraints affect both intertemporal and within-period households' choices of lifetime consumption. And the housing demand of doubly-constrained households may increase over time as the shadow price of lifting the collateral (borrowing) constraint exceeds the marginal utility of consumption. Consumption for borrower households is determined by their flow of funds (9).

Given that housing supply is assumed to be fixed and equal to 1, we can derive the real house price from (15) and we can thus study directly how asset prices interact with the consumption plans of borrowers (see the log-linearised equations in the Appendix):

## 3.2 Banking Sector

Banks collect deposits from savers and make loans to borrowers under monopolistic competition; this market power allows each individual bank to set its own interest rates on loans and deposits to maximize profits. In this section we outline the optimal lending and deposit rates and point to three parameters, the loan default rate,  $\lambda$ , the fraction of seizable collateral,  $\delta$ , and the loan-to-value ratio,  $\hat{\kappa}_t$ , that might be set to influence bank policy as part of a macroprudential framework. In our analysis of this model we will set these parameters to either a lax or restrictive level in order to understand the implications for monetary and fiscal policy.

### 3.2.1 Bank Profit Maximization

The representative bank  $j$  seeks to maximize profits:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t+s} \Pi_{j,t+s} \tag{17}$$

where  $\Pi_{j,t+s}$  denotes real profits and the nominal discount rate  $\Lambda_{t+s} = \beta^s \left( \frac{C_t}{C_{t+s}} \right)$  comes from the saver households' maximization problem. The coincidence of discount factors comes from the assumption that households (saver households) are the ultimate owner of banks and their profits.

Bank's profits,  $\Pi_j$ , expressed in real terms read:

$$\Pi_{j,t} = \left( \int_0^{s(\bar{\kappa}_t)} l_{j,t} R_{j,t}^L \phi(\kappa_t) d\kappa_t \right) + \left( \int_{s(\bar{\kappa}_t)}^{\infty} \delta \kappa_t q_t H_t \phi(\kappa_t) d\kappa_t \right) - R_{j,t}^D d_{j,t} + R_t^M rr_{j,t} \quad (18)$$

The representative bank maximizes the expected flow of profits subject to the ex-ante real budget constraint:

$$rr_{j,t} + l_{j,t} = d_{j,t} \quad (19)$$

where  $rr_{j,t}$  denotes high powered money (reserves) on which the Central Bank pays an interest rate equal to the policy rate,  $R_t^M$ , and since we assume a fractional reserves system then  $rr_{j,t} = rr d_{j,t}$  where  $rr$  is the reserve requirement coefficient.

In the profit function  $\phi(\kappa_t)$  is the probability density function of the shock on the LTV or, equivalently, on house prices while  $\delta$  is the fraction of collateral  $\kappa_t q_t H_t$  seized by the bank in case of borrower's insolvency. This latter term can, therefore, be interpreted as the collateral value net of monitoring cost faced by banks to assess and seize the collateral connected to the original loan, and this foreclosure cost is assumed to be constant (Bernanke *et al.*, 1999). Borrowers who default on their loan lose their housing holdings. Finally, let be  $\bar{\kappa}_t$  the threshold value of the shock for which borrowers are still willing to repay the loan, then the shortfall risk  $s(\bar{\kappa}_t)$  can be defined as follows:

$$s(\bar{\kappa}_t) = l_{j,t} - \delta \kappa_t q_t H_t \quad (20)$$

Assuming an exponential distribution of the probability function for  $\kappa_t$ , what we observe is that with positive housing price shock (i.e. higher household's equity) the loan to value ratio will be in the "safe" region,  $\kappa_t \in [0, \bar{\kappa}_t)$ , and the loans will be fully repaid, while with negative realizations of the house price shock (i.e. lower household's equity) the loan to value will lie in the "default" region,  $\kappa_t \in [\bar{\kappa}_t, \infty]$ , and loan default occurs.

**Optimal Loan Rate** Deposit and loan contracts bought by households are a composite basket of slightly differentiated products, loans and deposits, each supplied by a branch of a bank  $j$  with elasticities of substitution equal to  $\mu_L$  and  $\mu_D$  respectively.<sup>14</sup>

Given the assumption that banking intermediaries operate in a regime of monopolistic competition, each bank faces an upward sloping demand curve for deposits and a downward sloping demand for loans, as we will show below. This market power allows each individual bank to set its own interest rates on loans and deposits to maximize profits. As in Hüelsewig *et al.* (2009), we assume that the individual bank  $j$  that operates in an environment that is characterized by banker-customer relationships faces the following demand for lending from households:

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<sup>14</sup>Thus as in a standard Dixit-Stiglitz framework for goods markets, agents have to purchase loan (deposit) contracts by each banking intermediary in order to borrow (save) one unit of resources. This assumption allows to capture the existence of market power in the banking industry. In fact, leveraged households would allocate their borrowing among different banks so as to minimize the due total repayment. Saver households would allocate their savings in form of deposits among different banks so as to maximise the revenues.

$$l_{j,t} = \left( \frac{R_{j,t}^L}{R_t^L} \right)^{-\mu_L} l_t \quad (21)$$

where  $\mu_L > 1$  represents the interest rate elasticity of loan demand,  $R_{j,t}^L$  is the interest rate on the loan  $l_{j,t}$  provided by bank  $j$ , and  $l_t$  is the aggregate demand for loans. According to (21) we assume that banks provide differentiated loans as they act under monopolistic competition. Following Carletti *et al.* (2007), we interpret the parameter  $\mu_L$  as the household's willingness to modify the customer relationship with the bank in the event of a change in loan rates. The higher is  $\mu_L$  the weaker become the ties between the bank  $j$  and the customers, that is the market power measured by  $1/\mu_L$  decreases; and for values of  $\mu_L$  approaching infinity the loan market resembles perfect competition.

By replacing  $rr_{j,t}$  using the resource constraint (19) bank's profits can be rewritten as follows:

$$\Pi_{j,t} = l_{j,t} \left( \int_0^{s(\bar{\kappa}_t)} R_{j,t}^L \phi(\kappa_t) d\kappa_t - R_t^M \right) + \int_{s(\bar{\kappa}_t)}^\infty \delta \kappa_t q_t H_t \phi(\kappa_t) d\kappa_t - d_{j,t} (R_{j,t}^D - R_t^M) \quad (22)$$

By maximizing the expected flow of profits (22) subject to (21) we get the optimal loan rate:

$$R_{j,t}^L = \frac{\mu_L}{(\mu_L - 1)} \frac{1}{\int_0^{s(\bar{\kappa}_t)} \phi(\kappa_t) d\kappa_t} R_t^M \quad (23)$$

We assume that the probability function for  $\kappa_t$  has an exponential distribution<sup>15</sup>  $\phi(\kappa_t) = \lambda e^{-\lambda \kappa_t}$ . Hence, the cumulative function when  $\kappa_t \in [\bar{\kappa}_t, \infty]$ , i.e. loans are defaulted, is  $\Phi(\kappa_t) = \int_{s(\bar{\kappa}_t)}^\infty \lambda e^{-\lambda s(\kappa_t)} d\kappa_t = 1 - e^{-\lambda s(\bar{\kappa}_t)}$  while the cumulative function when  $\kappa_t \in [0, \bar{\kappa}_t)$ , i.e. loans are repaid, is  $1 - \Phi(\kappa_t) = \int_0^{s(\bar{\kappa}_t)} \lambda e^{-\lambda s(\kappa_t)} d\kappa_t = e^{-\lambda s(\bar{\kappa}_t)}$  where  $\lambda$  is the default rate, with  $\lambda = \frac{\phi(\cdot)}{\Phi(\cdot)} = \frac{\lambda e^{-\lambda \kappa}}{e^{-\lambda \kappa}}$ . And we can write (23) in a more compact form as follows:

$$R_{j,t}^L = X_{\mu_L} R_t^M e^{\lambda s(\bar{\kappa}_t)} \quad (24)$$

Note that the optimal loan rate,  $R_{j,t}^L$ , is given by a constant mark-up  $X_{\mu_L} = \frac{\mu_L}{(\mu_L - 1)}$  over the policy rate  $R_t^M$  plus a risk premium  $e^{\lambda s(\bar{\kappa}_t)}$ .

After imposing a symmetric equilibrium, log-linearizing and using the definition of the shortfall risk in (20), the optimal loan rate (24) reads:

$$\hat{R}_t^L = \hat{R}_t^M + \frac{\lambda}{1 - \delta} \left[ \hat{l}_t - \delta \left( \hat{\kappa}_t + \hat{q}_t + \hat{H}_t \right) \right] \quad (25)$$

Since the spread between  $\hat{R}_t^L$  and  $\hat{R}_t^M$  is the external finance premium *efp* we can rewrite the above relationship in a more compact form as follows:

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<sup>15</sup>So higher expected housing prices are reducing the failure rate of banks.

$$\hat{R}_t^L - \hat{R}_t^M = efp_t = F \left( \underbrace{\frac{l_t}{q_t H_t}}_{\text{Borrowers' Leverage}}, \underbrace{\kappa_t}_{\text{LTV}} \right) \quad (26)$$

According to (25) a fall in leveraging, that is a decrease in the level of households' liability to asset ratio, leads to an increase in the probability of repayment thus reducing the lending rate; and such a fall depends on the coefficient  $\lambda$ . We can note immediately that this default rate,  $\lambda$ , the fraction of seizable collateral,  $\delta$ , and the loan-to-value ratio,  $\hat{\kappa}_t$ , will each impact on the elasticity of the loan rate to the state of the economy.

The result in (25) introduces a 'financial accelerator' in monetary policy as lending will expand when the collateral value increases. In nominal terms, an increase in the house price as well as an increase in the fraction of the residential good that can be used as a collateral, raises the value of households' collateralized net worth relative to their stock of outstanding loans. In other words, in real terms the value of their outstanding loans falls relative to that of their collateral following an increase in house prices. The implication is that banks are willing to accept a lower risk premium, thus reducing the lending rate. The collateralized wealth could also act as a strict quantity constraint on bank borrowing, as for instance in the model of Kiyotaki and Moore (1997) and its variants (see, for instance, Krishnamurthy, 2003) where shocks to credit-constrained firms would then be amplified through changes in collateral values and transmitted to output.

**Optimal Deposit Rate** In a similar way followed by Hüelsewig *et al.* (2009), we also assume that the bank  $j$  faces the following demand for deposits:

$$d_{j,t} = \left( \frac{R_{j,t}^D}{R_t^D} \right)^{\mu_D} d_t \quad (27)$$

where  $d_{j,t}$  is the demand for bank  $j$  deposits,  $d_t$  is the economy-wide demand for deposits,  $R_t^D$  is the average deposit interest rate prevailing in the market, taken as given by the single bank when solving the problem and  $\mu_D$  is elasticity of substitution among deposit varieties. Banks exploit their market power to lower their marginal cost (deposit interest rates) in order to increase profits and  $\mu_D$  is a measure of the existing competition in the banking sector; the degree of competition in the banking sector is measured by the inverse of  $\mu_D$ .

