NOMINAL GDP GROWTH INDEXED BONDS: BUSINESS CYCLE AND WELFARE EFFECTS WITHIN THE FRAMEWORK OF NEW KEYNESIAN DSGE MODEL

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NIESR Discussion Paper No. 504

Date: 2nd May 2019

National Institute of Economic and Social Research

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This paper was first published in May 2019

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Keywords: New-Keynesian model, GDP-indexed bonds, Counter-cyclical fiscal policy

JEL Classifications: E62, E63, H63

Acknowledgements

I thank Jagjit Chadha, Katsuyuki Shibayama, Kevin Sheedy, and Alfred Duncan for very inspiring comments on this paper. The views express are those of the author and not necessarily those of the Bank of Korea.

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5th April 2019

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1 Introduction

After the Global Financial Crisis of 2007-2008, major central banks responded to the crisis by lowering short-term interest rates near zero and considerably expanding their balance sheets.

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At the same time, the crisis led to unprecedented level of fiscal expansions in the form of stimulus packages in many advanced countries. Even though these monetary and fiscal responses seem to have contributed significantly to the recovery from the Great Recession, a number of advanced countries are now suffering from rapidly increasing government debt. As of the end of 2018, the government debt of the U.S. is more than 100% of its GDP, which is much higher than its post-war average of 63%, and that of the U.K. has also rapidly increased and now approaching 90%. Under these circumstances, there are concerns that the possible policy options available to the central banks and the governments to cope with the next recession seem very limited.

Following these concerns, the interest in linking government debt cash flows to the growth rate of issuing country's GDP has been gradually growing both in academia and practitioners as an alternative fiscal policy tool to prepare for the next recession (see Barr et al. 2014; Bowman et al. 2016; Benford et al. 2016; Blanchard et al. 2016; Cabrillac et al. 2016; Kim and Ostry 2018.) The main advantage of issuing GDP-indexed bonds is that it helps reduce the upper tail risk of debt-to-GDP ratio by narrowing its distribution, and thus lowers the probability of sovereign default (Chamon and Mauro, 2006; Barr et al., 2014)[1]. For example, from the following debt-to-GDP dynamics,

$$d_{t+1} - d_t = \frac{(r_t - g_{t+1})}{1 + g_{t+1}} d_t - s_{t+1}, \tag{1.1}$$

we can see that a slow-down in growth, g_{t+1}, leads to a higher level of debt-to-GDP ratio, d_{t+1}, when the other variables - the interest rate, r_t, and primary surplus to GDP ratio, s_t - are unchanged. However, if the government finances its debt with GDP growth-indexed bonds, a slower growth also reduces the burden of interest payment, and thus mitigates the increase in debt-to-GDP ratio compared to the case where the conventional government debt is used.

Another advantage suggested by the literature is that the use of GDP growth-indexed bonds gives more room for conducting a counter-cyclical fiscal policy (Borensztein and Mauro 2004; Barr et al. 2014; Kim and Ostry 2018; Bonfim and Pereira 2018). If a government has very little or no fiscal space[2], the government has to increase the primary surplus to maintain debt-to-GDP ratio even when there is a negative shock on output (i.e., pro-cyclical fiscal policy).

[1]There also exit other benefits which are not covered in this paper. For example, for countries where pension liabilities are indexed to their GDP, GDP-indexed bonds can be particularly attractive investment vehicle for pension funds. Thus, such governments can benefit significantly in terms of borrowing costs. Examples can be found from the experiences of the UK government bond market where long-term inflation-indexed yields are depressed by strong demand from UK pension funds (Campbell and Viceira, 2009; Breedon and Chadha, 2003). Moreover, a rapid population ageing in developed countries may result in greater demand for GDP-indexed bonds, and the greater benefits in terms of borrowing costs as well.

[2]Ostry et al. (2010) has developed a concept of 'debt limit' which means an upper bound on how high debt-to-GDP ratio of a country can increase before the default risk becomes too high. The fiscal space means the gap between current debt-to-GDP ratio and the debt limit.