Therefore, by maximizing the flow of profits (22) with respect  $R^D$  subject to (27) we get the optimal deposit rate:

$$R_{j,t}^D = X_{\mu_D} R_t^M \quad (28)$$

where  $X_{\mu_D} = \frac{(1+\mu_D)}{\mu_D}$ . Therefore, with fully flexible deposit rates, the cost of deposits depends on the elasticity of substitution among deposit varieties,  $\mu_D$ , and the optimal deposit rate  $R_t^D$  would be determined as a mark-down,  $\frac{1}{X_{\mu_D}}$ , over the policy rate,  $R_t^M$ . Conversely, the policy rate

is simply given by a constant mark-up over the deposit rate, that is  $R_t^M = (X_{\mu_D})^{-1} R_{j,t}^D$ . This implies that the bank views households' deposits and reserves as perfect substitutes at the margin so the spread between the policy rate and the cost of deposits only depends on the elasticity of substitution among deposit varieties. The latter condition implies that money market credits and deposits are assumed to be perfect substitutes so that the deposit rate is then assumed to equal the policy rate (at least in log-linear form) and are therefore exogenous for the bank.

### 3.3 Real Sector

#### 3.3.1 Final Good Producers

In a perfectly competitive market, each firm producing final good uses a continuum of intermediate goods indexed by  $z \in [0, 1]$  according to the following CES technology:

$$Y_t = \left( \int_0^1 Y_{z,t}^{\frac{\psi-1}{\psi}} dz \right)^{\frac{\psi}{\psi-1}} \quad (29)$$

where  $Y_{z,t}$  is the demand by the final good producer of the intermediated good  $z$ , and  $\psi > 1$  is the elasticity of substitution between differentiated varieties of intermediate goods.

Profit maximization implies a downward sloping demand function for the typical intermediate good  $z$ :

$$Y_{z,t}^d = \left( \frac{P_{z,t}}{P_t} \right)^{-\psi} Y_t \quad (30)$$

where  $P_{z,t}$  denotes the price of the intermediate good,  $Y_{z,t}$ , and  $P_t$  is the price index of final consumption goods which is equal to:

$$P_t = \left( \int_0^1 P_{z,t}^{1-\psi} dz \right)^{\frac{1}{1-\psi}} \quad (31)$$

where the price index (31) is consistent with the maximization problem<sup>16</sup> of the final good producer earning zero profits and subject to the production function (29).

#### 3.3.2 Intermediate Goods Producers

There is a continuum of firms producing intermediate goods. Each firm has a monopolistic power in the production of its own good variety and therefore has a leverage in setting prices. The representative monopolistic firm,  $z$ , will choose a sequence of prices and labour inputs  $\{N_{z,t}, P_{z,t}\}$  to maximize expected discounted profits:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t+s} \Theta_{z,t+s} \quad (32)$$

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<sup>16</sup>Hence the problem of the final good producer is:  $\max P_{i,t} Y_{i,t} - \int_0^1 P_{i,t}(z) Y_{i,t}(z)$  subject to the demand function (30).

where  $\Lambda_{t,s} = \beta^s \left( \frac{C_{t+s}}{C_t} \right)^{-1} \left( \frac{P_t}{P_{t+s}} \right)$  is the relevant creditor household discount factor and  $\Theta$  denotes profits.

A Cobb-Douglas-type production function is adopted with decreasing return on labour which the only variable input, and with a fixed input represented by an endowment of wealth,  $H$ :

$$Y_{z,t} = A_t (N_{z,t})^{1-\gamma} (H_{z,t-1})^\gamma \quad (33)$$

where  $0 < \gamma < 1$  is a measure of decreasing returns,  $N_{z,t}$  denote firm's  $z$  demand of labor, and  $A_t$  is the productivity shock that is assumed to be common to all firms and evolves exogenously over time.<sup>17</sup>

The intratemporal demand function for good  $z$  is given by:

$$Y_{z,t}^d = \left( \frac{P_{z,t}}{P_t} \right)^{-\psi} Y_t \quad (34)$$

where  $\psi$  is the price elasticity of demand for individual goods faced by each monopolist and  $P$  is the general price level. Therefore,  $\frac{P_{z,t}}{P_t}$  is the relative price of good variety  $z$ .

Given the consumer's demand schedule (34) and taking wages as given, the cost minimization implies the following demand for labor:

$$w_t = \frac{MPL_t}{X_{\psi,t}} = \frac{1}{X_{\psi,t}} \frac{(1-\gamma) Y_t}{N_t} \quad (35)$$

where  $w_t = \frac{W_t}{P_t}$  is the real wage and  $X_{\psi,t}$  is the markup (or the inverse of the real marginal cost,  $MC_t = 1/X_{\psi,t}$ ) which in steady state is  $X_\psi \equiv \frac{\psi}{\psi-1}$  and  $MPL_t \equiv (1-\gamma) A_t N_t^{-\gamma}$  is the marginal product of labor.

**Inflation Dynamics.** As it is standard in the New Keynesian literature, we assume Calvo staggered nominal price adjustment. We assume that intermediate firms set nominal prices in a staggered fashion, according to a stochastic time dependent rule. The sale price can be changed in every period only with probability  $1 - \theta$ , independently of the time elapsed since the last adjustment. Following Galí (2008) we get a forward-looking Phillips curve that in log-linearised form reads as:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \eta \widehat{MC}_t + \xi_{mc,t} \quad (36)$$

where  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$  which is strictly decreasing in the price stickiness parameter  $\theta$ , in the demand elasticity  $\psi$  and in the measure of decreasing returns on labor  $1 - \gamma$ ; and  $\xi_{mc,t}$  denotes

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<sup>17</sup>We assume that the productivity shock evolves exogenously as follows:

$$\log A_t = \phi_a \log A_{t-1} + u_{a,t}$$

where  $\phi_a$  is the persistence of the productivity innovation, and the error term is i.i.d., with mean zero and variance  $\sigma_a$ .

a cost-push shock.<sup>18</sup> Relationship (36) states that inflation depends positively both on expected inflation and on the real marginal costs which according to (35) and ignoring the constant terms read as:

$$\widehat{MC}_t = \hat{w}_t - A_t \quad (37)$$

where  $\hat{w}_t$  is the real wage and  $A_t = \hat{Y}_t - \hat{N}_t$  is the technology (productivity) variable.

### 3.4 The Fiscal Rule

The government's budget constraint expressed in real terms is

$$b_t = \frac{R_{t-1}^B b_{t-1}}{\pi_t} + (g_t - tax_t) + \xi_{b,t} \quad (38)$$

where  $g_t$  is real net government spending (i.e., net of lump-sum taxes),  $b_t$  is the real value of one-period government liabilities issued at the end of period  $t-1$  and with maturity in  $t$ ,  $\frac{R_{t-1}^B b_{t-1}}{\pi_t}$  denotes real debt service on existing government debt,  $tax_t \equiv \tau_h q_t H_t + \tau_y Y_t$  are total tax revenues from (i) real property taxes,  $\tau_h q_t H_t$ , with  $q_t H_t$  denoting the real value of existing housing stock held by borrower households where  $\tau_h$  is the tax rate; and (ii) from savers  $\tau_y Y_t$ . And the term  $\xi_{b,t}$  is a sovereign shock.<sup>19</sup>

The log-linearization of (38) in per-capita terms reads as:

$$\frac{b}{Y} \hat{b}_t = \frac{b}{Y} R^B \left( \hat{R}_{t-1}^B + \hat{b}_{t-1} - \hat{\pi}_t \right) + \frac{g}{Y} \hat{g}_t - \frac{tax}{Y} \hat{tax}_t \quad (39)$$

where

$$\frac{tax}{Y} \hat{tax}_t = \tau_h \frac{qH}{Y} \left( \hat{q}_t + \hat{H}_t \right) + \tau_y \hat{Y}_t + \xi_{\tau_h,t} \quad (40)$$

We also assume a feedback rule on government spending:

$$\frac{g}{Y} \hat{g}_t = -f_y \hat{Y}_t + f_T \frac{tax}{Y} \hat{tax}_t \quad (41)$$

where  $f_T$  is a government spending feedback parameter<sup>20</sup> from tax revenues,  $f_y$  is the fiscal policy parameters on output. We are considering some feedback rules for fiscal policy which apply to

<sup>18</sup>We assume that the mark-up shock evolves exogenously as follows:

$$\log \xi_{mc,t} = \phi_{\xi_{mc}} \log \xi_{mc,t-1} + u_{\xi_{mc},t}$$

where  $\phi_{\xi_{mc}}$  is the persistence of the shock, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_{mc}}$ .

<sup>19</sup>We assume that the government spending shock evolves exogenously as follows:

$$\log \xi_{b,t} = \phi_{\xi_b} \log \xi_{b,t-1} + u_{\xi_b,t}$$

where  $\phi_{\xi_b}$  is the persistence of the shock, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_b}$ .

<sup>20</sup>The parameters  $f_T$  and  $f_y$  are set exogenously in the impulse response analysis. We then choose optimally these parameter values when performing the welfare analysis.

government spending over the cycle  $g_t = F(f_T, f_y)$ , which act on tax receipts both from housing sales and consumption.

### 3.5 Monetary Policy Rule

The model is closed by the Central Bank's reaction function. The Central bank is assumed to set the nominal interest rate according to a simple linear interest rate rule:

$$\hat{R}_t^M = \rho \hat{R}_{t-1}^M + (1 - \rho) (\alpha_\pi \hat{\pi}_t + \alpha_y \hat{Y}_t) + \xi_{m,t} \quad (42)$$

where  $R^M$  is the (net) policy rate and  $\rho$  captures the degree of interest rate smoothing, while  $\alpha_\pi$  and  $\alpha_y$  are the central bank's reaction coefficients with respect to expected consumer price index (CPI) and the output gap while  $\xi_{m,t}$  denotes the monetary policy shock.<sup>21</sup>

### 3.6 Macroprudential Rule

Among the three major macro-prudential policy tools (loan to value ratio, bank leverage ratio, tax tools to improve bank's net capital) we consider a (countercyclical) loan to value ratio (LTV) policy of the form:

$$\hat{k}_t = -f_k \hat{Y}_t + \xi_{k,t} \quad (43)$$

where  $f_k$  is the countercyclical macroprudential policy parameter that controls for credit and inflation volatility and can mitigate the negative impact of the real estate market shocks in the real economy. As one of the main tools for macro-prudential policy it not only has direct impacts on the credit growth and economic fluctuations, but also works as a very easy-to-understand tool as it clearly and accurately expresses the attitude of regulators towards credit risk.

### 3.7 Welfare Analysis

The aggregate welfare function depends on households' preferences over consumption, housing, labor, bonds<sup>22</sup> and public spending; the argument  $G_t$  represents government spending and is determined by the government in each period so that the representative consumer takes it as exogenously given. Thus social welfare is given by:<sup>23</sup>

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<sup>21</sup>The monetary policy shock evolves as follows:

$$\log \xi_{m,t} = \phi_{\xi_m} \log \xi_{m,t-1} + u_{\xi_m,t}$$

where  $\phi_{\xi_m}$  is the persistence of the monetary policy innovation, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_m}$ .

<sup>22</sup>We assume that households' utility function is separable in consumption, labor, housing and bond holdings, which implies that all the cross-derivatives are zero.

<sup>23</sup>Here we introduce a welfare function which is fairly standard in the literature (Woodford, 2003). The main difference in our framework stems from the introduction of government consumption.

$$U_t = \frac{c}{Y} (\log c_t + \chi_B \log b_t) + \frac{c^b}{Y} \left( \log c_t^b + \chi_H \log H_t - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) + \frac{g}{Y} (\log g_t) \quad (44)$$

where we attach weight coefficients equal to the steady state value of consumption over output,  $\frac{c}{Y}$ , and government spending over output,  $\frac{C^b}{Y}$ .

As shown in the Appendix the above welfare function can be also expressed in terms of aggregate welfare losses using the following purely quadratic loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + \mathcal{O}(\|a\|^3) \quad (45)$$

with  $L_t = \varphi_Y \sigma_Y^2 + \varphi_c \sigma_c^2 + \varphi_{c^b} \sigma_{c^b}^2 + \varphi_g \sigma_g^2 + \varphi_b \sigma_b^2 + \varphi_H \sigma_H^2 + \varphi_\pi \sigma_\pi^2$

where the hatted variable is the log-deviation from steady state,  $\mathcal{O}(\|a\|^3)$  collects all the terms of third order or higher, and “*t.i.p.*” denotes terms independent of policy. The weight coefficients are given by  $\varphi_Y \equiv \frac{\varsigma+\gamma}{1-\gamma}$ ,  $\varphi_c \equiv \frac{c}{Y}$ ,  $\varphi_{c^b} \equiv \frac{c^b}{Y}$ ,  $\varphi_b \equiv \frac{c}{Y} \chi_B$ ,  $\varphi_H \equiv \frac{c^b}{Y} \chi_H$ ,  $\varphi_g \equiv \frac{g}{Y}$  and  $\varphi_\pi \equiv \frac{\psi}{\eta}$  with  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$  and the welfare loss is given by a linear combination of the variances of output, inflation, consumption levels, housing and government bond holdings.

## 4 Model Solution

### 4.1 Steady States

Table 2 reports the parameter values used to calibrate this model and the steady-state values. The saver households’ intertemporal discount factor is set at the conventional (quarterly) value of  $\beta = 0.99$ . This implies a steady state real interest rate on (time) deposits of 4% in annual terms, as  $(1/\beta)^4 = 1.04$ . Lawrance (1991) estimates intertemporal discount factors for poor households between 0.95 and 0.98 so we set the borrower intertemporal discount factor at  $\check{\beta} = 0.96$ . Our calibration implies that borrowing constrained households using housing as collateral derive a higher per unit utility from housing services, so we set the housing parameter in the utility function  $\chi_H = 0.1$ . This value implies a steady-state ratio of total housing wealth to annualized GDP of 1.4. The corresponding ratio in U.S. data has ranged between 1.2 and 2.3 over the period 1952 to 2008, hence our steady state ratio is approximately an average of these values.<sup>24</sup>

The elasticity of substitution between varieties of nondurables,  $\psi$ , is set to 6 which yields a steady-state mark-up over the nominal costs of 20% in the real sector. We set the stickiness parameter for consumer prices,  $\theta$ , equal to the standard value 0.75 which implies a mean duration of price contracts of one year. We also assume that house prices are flexible.

We calibrate the loan-to-value ratio (LTV),  $\kappa$ , to 0.7 which is a value close to the euro area average. In fact, existing empirical studies show the presence of a substantial degree of

<sup>24</sup>Data in the National Income and Product Accounts (NIPA) for U.S. are provided by the Bureau of Economic Analysis.

heterogeneity within the euro area in terms of mortgage markets flexibility, with some countries as the Netherlands being close to the US where the value of the LTV ratio exceeds 0.9 for the period 1973-2006, and others, like for example Germany, France and Italy, displaying a much smaller degree of flexibility (IMF, 2008). The proposed calibration of  $\kappa$  thus provides an approximated average across European countries (Darracq-Pariès et al., 2008).

The inverse of the elasticity of labor supply,  $\varsigma$ , is set equal to 1; and we set the parameter associated with the elasticity of output with respect to labor at 0.3 implying that  $\gamma$  is 0.7. The policy parameters are chosen as follows: we set the interest rate smoothing parameter,  $\rho$ , to 0.85, the size of the response to inflation,  $\alpha_\pi$ , to 1.5, the response to output,  $\alpha_y$ , to 0.5 (see Darracq-Pariès and Notarpietro, 2008).

The steady-state<sup>25</sup> values of the endogenous variables as well as the value of the complex parameters are reported in Table 2. Starting with the interest rates, note that the steady state is computed at zero inflation so that we can interpret all the interest rates as real interest rates. Calibrations give a policy rate,  $R^M$ , at 1.01%, a saving deposit rate,  $R^D$ , at 1.06% and a loan rate,  $R^L$ , at 3.19% and a bond rate,<sup>26</sup>  $R^B$ , at 0.9% per annum. For borrowing constrained households the consumption to output ratio,  $C^b/Y$ , is 0.64 (against a value for  $C/Y$  of 0.20 for savers); also their level of housing wealth,  $qH^b/Y$ , is 1.4 times total output, while their level of indebtedness over consumption,  $l/Y$ , is 0.98. Finally, we set the steady state expected value of repayment of the leveraged households at 0.8 implying a value of  $\lambda$ , that is the default rate, of 1.25. The implied steady state value of housing tax is 1.08%.

## 4.2 Solution

For the impulse response analysis and simulation exercise we consider the real and financial shocks described in Table 3, which reports the volatility and persistence parameters chosen for the calibration and simulation exercise. These are standard parameters in the literature. The model is then solved using the solution methods of King and Watson (1998) to derive the impulse responses of the endogenous variables to different shocks, to obtain asymptotic variance and covariances of the variables and to simulate the data. This system of linear difference equations can be expressed as a singular dynamic system of the following form:

$$\check{\mathbf{A}}E_t\mathbf{y}_{t+1} = \check{\mathbf{B}}\mathbf{y}_t + \check{\mathbf{C}}\boldsymbol{\varepsilon}_t \quad \forall t \geq 0, \quad (46)$$

where  $\mathbf{y}_t$  is the vector of endogenous variables comprising both predetermined and non-predetermined variables including policy rules for the nominal interest rate,  $\boldsymbol{\varepsilon}_t$  is a vector of exogenous forcing variables, and  $\check{\mathbf{A}}$ ,  $\check{\mathbf{B}}$  and  $\check{\mathbf{C}}$  are matrices of fixed, time-invariant, coefficients.

<sup>25</sup>For details on the derivation of the steady-states see the Technical Appendix, available on request.

<sup>26</sup>We assume that government bonds are used by saver households in managing their liquidity. This implies that there is a spread between the consumption capital asset pricing model rate (i.e. the Euler for consumption) and the government bond rate, since government bonds command a liquidity premium.

$E_t$  is the expectations operator conditional on information available at time  $t$ . King and Watson (1998) demonstrate that if a solution exists and is unique then we may write any such solution in state-space form as follows,

$$\begin{aligned} \mathbf{y}_t &= \mathbf{\Pi} \mathbf{s}_t \\ \mathbf{s}_t &= \mathbf{M} \mathbf{s}_{t-1} + \mathbf{G} \mathbf{e}_t, \end{aligned} \tag{47}$$

where the  $\mathbf{s}_t$  matrix includes the state variables of the model (predetermined variables along with exogenous variables),  $\mathbf{e}_t$  is a vector of shocks to the state variables and  $\mathbf{\Pi}$ ,  $\mathbf{M}$  and  $\mathbf{G}$  are coefficient matrices. There are six shocks in  $\mathbf{G}$  and the variance-covariance as well as the autocorrelation matrices associated with these shocks are described in Table 3. The impulse responses of this system are given by:

$$\begin{aligned} E_t \mathbf{y}_{t+k} - E_{t-1} \mathbf{y}_{t+k} &= \mathbf{\Pi} (E_t \mathbf{s}_{t+k} - E_{t-1} \mathbf{s}_{t+k}) \\ &= \mathbf{M}^k \mathbf{G} \mathbf{e}_t. \end{aligned} \tag{48}$$

And the variance of the states,  $\mathbf{V}_{ss}$ , is given by:

$$vec(\mathbf{V}_{ss}) = (\mathbf{I} - \mathbf{M} \otimes \mathbf{M})^{-1} vec(\mathbf{G} \mathbf{V}_{ee} \mathbf{G}^T). \tag{49}$$

For the analysis of the optimal rule in the next section we minimize the variance of the arguments in the representative household's loss function,  $L$ :

$$L = \varphi_Y \sigma_Y^2 + \varphi_C \sigma_C^2 + \varphi_{C^b} \sigma_{C^b}^2 + \varphi_G \sigma_G^2 + \varphi_B \sigma_B^2 + \varphi_H \sigma_H^2 + \varphi_\pi \sigma_\pi^2 \tag{50}$$

In the appendix, this expression is shown to be an approximation of welfare loss and it is minimized subject to policy choice on monetary and/or fiscal policy such that  $[f_y, f_T, \tau_h, f_k] = \arg \min L$ .

## 5 Dynamic Model Results

In this section we report the results of specific exercises designed to understand this model's properties.<sup>27</sup> First, we examine the moments of the simulated model in Table 4. Then in Figures 3 to 8, we show the impulse responses of the benchmark calibration model to a set of forcing variables used to drive this model: a shock in the loan to value ratio offered by commercial banks, a goods productivity shock, a canonical monetary policy shock to the interest rate rule, a cost-push shock, an unanticipated shock to the house price and a government spending shock. The impulse response are drawn under a Taylor rule and under a coordinated rule. Finally, we undertake a welfare analysis of the model under restrictive and lax regimes.

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<sup>27</sup>The list of log-linearized model equations is reported in the Appendix.

## 5.1 Simulated model moments

The model can be solved for its moments to understand the basic relationships listed in the Appendix. Table 4 gives the relative standard deviation of key endogenous variables and their correlations with output for the benchmark simulation corresponding to the calibrated parameters in Tables 2-3. Both categories of consumption are volatile as well as aggregate consumption. Recall there is no investment *per se* in this model but trade in durable consumption of the housing stock drives up the standard deviation of consumption somewhat. Lending is more volatile than aggregate consumption type and is strongly procyclical. But the EFP is both volatile and procyclical, reflecting the dominance of demand shocks in the loans market. House prices are procyclical in this model and have a correlation of 0.58 with output.

## 5.2 Impulse responses

Figures 3 to 8 plot the responses of some key variables in this model to the forcing variables in each of three cases. For the model under a Taylor rule, which is indicated with dotted lines, and for the coordinated rule indicated as a straight line. Figure 3 shows a 1% shock to the loan to value ratio, a so-called collateral shock. The higher is the loan to value ratio the higher is the marginal amount of new lending that can be accorded for a given value of the collateral (housing). The quantity of loans to borrower households then jumps up as housing demand rises driving up the spread between the lending rate and the policy rate. Borrowers' consumption increases as a result of increased leverage and drives a temporary boom. But in this calibration the lending boom is short lived and in later periods, borrowers are paying back their loans and gradually reducing the level of loans, which depresses output. The savers in the benchmark scenario reduce current consumption following the increase in the saving deposit rate. Note that under the coordinated rule the quantity of lending is less and the lending rate considerably higher and house price actually fall as the higher LTV means that there is less incentive to use houses as collateral.