A government trapped in such a situation would get a larger room for conducting a counter-cyclical fiscal policy when its debts are fully or partially linked to the country's growth rate. Borensztein and Mauro (2004) showed this by conducting counterfactual simulations using the data from several advanced and emerging countries in 1990s[3], and Bonfim and Pereira (2018) also showed the similar results with recent data from France, Spain and Portugal.

One of the short-comings of the previous analyses on the benefits of GDP-indexed bond is that they rely their results on partial equilibrium models. More specifically, their results rely heavily on the assumptions about the joint process of the key variables in debt-to-GDP dynamics identity, Equation (1.1). On the contrary, within the framework of general equilibrium models, the relationships are obtained from the optimal choices of rational, forward-looking agents, not just by arbitrary assumptions. Unfortunately, however, we cannot use the standard medium-sized New Keynesian DSGE models such as Smets and Wouters (2007) for our analysis on the welfare and business cycle effects of GDP-indexed bonds. It is mainly because, in such models, we usually assume a rational, forward-looking representative household who is able to smooth his/her consumption intertemporally by trading in both financial and capital markets. Under such assumptions, the consumption of the representative household is a function of permanent income rather than current disposable income, and thus the structure of government finance (the choice between the two bonds) only affects the mix between outstanding debt and fiscal balance, and a particular mix is irrelevant to the household's decision on consumption and the business cycle. Generally speaking, since Ricardian equivalence holds in the standard DSGE models such as Smets and Wouters (2007), they are not suitable for analysing the effect of the counter-cyclicality of fiscal policy on business cycle and welfare.

However, there are plenty of empirical evidence which shows that consumption relies more strongly on current disposable income than the standard DSGE model suggests (Campbell and Mankiw, 1989; Mankiw, 2000). Based on such empirical evidence, Mankiw (2000) suggested a new model where some households follow the permanent income hypothesis and the rest of them are so-called rule-of-thumb households[4]. Galí et al. (2007) is the first paper that incorporated Mankiw's idea of rule-of-thumb households into the New Keynesian DSGE model with sticky-price in order to analyse the effect of government spending on consumption. Following the seminal paper of Galí et al. (2007), the idea of rule-of-thumb household has been widely used in the fiscal policy literature. Coenen and Straub (2004) extended one of the most famous medium scale New Keynesian DSGE model of Smets and Wouters (2003) by incorporating rule-of-thumb households, various distortionary and lump-sum taxes, and a fiscal policy rule

[3]They showed that the correlation betwen GDP growth and primary surplus-to-GDP ratio could have been much higher if those countries had linked all their government debts to their GDP growth. Such results held for both advanced and emerging market countries.

[4]His justification for the presence of such households can be either they are irrational, myopic, or have limited access to the financial or capital market. In this paper, we assumed the presence of "hand-to-mouth households" who are rational, forward-looking, but has no access to those markets.

that stabilises debt-to-GDP process. They estimated the share of rule-of-thumb households in the Euro area with the Bayesian estimation methodology. Their estimates for the share of rule of thumb households range from 24% to 37% depending on their assumptions on the fiscal policy rule. Similarly, Cogan et al. (2010) used extended version of Smets and Wouters (2007) model augmented with rule-of-thumb households to analyse the role of fiscal policy with the zero-lower-bound in nominal interest rate. They also estimated the model with the Bayesian methodology with the U.S. data, and their estimate of rule-of-thumb household share was around 29%. More recently, Drautzburg and Uhlig (2015) also relied on similar model to examine how and whether the presence of zero-lower-bound affects the sign and size of government spending multiplier.

These models relied on the idea of rule-of-thumb consumers mostly in order to examine the role of government spending on consumption. In this paper, we also adopted their assumption on the presence of rule-of-thumb households. However, the focus of this paper is different from the previous papers in that we are intended to examine whether and how the type of government bonds (conventional nominal bonds and GDP growth-indexed bonds) affects the business cycle and the welfare of the economy. The model in this paper is also based on Smets and Wouters (2003, 2007), augmented with hand-to-mouth households, lump-sum and distortionary taxes and Epstein and Zin type recursive preference.