Figure 4 plots the response of this economy to a 1% shock in goods productivity. Wages and output rise and borrowers' consumption jumps up and remains persistent as loans are offered against a falling EFP following an increase in the policy rate. This boom in borrowers' consumption is financed by a reduction in savers' consumption, as deposit rates rise to attract funds. Figure 5 plots the responses to a 1% shock in the policy rate. This shock leads to a temporary recession in saver households who reduce consumption in response to higher deposit rates. Borrower, or leveraged, households are insulated temporarily as they benefit from the reduction in the cost of external finance, but in the following periods there is a longer recession for these borrowers as they repay back their loans. In response to a 1% mark-up shock rising inflation (Figure 6) the real house price drops driving down borrowers' consumption, lending and the EFP. Figure 7 shows that an unanticipated 1% shock to house prices brings about a delayed boom as deposit rates fall and lending stimulates consumption. Note that initially the EFP reduces as the increase in the value of households' collateralized net worth relative to the stock of outstanding loans drives down the loan rate since the risk premium asked by lenders decreases. The reduction in the interest rate spread

is weak and short-termed and then it starts increasing as demand for loans outstrips supply. In fact, the increase in house prices relaxes the credit constraint of the leveraged households, allowing them to borrow and consume more.

Finally, when government spending increases, see Figure 8, it acts, as we would expect, like a demand shock raising output and inflation temporarily. The increase in the policy and deposit rates reduces savers' consumption whereas the increase in borrowers' consumption drives up the demand for additional loans and so the EFP rises. Figure 9 shows the middle segment, as an illustration, from a simulation of 10,000 data points, discarding the first 500 observations, of the benchmark model. The simulated data are HP filtered ( $\lambda = 1600$ ). Borrower's consumption covaries positively with house prices and it is negatively correlated with saver's consumption. This replicates the empirical evidence from the BHPS data for UK where a much stronger house-price effect was found for credit constrained households as well as a negative correlation between consumption of borrowers and savers.

### 5.3 Fiscal, Monetary and Macro-prudential Policy

Table 5 shows the optimal policy fiscal and coordinated policy parameters. The model with an optimal fiscal rule has choices of the feedback from taxation receipts, output and money growth, set by parametric choice of minimization of the loss function. But note that fiscal policy acts on the tax and spends from the receipts of house purchases, and these purchases are the counterpart of bank lending in this model - recall that loans are used exclusively to finance borrower household consumption. Whereas a coordinated policy, by setting a countercyclical loan to value ratio, acts on the borrower household first by impacting on loans and the loan rate and so when the policy maker also reacts to the financial condition in the market, she is acting on the consumption plans of borrower whose optimal plans are directly affected by interest rates. Table 6 shows that under a coordinated policy rule the correlation of the loan rate,  $R^L$ , and of the loan to value turns acyclical reducing the volatility of borrower household consumption. Using the welfare criterion, we can analyze, as an illustrative calculation, the implications of varying the relative standard deviation of the house price shock with respect to the collateral shock. Figure 10 shows that the Taylor rule delivers the highest loss. When some optimal fiscal response is introduced within this economy the loss are much ameliorated. Finally, a coordination of fiscal and macroprudential policies reduces the overall volatility in the economy. In this sense fiscal policy seem to be trying to offset more restrictive behavior of banks when their external finance premium becomes more sensitive to the state of the economy by injecting more demand into the economy and acting to stabilize changes in house prices. We do not want to introduce a too normative conclusion here but this would seem to imply that the correct parametric response of stabilization policy is not independent of the lending behavior of banks.

## 5.4 Evaluating Policy Action under Uncertainty

In the current set-up, the Central Bank sets the policy rate to pursue the inflation target and it will be the Financial Policy Committee (FPC) that will have additional MPI at its disposal to pursue financial stability. When it comes to MPIs, to the extent that we cannot be sure about the impact of an instrument, Brainard uncertainty introduces a trade-off between the achievement of the target and the minimisation of uncertainty induced by the use of an instrument. There are two problems in the case of MPIs: (i) there is likely to be considerably more uncertainty with a set of untried instruments (ii) since a countercyclical macroprudential policy is linked to other policies that moderate cyclical fluctuations – above all monetary policy, which also affects macroprudential variables as asset prices and credit - macroprudential policy is likely to influence the transmission mechanism of monetary policy. Ideally we want to think about which instruments may be used and how they might be used together in a manner that does not induce greater uncertainty into the operation of monetary policy. Figure 11 shows how the use of MPIs may improve the trade-off in uncertainty-space available to policymakers, so that rather than the line ABC the policymaker will be able to choose along AB'C' and would prefer a point fairly near to C', where inflation is at target and an undershoot, such as B, is no longer the preferred point on the trade-off. To the extent that changing the constraints faced by financial intermediaries will alter the financial conditions, there may not only be an impact on the appropriate stance of monetary and fiscal policy but also an impact on the appropriate MPIs conditioned on the monetary policy stance. Consider a world in which the monetary policymaker wishes to smooth the response of consumption to a large negative shock to aggregate demand and reduces interest rates faced by collateral-constrained consumers. Simultaneously, financial stability may be considered to be threatened and various MPIs may be tightened, which would act against the interest rate changes made by the monetary policymaker and may need further or extended lower rates of interest rates. If, on the other hand, sufficient precautionary moves had been made by the FPC (and possibly the Fiscal Authorities) in advance there may be no immediate conflict (Chadha, 2014).

We calculate the welfare from a particular policy simulation of a model,  $M$ , with fixed structural parameters and shocks:

$$E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) | M \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t + \dots \quad (51)$$

So that we can evaluate the expected losses as some function of a policy function:

$$\Gamma(\text{policy}) | M = \Gamma(f_y, f_T, \tau_h, f_k) \quad (52)$$

where we can hold any given parameter constant. We can also seek to evaluate the optimal rule by seeking a solution to the following minimisation problem:

$$\Gamma(f_y, f_T, \tau_h, f_k) = \arg \min L(f_y, f_T, \tau_h, f_k) \mid M. \quad (53)$$

What we do not deal with is the possibility that the parameters within  $M$  are uncertain and so we may wish to assess the likely loss from uncertainty in the multiplicative parameter e.g. the standard model we use takes the following form:

$$y = \gamma_1 + \gamma_2 r_t + \varepsilon_t \quad \varepsilon_t \sim (0, \sigma_\varepsilon^2) \quad (54)$$

But we want to consider:

$$y = \gamma_1 + \hat{\gamma}_2 r_t + \varepsilon_t \quad \varepsilon_t \sim (0, \sigma_\varepsilon^2) \quad \text{and} \quad \hat{\gamma}_2 \sim (\mu_\gamma, \sigma_\gamma^2) \quad (55)$$

Now clearly there are a number of possible uncertain parameters in our model. But if we look carefully at the endogenous pricing of the loan rate, it seems that  $\lambda$  the loan default rate parameter is key. As it determines the slope of the supply curve of bank loans and we can imagine that it might be flat (good times) or steep (bad times). In order to characterise the impact of the various rules under parameter uncertainty we follow the following steps:

1. We calculate the welfare loss for the benchmark model using a value of lambda in normal times i.e.  $E_0 L \mid M \cap \lambda^B = 1.05$  which corresponds to a 95% probability of getting paid back.
2. Now choose some range for lambda and some supporting distribution e.g.  $\lambda \in [1, \dots, 2]$  and suppose the median is 1.05 i.e. 95% probability of getting paid back and we have a diffuse prior with equal chances from 100% to 50% i.e.  $1/0.5=2$ .
3. We can calculate the losses under the Benchmark for each possible  $\lambda$  i.e.  $\lambda_{i=1}^n [1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0]$ .
4. We can then assess the mean absolute error across this lambda uncertainty relative to the certainty equivalent loss,  $E_0 L \mid M \cap \lambda^B$ , where we know that  $\lambda = 1.05$ :  $\frac{|E_0 L \mid M \cap \lambda^i - E_0 L \mid M \cap \lambda^B|}{n}$ .
5. We can then assess how a co-ordinated or optimal rule for fiscal policy fares relative to a benchmark in the presence of parameter uncertainty. In Figure 12 we report the mean absolute error across this lambda uncertainty relative to the certainty equivalent loss and we show that a coordinated solution can minimise the overall losses within the economy.

## 6 Conclusions

In this paper we model two types of households, ones that are savers and ones that face both a liquidity and a borrowing constraint. The second type of households take on debt to finance their consumption. In this model banks have a specific role in pricing the risk of these leveraged

households by setting the margin for lending rates as a spread over the deposit rates. We find that compared to a standard representative agent model, consumption displays considerably more volatility in this set-up, as access to credit pushes consumption up and the requirement for debt repayments pulls consumption back again with countercyclical lending spreads. A complementary cycle also emerges for savers. So a deeper and more persistent business cycle emerges endogenously from this model.

Households' welfare is shown not necessarily to be maximized under a standard interest rate rule. In fact, stabilizing inflation and output with a jointly determined macroprudential and fiscal policy responses seems preferable. We analyze the aggregate welfare of households when some form of macroprudential policy also operates, which limits the lending, capital returns and perceived default rate of loans, and these produce lower losses for the representative household. Therefore, some augmentation of policy to include an additional role for macroprudential policy may improve welfare even in the case of uncertainty over some of the parameters governing banking decision, such as the loan default rate. Two tentative normative points emerge from this analysis. First, there may be more scope for countercyclical fiscal and macroprudential policy to stabilize the economy than we thought when models were unable to speak about intermediation. Secondly, when banks do not price risk in a substantive manner the policy frontier deteriorates and this would imply there is an ongoing need to calibrate macroprudential policies, particularly in a world where lending is driven by home ownership.

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# Appendix

## A List of Steady-State Relationships

Assuming that the steady state gross inflation rate  $\pi \equiv \frac{P_t}{P_{t-1}}$  is equal to 1, that is inflation is zero in the steady state and that housing supply which is fixed and equal to 1,  $\bar{H} = 1$ , then the steady state equilibrium will be described by:

$$R^D = \frac{1}{\beta} \quad (56)$$

$$\frac{c^b}{Y} = 1 - \frac{c}{Y} - \frac{g}{Y} \quad (57)$$

$$R^B = \frac{1}{\beta} \left[ 1 - \chi_B \frac{c}{b} \right] \quad (58)$$

$$R_M = (X_{\mu_D})^{-1} R^D \quad (59)$$

$$R^L = X_{\mu_L} \frac{1}{1 - \Phi} R^M \quad (60)$$

$$\tau_h = \left[ \frac{g}{Y} - \frac{b}{Y} (1 - R^B) - \tau_y \right] \left( \frac{qH}{Y} \right)^{-1} \quad (61)$$

$$\frac{c^b}{qH} = \frac{\frac{c^b}{Y}}{\frac{qH}{Y}} \quad (62)$$

$$\frac{l}{c^b} = \frac{\kappa qH}{c^b} \quad (63)$$

$$l = (1 - rr)d \quad (64)$$

$$\frac{wN}{Y} = \frac{(1 - \gamma)}{X_{\psi,t}} \quad (65)$$

where  $X_{\mu_L} \equiv \frac{\mu_L}{(\mu_L - 1)}$  is the loan interest rate markup and  $1/X_{\mu_D} = \frac{1 + \mu_D}{\mu_D}$  denotes the policy rate markup.

## B The Log-Linearized Model

The model can be reduced to the following log-linearized system in which all the hatted variables denote percent changes from the steady state, and those without subscript denote steady-state values:<sup>28</sup>

<sup>28</sup>For details on the derivation of the log-linearised equations and steady-states see Technical Appendix, available on request.