We show that, under certain conditions, the use of GDP growth-indexed bond may help stabilise the business cycle and improve the welfare of hand-to-mouth households. In our model, the hand-to-mouth households are assumed to be rational and forward-looking, and they have desires for consumption smoothing, but they do not have access to either financial or capital market. That is to say, they cannot save, borrow, and invest in capital. As mentioned above, when there exist only Ricardian households, even if the government's choice on the type of bonds can affect the fiscal balance, it has no impact on business cycle and welfare. However, when there exist non-Ricardian or hand-to-mouth households, their consumption is directly affected by the changes in primary surplus.

Furthermore, in our model, the consumption and leisure choices of the two types of households are interconnected via labour market. More specifically, the increase in current disposable income of hand-to-mouth households leads to an increase in aggregate demand, and at the same time, to a decrease in labour supply from the hand-to-mouth households (i.e., intratemporal consumption smoothing). Therefore, the increased demand in aggregate labour should be met by an increase in labour supply from the Ricardian households. This is a more realistic assumption than the previous papers where the two types of households supply identical amount of labour and the hand-to-mouth households can smooth their consumption neither intertemporally nor intratemporally. Through this channel, the changes in fiscal balance affects not only the consumption/leisure choices of the hand-to-mouth households, but also those of the Rational households.

The remainder of this paper is organised as follows. Section 2 briefly outlines the model focusing on the differences from existing models, and the model parameters are discussed in section 3. The results from the baseline model, and the mechanism behind the results are in Section 4 with the sensitivity analysis with different key parameter values. Section 5 concludes.

2 The DSGE model

To analyse the effect of the use of GDP growth-indexed bonds on the business cycle and welfare, we built our DSGE model based on the medium scale New Keynesian DSGE model of Smets and Wouters (2003, 2007). We kept most of the key features of the Smets and Wouters (2003, 2007) models, which include two nominal frictions: sticky prices and wages; four real rigidities: external consumption habit, investment adjustment cost, variable capital utilisation, monopolistically competitive goods and labour markets; and seven exogenous shocks on productivity, preference, government spending, investment, monetary policy, price markup and wage markup shocks. On top of them, we further assumed that the households in our model have a recursive preference following Rudebusch and Swanson (2012) to better reflect the difference in the government's borrowing cost between the two type of bonds. We also assumed the presence of hand-to-mouth households following Galí et al. (2007) so that the Ricardian equivalence does not hold anymore. Lastly, we assumed that the government finances exogenous government spending and lump-sum government transfer via debts and distortionary taxes on labour and capital income.

2.1 Households

There exist a continuum of households with a unit mass indexed by $j \in [0, 1]$ grouped into two types - Ricardian and hand-to-mouth households - in this model economy. A fraction $1 - \omega$ of the households are Ricardian household who are rational, forward-looking and able to access to both financial and capital markets. The rest of the households, a fraction of ω, are hand-to-mouth households. They are also rational and forward-looking, but they have no vehicle to save or borrow as they cannot access those markets. That is to say, the hand-to-mouth households in this model have a desire to smooth their consumption, but their ability to do this is severely restricted as they cannot do it intertemporally. They can smooth their consumption only through changes in their labour supply. Therefore, in each period, they consume all their disposable income (= after-tax labour income plus government transfer). Such assumption is a bit different from the previous papers in the fiscal policy literature (Galí et al., 2007; Coenen and Straub, 2004; Drautzburg and Uhlig, 2015). In those papers, non-Ricardian households are assumed to simply take wages and working hours determined by the labour union, and they optimise neither intertemporally nor intratemporally. This also means that the labour supplies of the two household groups are identical at all times even though the consumption level of the two groups can be different.[5] On the contrary, in our model, we make a bit more realistic assumption. That is, the hand-to-mouth households optimise at least intratemporally, and thus the labour supplies of the two households need not be the same.