## B.1 Saver Household

- Consumption Demand:

$$\hat{c}_t = (\hat{c}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t^D) \quad (66)$$

- Bonds Demand:

$$\hat{b}_t = \chi_B^{-1} \left( \frac{b}{c} \right) \left[ \hat{c}_t + \beta R^B \left( \hat{R}_t^B - \hat{c}_{t+1} - \hat{\pi}_{t+1} \right) \right] \quad (67)$$

## B.2 Borrower Household

- Budget Constraint:

$$\begin{aligned} & \check{c}_t + \frac{qH}{c^b} (1 + \tau_h) \left( \hat{q}_t + \hat{H}_t \right) - \frac{qH}{c^b} \left( \hat{q}_{t-1} + \hat{H}_{t-1} - \hat{\pi}_t \right) + \\ & + \frac{R^L l}{c^b} \left( \hat{R}_{t-1}^L + \hat{l}_{t-1} - \hat{\pi}_t \right) - \frac{l}{c^b} \hat{l}_t - \frac{wN}{c^b} \left( \hat{w}_t + \hat{N}_t \right) = 0 \end{aligned} \quad (68)$$

- Housing Demand:

$$\hat{H}_t = \frac{1}{\chi_H} \frac{qH}{c^b} \left\{ \begin{aligned} & (1 + \tau_h) \check{c}_t - \check{\beta} (\check{c}_{t+1} + \pi_{t+1}) \\ & - \left( 1 + \tau_h - \check{\beta} \right) \hat{q}_t + \kappa \frac{(1 - \check{\beta} R^L)}{R^L} \left( \hat{\nu}_t + \hat{\kappa}_t + \hat{q}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t^L \right) \end{aligned} \right\} \quad (69)$$

where  $vc^b = \frac{(1 - \check{\beta} R^L)}{R^L}$ .

- Real House Price:

Given the market clearing condition  $\hat{H}_t = 0$  under a fixed supply of housing  $\hat{H}$  and using the residential goods demand relationship, the log-linearized form of (69) reads:

$$\hat{q}_t = \left( 1 + \tau_h - \check{\beta} \right)^{-1} \left\{ \begin{aligned} & (1 + \tau_h) \check{c}_t - \check{\beta} (\check{c}_{t+1} + \pi_{t+1}) \\ & + \kappa \frac{(1 - \check{\beta} R^L)}{R^L} \left( \hat{\nu}_t + \hat{\kappa}_t + \hat{q}_{t+1} + \hat{\pi}_{t+1} - \hat{R}_t^L \right) \end{aligned} \right\} + \xi_{q,t} \quad (70)$$

where  $\xi_{q,t}$  denotes a non-fundamental shock to house prices.<sup>29</sup>

<sup>29</sup>We assume that the house price shock evolves exogenously as follows:

$$\log \xi_{q,t} = \phi_{\xi_q} \log \xi_{q,t-1} + u_{\xi_q,t}$$

where  $\phi_{\xi_q}$  is the persistence of the shock, and the error term is i.i.d., with mean zero and variance  $\sigma_{\xi_q}$ .

- Loan Demand from Leveraged Households:

$$\hat{l}_t = \hat{k}_t + \hat{q}_{t+1} + \hat{\pi}_{t+1} + \hat{H}_t - \hat{R}_t^L \quad (71)$$

- Labor Supply:

$$\hat{w}_t = \hat{c}_t^b + \varsigma \hat{N}_t \quad (72)$$

### B.3 Banking Sector

- Loan Rate:

$$\hat{R}_t^L = \hat{R}_t^M + \frac{\lambda}{1 - \delta} \left[ \hat{l}_t - \delta (\hat{k}_t + \hat{q}_t + \hat{H}_t) \right] \quad (73)$$

- Deposit Rate:

$$\hat{R}_t^D = \hat{R}_t^M \quad (74)$$

### B.4 Real Sector

- Phillips Curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \eta \widehat{MC}_t + \xi_{mc,t} \quad (75)$$

where  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$ .

- Output:

$$\hat{Y}_t = A_t + (1 - \gamma) \hat{N}_t \quad (76)$$

- Labor Demand:

$$\hat{w}_t = (\hat{Y}_t - \hat{N}_t) + \widehat{MC}_t \quad (77)$$

### B.5 Monetary Policy

$$\hat{R}_t^M = \rho \hat{R}_{t-1}^M + (1 - \rho) \left( \alpha_\pi \hat{\pi}_t + \alpha_y \hat{Y}_t \right) + \xi_{m,t} \quad (78)$$

The augmented Taylor rule specification:

$$\hat{R}_t^M = \rho \hat{R}_{t-1}^M + (1 - \rho) \left( \alpha_\pi \hat{\pi}_t + \alpha_y \hat{Y}_t \right) + \xi_{m,t} \quad (79)$$

## B.6 Fiscal Policy

- Government Budget Constraint:

$$\frac{b}{Y}\widehat{b}_t = \frac{b}{Y}R^B \left( \widehat{R}_{t-1}^B + \widehat{b}_{t-1} - \widehat{\pi}_t \right) + \frac{g}{Y}\widehat{g}_t - \frac{tax}{Y}\widehat{tax}_t + \xi_{b,t} \quad (80)$$

where taxation is:

$$\frac{tax}{Y}\widehat{tax}_t = \tau_h \frac{qH}{Y} \left( \widehat{q}_t + \widehat{H}_t \right) + \tau_y \widehat{Y}_t \quad (81)$$

The feedback rule on government spending is:

$$\frac{g}{Y}\widehat{g}_t = -f_y \widehat{Y}_t + f_T \frac{tax}{Y}\widehat{tax}_t \quad (82)$$

## B.7 Macroprudential Rule (LTV)

$$\widehat{k}_t = -f_k \widehat{Y}_t + \xi_{k,t} \quad (83)$$

## B.8 Market Clearing

$$\widehat{d}_t = \widehat{l}_t \quad (84)$$

$$\widehat{Y}_t = \frac{c}{Y}\widehat{c}_t + \frac{c^b}{Y}\widehat{c}_t^b + \frac{g}{Y}\widehat{g}_t \quad (85)$$

$$\widehat{H}_t = 0 \quad (86)$$

## C Welfare Analysis

The aggregate welfare function depends on households' preferences over consumption, housing, labor, bonds and public spending; the argument  $G_t$  represents government spending and is determined by the government in each period so that the representative consumer takes it as exogenously given. Thus social welfare is given by:

$$U_t = \frac{c}{Y} (\log c_t + \chi_B \log b_t) + \frac{c^b}{Y} \left( \log c_t^b + \chi_H \log H_t - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) + \frac{g}{Y} (\log g_t) \quad (87)$$

where we attach weight coefficients equal to the steady state value of consumption over output and government spending over output. All derivations are calculated at the steady state values  $C$ ,  $C^b$ ,  $N$ ,  $g$  and the variables signed with a “ $\sim$ ” denote second-order approximations in terms of log-deviations:

$$\tilde{X} = X_t - X \simeq X \left( \widehat{X}_t + \frac{1}{2}\widehat{X}_t^2 \right) \quad (88)$$

where  $\widehat{X}$  is the log-deviation from steady state for a generic variable  $X_t$ .

## C.1 Deriving the second order approximation

Since the utility is additively separable between consumption, labor, housing then we can consider the second-order approximations to each term in (87) separately.

- The second-order approximation to saver's consumption is given by:

$$\begin{aligned}
\log c_t &\approx U_c \tilde{c}_t + \frac{U_{cc}^2 \tilde{c}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{1}{c} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) - \frac{1}{c^2} \frac{c^2 \tilde{c}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \hat{c}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{89}$$

where  $\mathcal{O}(\|a\|^3)$  collects all the terms of third order or higher, in the bound  $\|a\|$  on the amplitude of the relevant shocks.

- The second-order approximation to bond holdings is given by:

$$\begin{aligned}
\chi_B \log b_t &= \chi_B U_b \tilde{b}_t + \chi_B U_{bb} \frac{\tilde{b}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&= \frac{\chi_B}{b} \left( \hat{b}_t + \frac{1}{2} \hat{b}_t^2 \right) - \frac{\chi_B}{b^2} \frac{b^2 \tilde{b}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&= \chi_B \hat{b}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{90}$$

- The second-order approximation to borrower's consumption is given by:

$$\begin{aligned}
\log c_t^b &\approx U_{c^b} \tilde{c}_t^b + \frac{U_{c^b c^b} (\tilde{c}_t^b)^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{1}{c^b} c^b \left( \hat{c}_t^b + \frac{1}{2} (\hat{c}_t^b)^2 \right) - \frac{1}{(c^b)^2} \frac{(c^b)^2 (\tilde{c}_t^b)^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \hat{c}_t^b + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{91}$$

- The second-order approximation to real estate holdings is given by:

$$\begin{aligned}
\chi_H \log H_t &\approx \chi_H U_H \tilde{H}_t + \frac{\chi_H U_{HH} \tilde{H}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \frac{\chi_H}{H} H \left( \hat{H}_t + \frac{1}{2} \hat{H}_t^2 \right) - \frac{\chi_H}{H^2} \frac{H^2 \tilde{H}_t^2}{2} + \mathcal{O}(\|a\|^3) \\
&\approx \chi_H \hat{H}_t + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{92}$$

- The second-order approximation to labor is given by:

$$\frac{N_t^{1+\varsigma}}{1+\varsigma} \approx U_N \tilde{N}_t + U_{NN} \frac{\tilde{N}_t^2}{2} + \mathcal{O}(\|a\|^3) \quad (93)$$

Given that  $U_{NN} = \varsigma N U_N$  with  $U_N = N^\varsigma$  we can rewrite the above relationship as follows:

$$\begin{aligned} \frac{N_t^{1+\varsigma}}{1+\varsigma} &\approx U_N N \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \mathcal{O}(\|a\|^3) \\ &\approx N^{\varsigma+1} \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \mathcal{O}(\|a\|^3) \end{aligned} \quad (94)$$

Finally using the labor market clearing when the economy remains in a neighborhood of an efficient steady state yields  $N^\varsigma c^b = (1-\gamma) \frac{Y}{N}$  from which we get  $N^{\varsigma+1} = \frac{(1-\gamma)Y}{c^b}$ . Substituting this into the above relationship gives:

$$\frac{N_t^{1+\varsigma}}{1+\varsigma} = \frac{(1-\gamma)Y}{C^b} \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \mathcal{O}(\|a\|^3) \quad (95)$$

- The second-order approximation to government spending is given by:

$$\begin{aligned} \log g_t &\approx U_g \tilde{g}_t + \frac{U_{gg} \tilde{g}_t^2}{2} + \mathcal{O}(\|a\|^3) \\ &\approx \frac{1}{g} \left( \hat{G}_t + \frac{1}{2} \hat{g}_t^2 \right) - \frac{1}{g^2} \frac{g^2 \hat{g}_t^2}{2} + \mathcal{O}(\|a\|^3) \\ &\approx \hat{g}_t + \mathcal{O}(\|a\|^3) \end{aligned} \quad (96)$$

## C.2 Simplifying the Welfare Function

We now add equations (89), (91), (92), (95) in order to get a second-order approximation to the welfare function (87):

$$U_t - U \simeq \frac{c}{Y} \left( \hat{c}_t + \chi_B \hat{b}_t \right) + \frac{c^b}{Y} \left( \hat{c}_t^b + \chi_H \hat{H}_t \right) - (1-\gamma) \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \frac{g}{Y} \hat{g}_t + \mathcal{O}(\|a\|^3) \quad (97)$$

(i) From the second order approximation of the housing market clearing condition we have  $H \left( \hat{H}_t + \frac{1}{2} \hat{H}_t^2 \right) = 0$  which implies that  $\hat{H}_t = -\frac{1}{2} \hat{H}_t^2$  so that (97) can be rewritten as follows:

$$U_t - U \simeq \frac{c}{Y} \left( \hat{c}_t + \chi_B \hat{b}_t \right) + \frac{c^b}{Y} \hat{c}_t^b - \frac{1}{2} \frac{c^b}{Y} \chi_H \hat{H}_t^2 - (1-\gamma) \left( \hat{N}_t + \frac{1}{2} (1+\varsigma) \hat{N}_t^2 \right) + \frac{g}{Y} \hat{g}_t + \mathcal{O}(\|a\|^3) \quad (98)$$

(ii) From the second-order approximation of the aggregate resource constraint:

$$\left(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) = \frac{c}{Y} \left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \frac{c^b}{Y} \left(\hat{c}_t^b + \frac{1}{2}(\hat{c}_t^b)^2\right) + \frac{g}{Y} \left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2\right) + \mathcal{O}(\|a\|^3) \quad (99)$$

which implies

$$\frac{c}{Y}\hat{c}_t + \frac{c^b}{Y}\hat{c}_t^b + \frac{g}{Y}\hat{g}_t = \left(\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2\right) - \frac{1}{2} \left(\frac{c}{Y}\hat{c}_t^2 + \frac{c^b}{Y}(\hat{c}_t^b)^2 + \frac{g}{Y}\hat{g}_t^2\right) + \mathcal{O}(\|a\|^3) \quad (100)$$

(iii) To eliminate  $\hat{N}_t$  which allows us to express the welfare function in terms of output, we use both the production function (33) and the aggregate intratemporal demand function (34) to rewrite  $\hat{N}_t$  in terms of output:

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\gamma}} \int_0^1 \left(\frac{P_{z,t}}{P_t}\right)^{-\frac{\psi}{1-\gamma}} \quad (101)$$

which in log-linear form can be rewritten as follows

$$(1-\gamma)\hat{N}_t = \hat{Y}_t - A_t + \widehat{dp}_t \quad (102)$$

where  $\widehat{dp}_t \equiv (1-\gamma) \log \left[ \int_0^1 \left(\frac{P_{z,t}}{P_t}\right)^{-\frac{\psi}{1-\gamma}} dz \right]$  is a measure of consumption goods price dispersion in the intermediate sector with  $z$  denoting the price of good variety  $z$ .

**Lemma 1** *In a neighborhood of a symmetric steady state and up to a second-order approximation  $\widehat{dp}$  is proportional to the cross-sectional variance of relative prices,  $\widehat{dp}_t \equiv \frac{1}{2} \frac{\psi}{\varrho} \text{var} \{P_{z,t}\} + \mathcal{O}(\|a\|^3)$  where  $\varrho \equiv \frac{1-\gamma}{1+\gamma(\psi-1)}$*

**Proof.** See Galí (2008), Chapter 4. ■

**Lemma 2**  $\sum_{t=0}^{\infty} \beta^t \text{var} \{P_{z,t}\} = \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$

**Proof.** See Woodford (2003), Chapter 6. ■

iv) In a stationary equilibrium money grows at a constant rate  $\Delta m = 0$  which implies zero inflation in steady state and the primary fiscal position is balanced  $PD_t = G_t - T_t = 0$ . Therefore, the second order approximation to the per capita government budget constraint reads as:

$$\frac{b}{Y} \left(\hat{b}_t + \frac{1}{2}\hat{b}_t^2\right) = \frac{bR^B}{Y} \left(\hat{b}_{t-1} + \frac{1}{2}\hat{b}_{t-1}^2\right) + \frac{bR^B}{Y} \left(\hat{R}_{t-1}^B + \frac{1}{2}(\hat{R}_{t-1}^B)^2\right) + \mathcal{O}(\|a\|^3) \quad (103)$$

that can be rewritten as follows:

$$\hat{b}_t = -\frac{1}{2}\hat{b}_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \quad (104)$$

where “*t.i.p.*” denotes terms independent of policy.

Substituting (100) and (102) and (104) we can rewrite the welfare function (98) as follows:

$$\begin{aligned}
U_t - U &\simeq \left( \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 \right) - \frac{1}{2} \left( \frac{c}{Y} \hat{c}_t^2 + \frac{c^b}{Y} (\hat{c}_t^b)^2 + \frac{g}{Y} \hat{g}_t^2 \right) - \frac{1}{2} \frac{c^b}{Y} \chi_H \hat{H}_t^2 + \\
&+ \frac{c}{Y} \chi_B \left( -\frac{1}{2} \hat{b}_t^2 \right) - \left[ \left( \hat{Y}_t - A_t + \widehat{dp}_t \right) + \frac{1}{2} \left( \frac{1+\varsigma}{1-\gamma} \right) \left( \hat{Y}_t - A_t + \widehat{dp}_t \right)^2 \right] + \mathcal{O}(\|a\|^3)
\end{aligned} \tag{105}$$

By simplifying and using Lemma 1 and 2 in order to rewrite the terms involving the price dispersion as a function of inflation and knowing that  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$  and collecting “t.i.p.” terms the above relationship can be rewritten as follows:

$$(U_t - U) \simeq \frac{1}{2} \left[ \left( 1 - \frac{1+\varsigma}{1-\gamma} \right) \hat{Y}_t^2 - \left( \frac{c}{Y} \hat{c}_t^2 + \frac{c^b}{Y} (\hat{c}_t^b)^2 + \frac{g}{Y} \hat{g}_t^2 \right) - \frac{c}{Y} \chi_B \hat{b}_t^2 - \frac{c^b}{Y} \chi_H \hat{H}_t^2 - \frac{\psi}{\eta} \hat{\pi}^2 \right] + t.i.p. + \mathcal{O}(\|a\|^3) \tag{106}$$

The welfare function (106) can be also expressed in terms of aggregate welfare losses using the following purely quadratic loss function:

$$E_0 \sum_{t=0}^{\infty} \beta^t (U_t - U) = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + \mathcal{O}(\|a\|^3) \tag{107}$$

$$\text{with } L_t = \varphi_Y \sigma_Y^2 + \varphi_c \sigma_c^2 + \varphi_{c^b} \sigma_{c^b}^2 + \varphi_g \sigma_g^2 + \varphi_b \sigma_b^2 + \varphi_H \sigma_H^2 + \varphi_\pi \sigma_\pi^2$$

where the weight coefficients are given by  $\varphi_Y \equiv \frac{\varsigma+\gamma}{1-\gamma}$ ,  $\varphi_c \equiv \frac{c}{Y}$ ,  $\varphi_{c^b} \equiv \frac{c^b}{Y}$ ,  $\varphi_b \equiv \frac{c}{Y} \chi_B$ ,  $\varphi_H \equiv \frac{c^b}{Y} \chi_H$ ,

$\varphi_g \equiv \frac{g}{Y}$  and  $\varphi_\pi \equiv \frac{\psi}{\eta}$  with  $\eta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\gamma}{1+\gamma(\psi-1)}$ .

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LEGEND. PARAMETERS AND COEFFICIENTS

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Parameter	Description
$\beta$	Intertemporal discount rate for savers
$\check{\beta}$	Intertemporal discount rate for borrowers
$\chi_B$	Financial assets weight for savers
$\chi_H$	Housing services weight for borrowers
$\varsigma$	Labour supply aversion
$\kappa$	Loan-to-Value
$\omega$	Fraction of savers
$\delta$	Fraction of seizable collateral
$\psi$	Elasticity of substitution between goods
$X_\psi$	Price markup
$\theta$	Index of price stickiness
$\gamma$	Decreasing returns to labor
$X_{\mu_D}$	Markup on deposit rate
$X_{\mu_L}$	Markup on loan rate
$rr$	Fractional reserve coefficient
$g/Y$	Government Spending to Output
$b/Y$	Debt to Output Ratio
$t/Y$	Taxation to output
$f_T$	Fiscal feedback parameter on housing tax
$f_y$	Fiscal feedback parameter on output
$f_k$	LTV feedback parameter
$f_m$	Fiscal feedback parameter on seignorage revenues
$\tau_h$	Housing tax
$\tau_y$	Lump-sum tax
$\rho$	Degree of interest rate smoothing
$\alpha_\pi$	Central Bank's reaction coefficient to expected inflation
$\alpha_y$	Central Bank's reaction coefficient to output gap
$\lambda$	Default rate
$1 - \Phi$	Loan repayment probability

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TABLE 1. CORRELATION BETWEEN CHANGE IN CONSUMPTION  
AND CHANGE IN HOUSE VALUE

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	Borrower	Saver	All
Nondurable Consumption:	0.0408	-0.0057	0.0200
Total Consumption:	0.0167	-0.0103	0.0042

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Source: BHPS 1997-2008.

\* Values refer respectively to the Mean for Saver, Borrowers and Mean Consumer (All Sample). Total Consumption is the sum of Durable and Non Durable Consumption.

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TABLE 2. CALIBRATED PARAMETERS

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Parameter	Description	Value
<i>Preference Parameters and Collateral</i>		
$\beta$	Intertemporal discount rate for savers	0.99
$\tilde{\beta}$	Intertemporal discount rate for borrowers	0.96
$\chi_B$	Financial assets weight for savers	0.112
$\chi_H$	Housing weight for borrowers	0.1
$\varsigma$	Labour supply aversion	1.01
$\kappa$	Loan-to-Value	0.7
$\delta$	Fraction of seizable collateral	0.6
<i>Sticky Prices</i>		
$\psi$	Elasticity of substitution between goods	6
$X_\psi$	Price markup	1.2
$\eta$	Elasticity of inflation to marginal cost	0.45
<i>Technology</i>		
$\gamma$	Production function parameter	0.7
<i>Loans, Deposits and Fractional Reserves</i>		
$X_{\mu_D}$	Markup down of deposit rate	0.95
$X_{\mu_L}$	Markup on loan rate	3
$rr$	Fractional reserve coefficient	0.005
$\lambda$	Default rate	1.25
$1 - \Phi$	Loan repayment probability	0.8
<i>Savers</i>		
$c/Y$	Consumption to output ratio	0.2
<i>Fiscal Policy</i>		
$g/Y$	Government spending to output	0.16
$tax/Y$	Taxation to output	0.10
$\tau_y$	Income tax	0.15
<i>Monetary Policy</i>		
$\rho$	Degree of interest rate smoothing	0.85
$\alpha_\pi$	Central Bank reaction to expected inflation	1.5
$\alpha_y$	Central Bank reaction to output gap	0.5

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TABLE 2 (CONTINUED) IMPLIED STEADY STATES

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	Description	Value
Banking Sector		
$R^D - 1$	Deposit rate	0.0101
$R^M - 1$	Policy rate	0.0106
$R^L - 1$	Loan rate	0.0319
Public Sector		
$\tau_h$	Housing tax	0.0108
$b/Y$	Public debt to output	0.56
$R^B$	Bond rate	0.009
Borrowers		
$c^b/Y$	Consumption to output ratio	0.64
$l/Y$	Loan to output	0.98
$qH/Y$	Housing wealth to output ratio	1.4
$WN/Y$	Wage to output	0.25

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TABLE 3. BASELINE MODEL SHOCKS\*