An individual household j in this model is assumed to have the following non-separable period utility function[6]:

$$U_{t,j}^{X} = \varepsilon_t^b \left[\frac{1}{1-\sigma_c} \left(C_{t,j}^{X} - \lambda C_{t-1}^{X} \right)^{1-\sigma_c} \right] \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_{t,j}^{X} \right)^{1+\sigma_l} \right] \tag{2.1}$$

where

$$X = \begin{cases} H & \text{if } j \in [0, \omega] \\ R & \text{if } j \in [\omega, 1] \,. \end{cases}$$

The superscript H and R denote hand-to-mouth and Ricardian households, respectively. The household obtains utility from the difference between individual current consumption, $C_{t,j}^{X}$, and the group-wise aggregate consumption in the previous period, C_{t-1}^{X}. That is to say, there exists an external habit in consumption and each household in this model tries to keep up with the other households only in the same group. The household also obtains disutility from supplying homogeneous labour, $h_{t,j}^{X}$, to the union. As the two types of households are identical except for their ability to access financial and capital markets, the quality of their labours are homogeneous regardless of the household type, and thus same hourly wage rate, $\overline{W}_t$, are applied. Following Smets and Wouters (2003), we assume the preference shock ε_t^b that affects the intertemporal substitution of households follows a simple AR(1) process.

Following Rudebusch and Swanson (2012)[7], we assume that an individual household in this model maximises its welfare $V_{t,j}^{X}$ recursively given as below:

$$V_{t,j}^{X} = U_{t,j}^{X} + \beta^X E_t \left[\left(V_{t+1,j}^{X} \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}} , \tag{2.2}$$

where β^X is a discount factor for household type X, but we assume that the two household groups share the same discount factor, $\beta = \beta^R = \beta^H$. This assumption is also consistent with

[5]In some papers, it is simply assumed with no justification (Coenen and Straub, 2004; Drautzburg and Uhlig, 2015), and in other papers, this is guaranteed by the assumption that the wage markup is large enough such that the wage is always higher than the MRS of both households (Galí et al., 2007).

[6]The functional form of the period utility is same as one in Smets and Wouters (2007).

[7]Rudebusch and Swanson (2012) rewrote the recursive preference suggested by Epstein and Zin (1991) as in Equation (2.2) for notational clarification.

our key assumption that the two types of households are heterogeneous only in terms of their ability to save or borrow, not in terms of their preference.

(Ricardian households) A Ricardian household j faces the following intertemporal budget constraint:

$$\begin{aligned} & C_{t,j}^R + I_{t,j} + \frac{B_{t,j}}{R_t P_t} + \frac{Q_t^G B_{t,j}^G}{P_t} \\ \leq \quad & \frac{B_{t-1,j}}{P_t} + \frac{B_{t-1,j}^G}{P_t}\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right) + (1-\tau_w)\frac{\overline{W}_t h_{t,j}^R}{P_t} + (1-\tau_r)\frac{R_t^k}{P_t} z_{t,j} K_{t-1,j} \\ & + \tau_r \delta K_{t-1,j} - a(z_{t,j}) K_{t-1,j} + D_{t,j}^f + (1-\tau_w) D_{t,j}^u + T_{t,j}. \end{aligned} \tag{2.3}$$

In the left-hand side, the household j consumes, invests, and saves by purchasing bonds. As only Ricardian households can invest or save, we abstract superscripts R from real investment $I_{t,j}$, capital $K_{t,j}$, the two bonds $B_{t,j}$ and $B_{t,j}^G$, and related variables such as their prices. $B_{t,j}$ is the units of 1-period nominal conventional government bond purchased at t at the unit price of $1/R_t$, and $B_{t,j}^G$ is the units of nominal GDP growth-indexed bonds (NGDP-indexed bond) purchased at the price of Q_t^G. On the right-hand side, the household finances its expenditure from the repayment of the bonds purchased from the previous period, after-tax labour and capital rental incomes, profits from intermediate firms and labour unions, and lump-sum transfer from the government. The bonds purchased in the previous period pays $\frac{B_{t-1,j}}{P_t}$ or $\frac{B_{t-1,j}^G}{P_t}\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right)$ back at t in terms of the final goods[8]. The distortionary taxes are levied on labour and capital rental income, and the same constant tax rates, τ_w and τ_r, are applied to both household groups. $D_{t,j}^f$ is the real profit from the intermediate firms, and it is evenly distributed among the Ricardian households because we assume that the firms are owned by only the Ricardian households. $D_{t,j}^u$ denotes real profit from the unions, and it is evenly distributed to all the households regardless of the household type. Note that the profit from the labour union is also taxed at the rate of τ_w; and tax allowance is assumed to apply to costs due to depreciation of capital, $\tau_r \delta K_{t-1,j}$. T_t is lump-sum government transfer in terms of final goods, which is also evenly distributed to all the households regardless of the household types. $z_{t,j}$ is the level of capital utilisation, and $a(z_{t,j})$ is the quadratic cost of capital utilisation given as below:

$$a(z_t) = \delta_1 (z_t - 1) + \frac{\delta_2}{2}(z_t - 1)^2. \tag{2.4}$$

[8]Note that a unit of NGDP-indexed bond purchased at $t-1$ pays $\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right)$ units of money when it matures.

Lastly, the Ricardian household accumulates its capital following the law of motion based on Christiano et al. (2005) as below :

$$K_{t,j} = (1-\delta)\,K_{t-1,j} + \varepsilon_t^I \left[1 - \frac{\phi}{2}\left(\frac{I_{t,j}}{I_{t-1,j}} - \gamma\right)^2\right] I_{t,j}, \tag{2.5}$$

where γ is trend productivity growth, and the investment shock, ε_t^I, follows a simple AR(1) process.

(Hand-to-mouth households) The hand-to-mouth households can neither trade bonds nor accumulate capital, and do not have the ownership of intermediate firms. Thus, the sources of their income are after-tax wages, profits from the unions, and the lump-sum transfer from the government only. This gives the following simple budget constraint of hand-to-mouth households:

$$C_{t,j}^H \leq (1-\tau_w)\left(\frac{\overline{W}_t h_{t,j}^H}{P_t} + D_{t,j}^u\right) + T_{t,j}. \tag{2.6}$$

(First order conditions: Ricardian households) The Ricardian households maximise Equation (2.2) by choosing $C_{t,j}^R$, $B_{t,j}$, $B_{t,j}^G$, $h_{t,j}^R$, $I_{t,j}$, $K_{t,j}$ and $z_{t,j}$ subject to Equation (2.3) to (2.5). This gives the following seven first order conditions[9]:

$$\Xi_t^R = \varepsilon_t^b \left(C_t^R - \lambda C_{t-1}^R\right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^R\right)^{1+\sigma_l}\right] \tag{2.7}$$

$$\frac{1}{R_t} = E_t\left[M_{t,t+1}^R \Pi_{t+1}^{-1}\right] \tag{2.8}$$

$$Q_t^G = E_t\left[M_{t,t+1}^R\left(\frac{Y_{t+1}}{Y_t}\right)\right] \tag{2.9}$$

$$(1-\tau_w)\frac{\overline{W}_t}{P_t} = \left(C_t^R - \lambda C_{t-1}^R\right)\left(h_t^R\right)^{\sigma_l} \tag{2.10}$$

$$\begin{aligned} 1 &= q_t \varepsilon_t^I \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - \gamma\right)\frac{I_t}{I_{t-1}}\right] + \\ &\quad E_t\left[M_{t,t+1}^R q_{t+1}\varepsilon_{t+1}^I \phi\left(\frac{I_{t+1}}{I_t} - \gamma\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{aligned} \tag{2.11}$$

$$q_t = E_t\left[M_{t,t+1}^R\left\{\begin{array}{c}(1-\tau_r)\frac{R_t^k}{P_t}z_t + \delta\tau_r \\ -a\left(z_{t+1}\right) + q_{t+1}(1-\delta)\end{array}\right\}\right] \tag{2.12}$$

$$(1-\tau_r)\frac{R_t^k}{P_t} = \delta_1 + \delta_2\left(z_t - 1\right) \tag{2.13}$$

[9]The subscript index j is dropped as the household decisions within the group are symmetric.

where $\Pi_{t+1} \equiv P_{t+1}/P_t$, and $M^R_{t,t+1}$ is the real stochastic discount factor for the Ricardian households defined as

$$M^R_{t,t+1} \equiv \beta \left(\frac{V^R_{t+1}}{E_t \left[\left(V^R_{t+1} \right)^{1-\sigma_{EZ}} \right]^{\frac{1}{1-\sigma_{EZ}}}} \right)^{-\sigma_{EZ}} \frac{\Xi^R_{t+1}}{\Xi^R_t}, \tag{2.14}$$

$q_t \equiv \frac{\Xi^k_t}{\Xi^R_t}$ is so-called Tobin's q, and Ξ^k_t is the Lagrangian multiplier for the law of motion of capital.