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	Description	Value
$\rho_\kappa$	Autocorrelation of the collateral shock	0.95
$\rho_{\xi_q}$	Autocorrelation of house price shock	0.20
$\rho_a$	Autocorrelation of productivity shock	0.70
$\rho_{\xi_m}$	Autocorrelation of monetary policy shock	0.10
$\rho_{\xi_g}$	Autocorrelation of fiscal policy shock	0.70
$\rho_{\xi_{mc}}$	Autocorrelation of the cost-push shock	0.92
$\sigma_\kappa$	Standard deviation of the loan to value shock	0.01
$\sigma_{\xi_q}$	Standard deviation of house price shock	0.01
$\sigma_a$	Standard deviation of the productivity shock	0.01
$\sigma_{\xi_m}$	Standard deviation of the monetary policy shock	0.01
$\sigma_{\xi_g}$	Standard deviation of the fiscal policy shock	0.01
$\sigma_{\xi_{mc}}$	Standard deviation of the cost-push shock	0.01

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\*Source: Chadha et al. (2009)

TABLE 4. MOMENTS		
	Rel stdev ( $Y$ )	Corr ( $Y$ )
Benchmark Policy/Regime		
$C^*$	1.20	0.90
$C$	5.35	-0.77
$C^b$	2.94	0.92
$l$	2.48	0.44
$R^L$	0.54	0.66
$EFP$	0.41	0.27
$q$	2.05	0.58
$\pi$	0.71	0.54

TABLE 5. OPTIMAL POLICY PARAMETERS				
	$f_y$	$f_T$	$\tau_h$	$f_k$
Taylor + Fiscal Policy	0.26	0.14	0.02	–
Coordinated	0.20	0.20	0.10	0.40

TABLE 6. RELATIVE STANDARD DEVIATION AND CORRELATION WITH OUTPUT				
	$R^L$		$\kappa$	
Taylor + Fiscal Policy	0.70	0.35	3.20	0.18
Co-ordinated	0.73	0.01	3.41	0.07

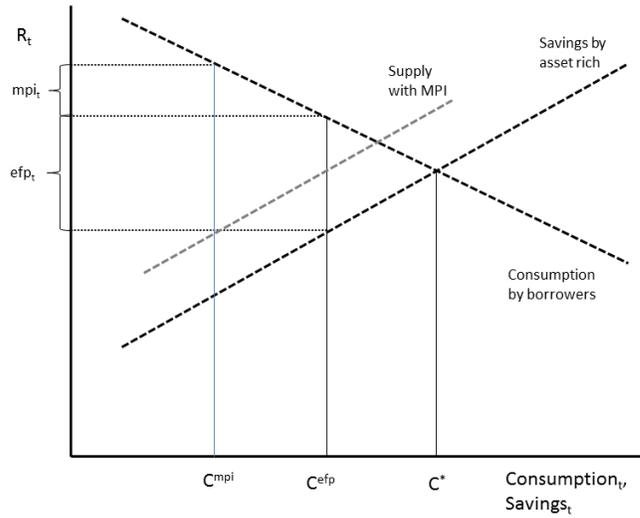


Figure 1: Negative Correlation between Consumption of Savers (Asset Rich) and Borrowers.

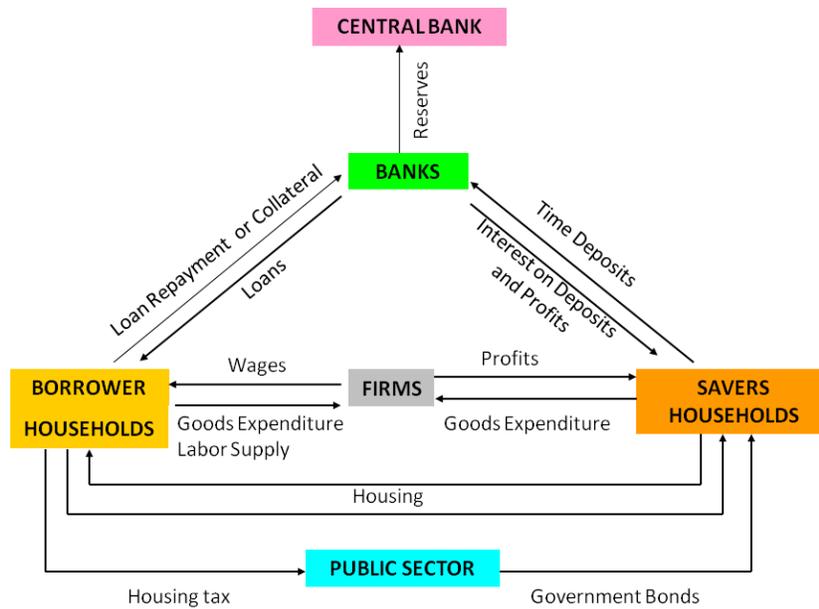


Figure 2: The Model.

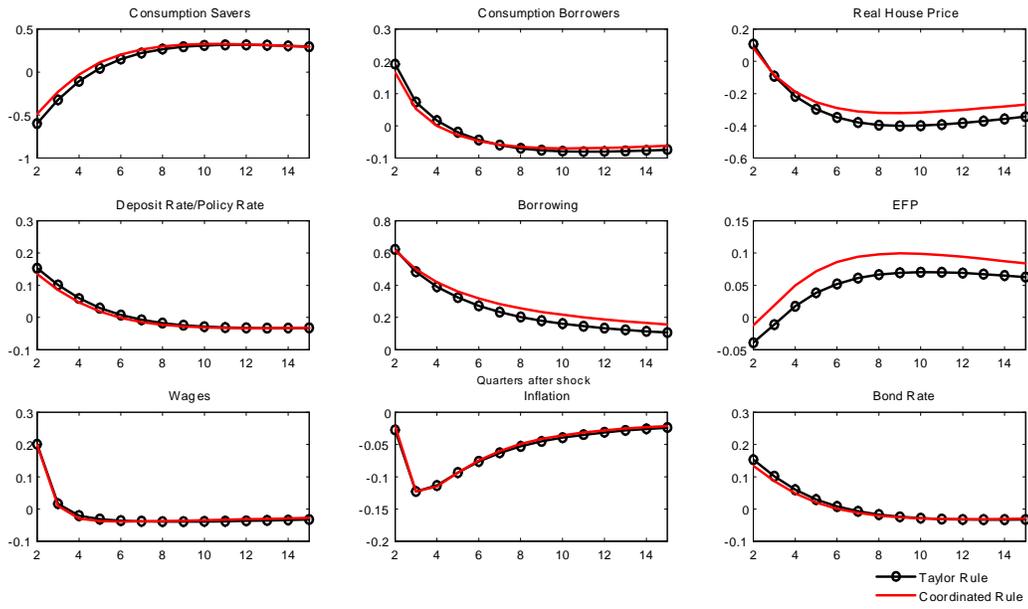


Figure 3: Loan-to-Value Shock.

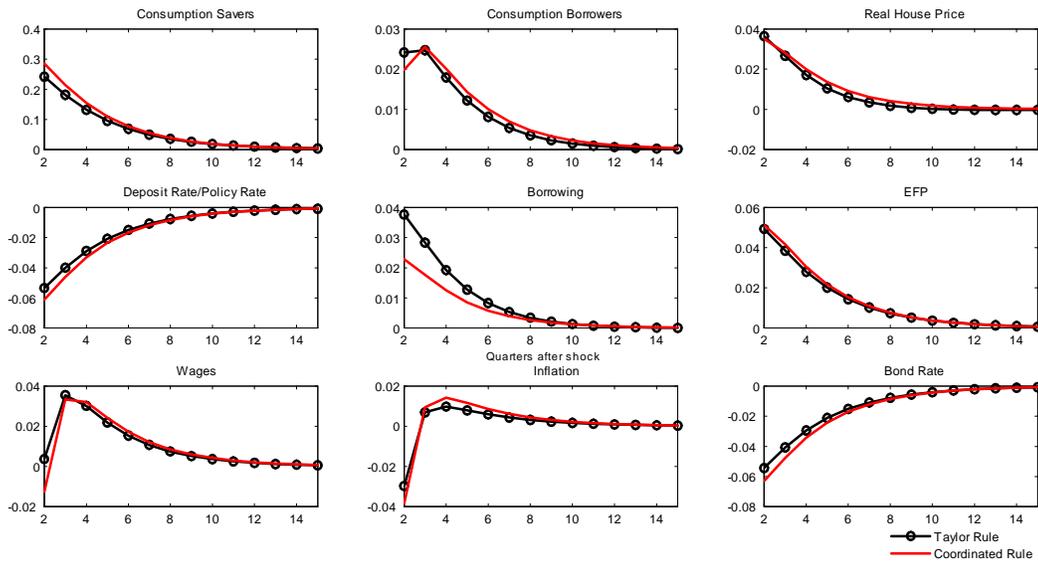


Figure 4: Productivity Shock.

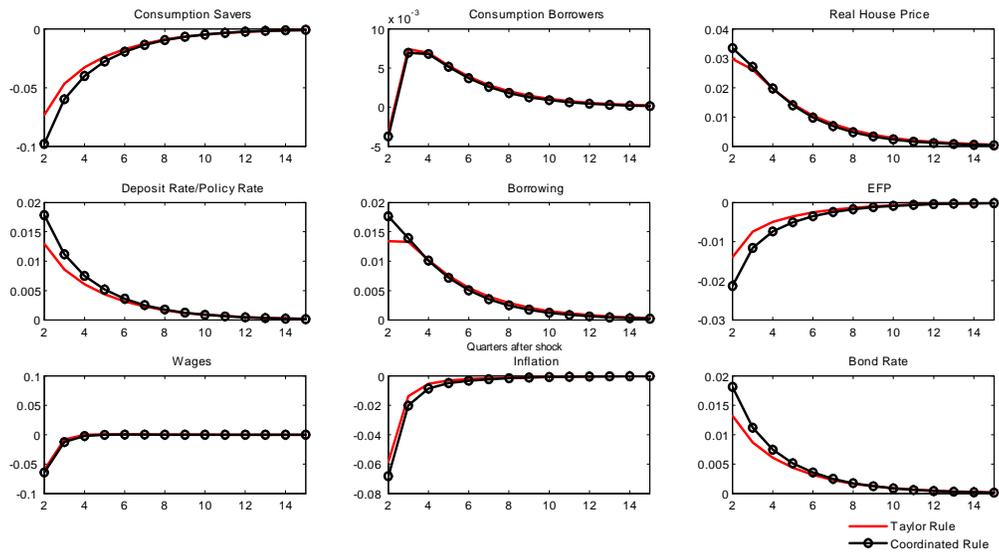


Figure 5: Monetary Shock.