(First order conditions: hand-to-mouth households) The hand-to-mouth households also maximises Equation (2.2), but by choosing only consumption $C^H_{t,j}$ and labour supply $h^H_{t,j}$ subject to Equation (2.6). This gives the two first order conditions:

$$\Xi^H_t = \varepsilon^b_t \left(C^H_t - \lambda C^H_{t-1} \right)^{-\sigma_c} \exp \left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h^H_t \right)^{1+\sigma_l} \right] \tag{2.15}$$

$$(1-\tau_w) \frac{\overline{W}_t}{P_t} = \left(C^H_t - \lambda C^H_{t-1} \right) \left(h^H_t \right)^{\sigma_l} \tag{2.16}$$

2.2 Producers

The production sector in this model is very similar to that of Smets and Wouters (2003) except for some simplications. In order to recursively express the non-linear equilibrium conditions for the price setting (and wage setting as well), we made a modification on the price markup shock[10]. The perfectly competitive final good producer aggregates intermediate goods using the standard Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 Y_{t,i}{}^{\frac{1}{1+\lambda_P}} di \right)^{1+\lambda_P}, \tag{2.17}$$

and the optimisation problem of the final good producer gives the following demand schedule for the ith intermediate good:

$$Y_{t,i} = \left(\frac{P_{t,i}}{P_t} \right)^{-\left(\frac{1+\lambda_p}{\lambda_p}\right)} Y_t, \tag{2.18}$$

where $P_{t,i}$ denotes the price of the ith intermediate good.

There exists a continuum of firms indexed by $i \in [0,1]$ operating under monopolistic competi-

[10] Smets and Wouters (2003) assumed that the substitutability parameter, $\lambda_{p,t}$, is time-varying in order to incorporate price markup shocks into the model. Instead, we assumed the parameter to be constant, but added a wedge type markup shock, ε^p_t, to the price setting problem of intermediate good producers (see Equation 2.20). In both cases, the steady state level of price markup is given by λ_p.

tion, and an individual firm i produces its intermediate good using the technology below:

$$Y_{t,i} = \varepsilon_t^a \left(K_{t,i}^S\right)^\alpha \left(\gamma^t l_{t,i}\right)^{1-\alpha} - \gamma^t \Phi, \tag{2.19}$$

where $K_{t,i}^S \equiv z_t K_{t-1,i}$ is the capital service rented from the Ricardian households, $l_{t,i}$ is the labour index supplied by the labour packer, and Φ is the fixed costs in production. The intermediate firm i maximises the sum of stochastically discounted future profits by choosing optimal price of ith good, $\hat{P}_{t,i}$. Following Calvo (1983) pricing scheme, we assume only $1-\zeta_p$ of them are allowed to re-optimise their prices and the rest of the firms just partially index their prices by past inflation. That is to say, each individual intermediate good producer solves the following problem when given a chance of re-optimising:

$$\max_{\hat{P}_{t,i}} E_t \sum_{s=0}^{\infty} \zeta_p^s M_{t,t+s}^R \left(\frac{1}{P_{t+s}}\right) \left[X_{t,s}^p \hat{P}_{t,i} - \varepsilon_{t+s}^p MC_{t+s}\right] Y_{t+s,i} \tag{2.20}$$

subject to the demand schedule given in Equation (2.18). Note again that the intermediate good firms are owned only by the Ricardian households, thus all the profits are given only to them. Therefore, the future profits from the firms are discounted using the stochastic discount factor of the Ricardian households. MC_{t+s} denotes the nominal marginal cost for intermediate good production, and $X_{t,s}^p$ denotes the indexation factor defined as below:

$$X_{t,s}^p \equiv \begin{cases} 1 & \text{if } s = 0 \\ \prod_{l=1}^s \left(\Pi_{t+l-1}^{\iota_p} \Pi_*^{1-\iota_p}\right) & \text{if } s \geq 1, \end{cases}$$

where Π_* is the steady state level of gross inflation. Note also that there exists a price markup shock, ε_t^p, that follows an AR(1) process.

2.3 Labour market

The assumptions on the labour market structure are not much different from the standard New Keynesian DSGE model with sticky wages. We assumed that there exists a continuum of monopolistically competitive labour unions indexed by $z \in [0,1]$. Each union differentiates the homogeneous labours purchased from the households at the wage of $\overline{W}_t$, and provides the differentiated labour, $l_{t,z}$, to the labour packer at the wage of $W_{t,z}$. The household types and the labour types are independent each other[11], and the union cannot tell the household type. That is why all the households gets the same hourly wages and there is no superscript R or H on the differentiated labour. The labour packer aggregates the differentiated labour into the

[11] In other words, the fraction of hand-to-mouth households and Ricardian households is uniformly distributed across unions.

aggregate labour index, l_t, using the following Dixit-Stiglitz aggregator:

$$l_t \quad = \quad \left(\int_0^1 l_{t,z}^{\frac{1}{1+\lambda_w}} \, dz \right)^{1+\lambda_w}. \tag{2.21}$$

Analogous to the final good producer, the optimisation problem of labour packer gives the following demand schedule for z-type of labour:

$$l_{t,z} \quad = \quad \left(\frac{W_{t,z}}{W_t} \right)^{-\frac{1+\lambda_w}{\lambda_w}} l_t. \tag{2.22}$$

A union for type-z labour solves the following optimisation problem to maximise the stochastically discounted future profits by choosing optimal wage, $\hat{W}_{t,z}$:

$$\max_{\hat{W}_{t,z}} E_t \sum_{s=0}^{\infty} \zeta_w^s M_{t,t+s}^R \left(\frac{1}{P_{t+s}} \right) \left[\gamma^s X_{t,s}^w \hat{W}_{t,z} - \varepsilon_{t+s}^w \overline{W}_{t+s} \right] l_{t+s,z}, \tag{2.23}$$

subject to the demand schedule of Equation (2.22), where $X_{t,s}^w$ is the indexation factor defined as:

$$X_{t,s}^w \equiv \begin{cases} 1 & \text{if } s = 0 \\ \prod_{l=1}^s \left(\Pi_{t+l-1}^{\iota_w} \Pi_*^{1-\iota_w} \right) & \text{if } s \geq 1. \end{cases}$$

As mentioned earlier, the union cannot tell from which group the individual household comes. However, the Ricardian households account for the majority of the population, we assume that the unions discount future profits using the stochastic discount factor of the Ricardian households.[12] There exists a wage markup shock, ε_t^w, that follows AR(1) process as well.

2.4 Monetary and fiscal policy

The central bank is assumed to set its policy rate following the monetary policy rule below:

$$\frac{R_t}{R_*} = \left(\frac{R_{t-1}}{R_*} \right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_*} \right)^{\psi_1} \left(\frac{Y_t}{Y_t^*} \right)^{\psi_2} \right]^{1-\rho_R} \left(\frac{Y_t / Y_{t-1}}{\gamma} \right)^{\psi_3} \varepsilon_t^r, \tag{2.24}$$

where R_* and y_* are the steady state levels of nominal short-term interest rate and detrended output, respectively, and $Y_t^* \equiv y_* \gamma^t$ is trend level of output. This monetary policy rule is same as that in Smets and Wouters (2007) except that the output gap in this model is defined as the deviation from trend output rather than a deviation from the flexible-price-economy output. There exists the monetary policy shock, ε_t^r, that follows an AR(1) process.

[12] This assumption is following Drautzburg and Uhlig (2015) who justified their assumption with a median-voter decision rule.