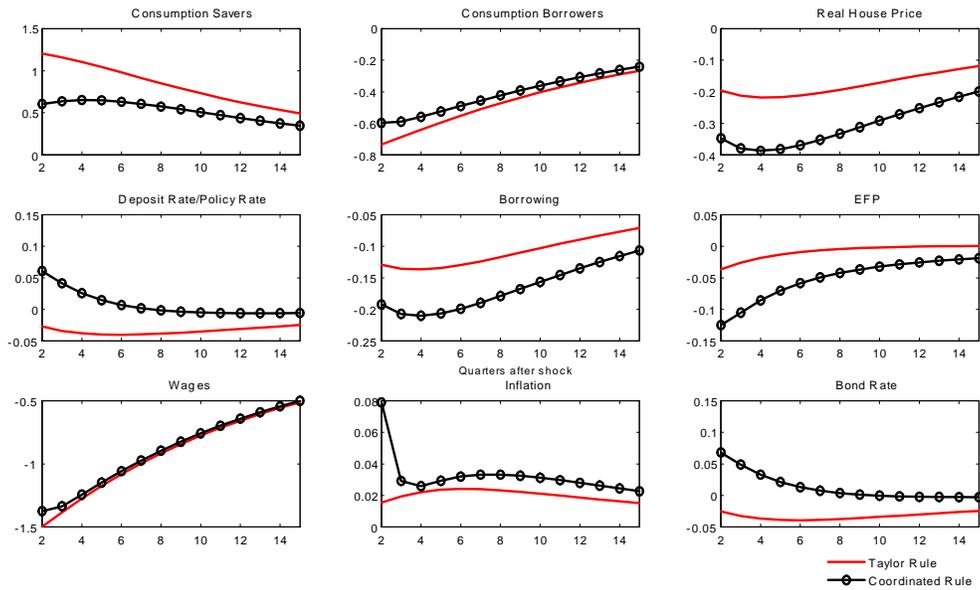


Figure 6: Mark-up Shock.

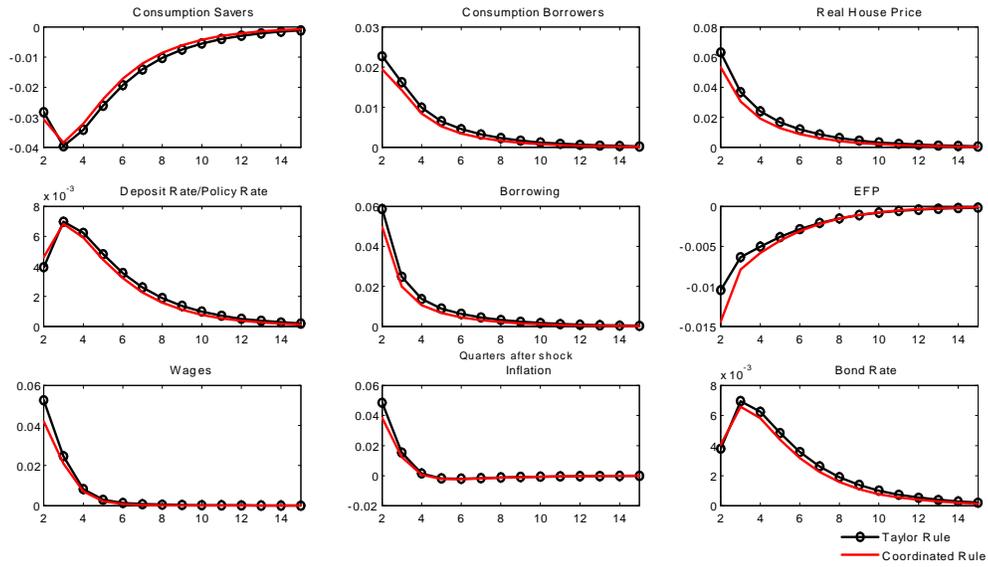


Figure 7: House Price Shock.

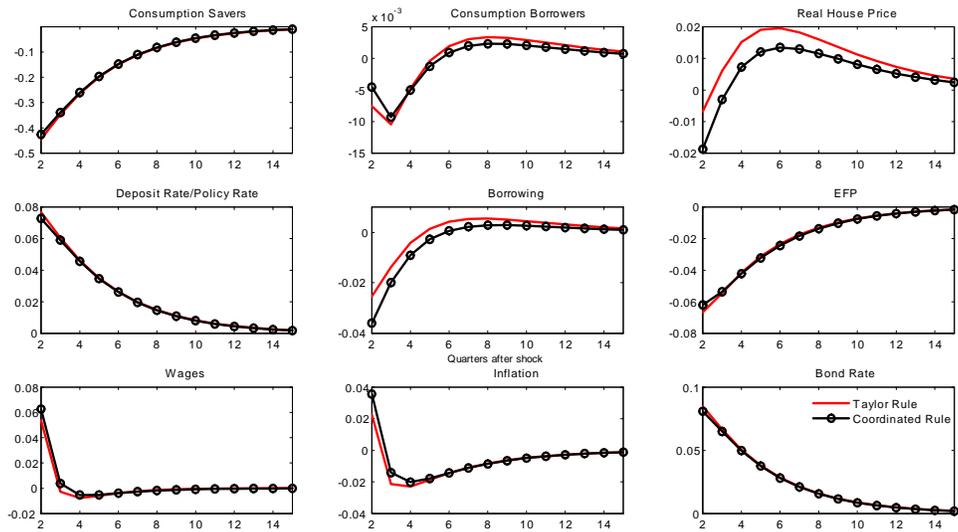


Figure 8: Government Spending Shock.

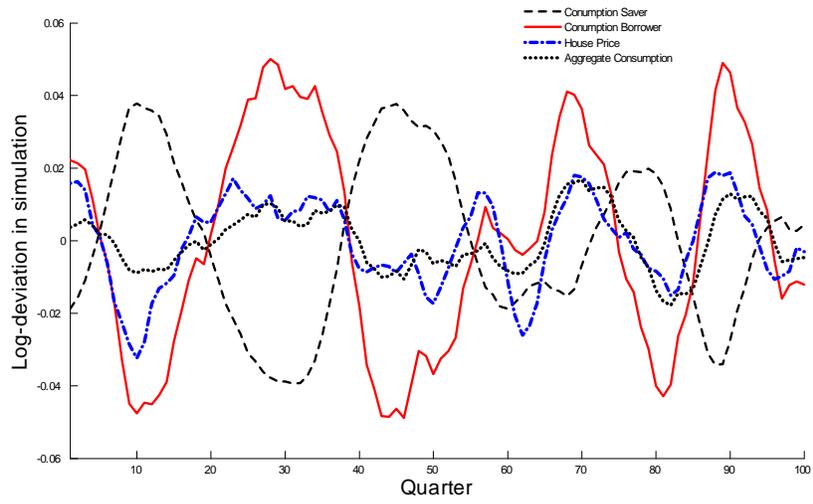


Figure 9: Simulated Data.

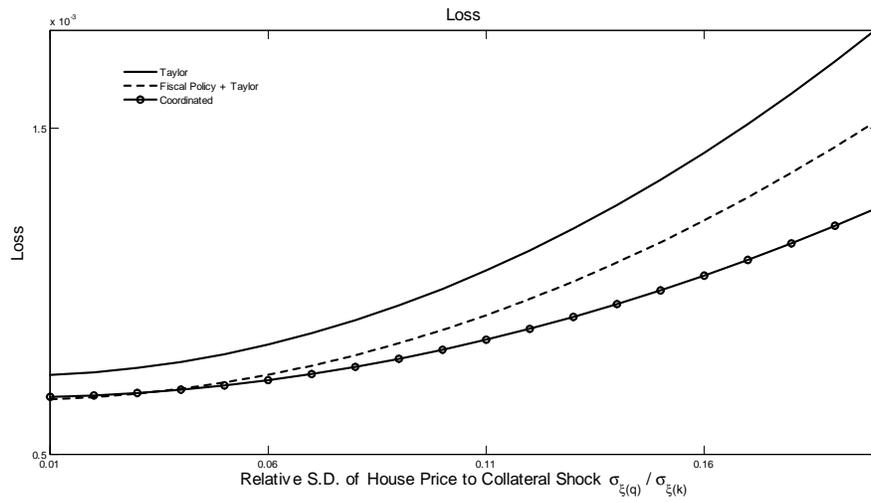


Figure 10: Loss under Various Policies Varying the Relative S.D. of House Price Shock to Collateral Shock.

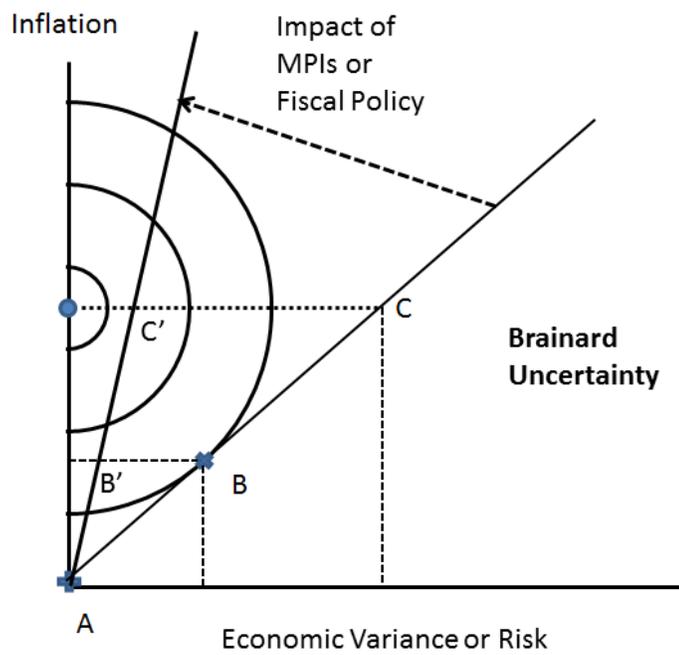


Figure 11: Brainard Uncertainty.

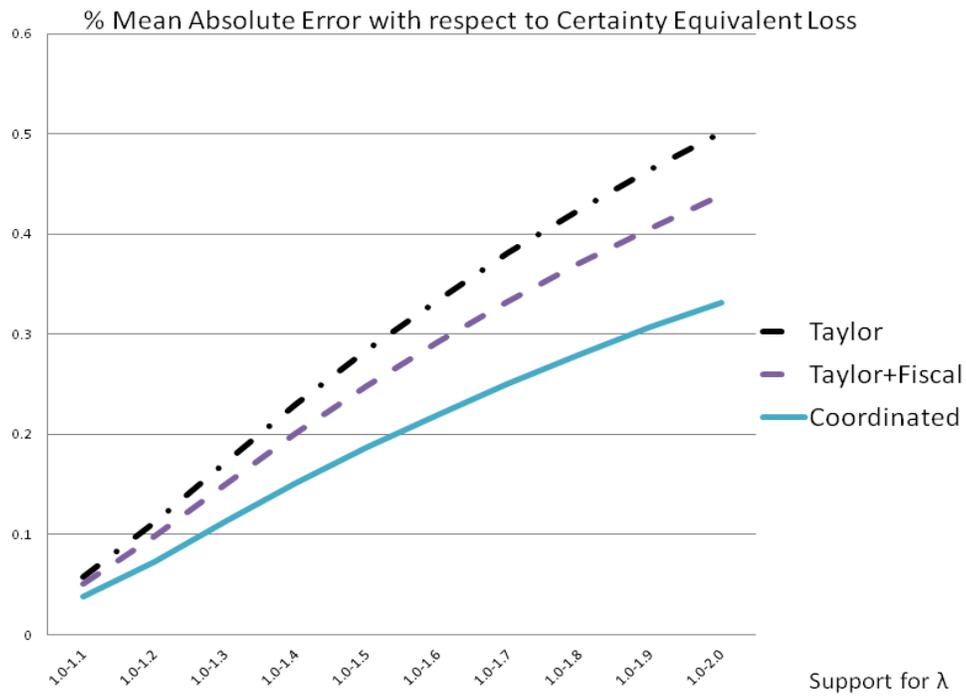


Figure 12: Loss of Various Rules under Parameter Uncertainty over  $\lambda$  Compared to the Certainty Equivalent Loss.