The government faces the following intertemporal budget constraint:

$$G_t + T_t + \frac{B_{t-1}}{P_t} + \left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right)\frac{B^G_{t-1}}{P_t} = \frac{B_t}{R_t P_t} + \frac{Q^G_t B^G_t}{P_t} + \tau_w \frac{W_t l_t}{P_t} + \tau_r \frac{z_t K_{t-1} R^k_t}{P_t} - \tau_r \delta K_{t-1}, \quad (2.25)$$

where $G_t \equiv \varepsilon^g_t Y^*_t$ is the level of real government spending. In other words, the government consumes G_t units of final good in each period. Following Smets and Wouters (2007), we assume that the government spending is also affected by the productivity shock as follows:

$$\log\left(\frac{\varepsilon^g_t}{\varepsilon^g_*}\right) = \rho_g \log\left(\frac{\varepsilon^g_{t-1}}{\varepsilon^g_*}\right) + \eta^g_t + \rho_{ga}\eta^a_t, \quad (2.26)$$

where ε^g_* denotes the steady state government spending over output ratio. Note also that $W_t l_t$ is the tax base for labour income, which equals the sum of the wages paid to the households, $\overline{W}_t\left(\omega h^H_t + (1-\omega)\, h^R_t\right)$, and the unions' nominal profits, $P_t D^u_t = W_t l_t - \overline{W}_t\left(\omega h^H_t + (1-\omega)\, h^R_t\right)$.[13]

We further assume that the fiscal authority is constrained to keep its debt-to-GDP ratio at a constant level, $\overline{D}$. Under this assumption, the model government has no autonomy in fiscal policy since the distortionary tax rates are constant, government spending is exogenous, the debt-to-GDP ratio is constant, and these three determine the size of lump-sum government transfer. This assumption seems a bit extreme. However, as the goal of this paper is to examine how and whether the government can rely on NGDP-indexed bonds as an alternative fiscal policy tool for stabilising business cycle (or improving welfare) when all the other fiscal policy tools are lost, such an extreme assumption can help us to see the effect more clearly. Furthermore, the experiences after the financial crisis of 2007 may support this assumption as well. After the crisis, the debt-to-GDP ratios in many advanced countries have approached closely to their debt limits[14] (Ostry et al., 2010), and this forced many countries to use austerity measures even when they were in recession. From the assumption of constant debt-to-GDP ratio, the value of newly issued debt at t should always be equal to $\overline{D}Y_t$ such that

$$\frac{B_t}{R_t P_t} = \left(1-\omega^G\right)\overline{D}Y_t \quad (2.27)$$

$$\frac{Q^G_t B^G_t}{P_t} = \omega^G \overline{D}Y_t, \quad (2.28)$$

where $\omega^G = 0$ or 1 is the share of NGDP-indexed bonds.

[13] The labour packer receives $W_t l_t$ from the intermediate good producers by supplying labour, and as the labour packer earns zero profit in the perfectly competitive labour market, the total revenue of the labour unions from the labour packer should be $W_t l_t$ as well, or $W_t l_t = \int_0^1 w_{t,z} l_{t,z} dz$.

[14] The empirical analysis by Ostry et al. (2010) shows that many advance countries have already or almost reached their debt limits, which are defined as the theoretical threshold level of debt-to-GDP where a government with debt-to-GDP ratio higher than this level is excluded from the bond market.

2.5 Aggregation and Equilibrium

Aggregating the individual households' budget constraints, Equation (2.3) and (2.6), within each group gives the two group-wise aggregate budget constraints below:

$$
\begin{aligned}
(1-\omega)\,C_t^R + I_t + \frac{B_t}{R_t P_t} + \frac{Q_t^G B_t^G}{P_t} \quad = \quad & \frac{B_{t-1}}{P_t} + \frac{B_{t-1}^G}{P_t}\left(\frac{Y_t P_t}{Y_{t-1}P_{t-1}}\right) \\
& + (1-\tau_r)\,\frac{R_t^k K_t^s}{P_t} + \tau_r \delta K_{t-1} + (1-\tau_w)\,\frac{\overline{W}_t\,(1-\omega)\,h_t^R}{P_t} \\
& + D_t^f + (1-\tau_w)\,(1-\omega)\,D_t^u - a\,(z_t)\,K_{t-1} + (1-\omega)\,T_t
\end{aligned}
\tag{2.29}
$$

$$
\omega C_t^H \quad = \quad (1-\tau_w)\left\{\frac{\overline{W}_t \omega h_t^H}{P_t} + \omega D_t^u\right\} + \omega T_t, \tag{2.30}
$$

where D_t^f and D_t^u are aggregate real profits from the intermediate firms and labour unions, respectively:

$$
\begin{aligned}
D_t^f \quad &= \quad \frac{1}{P_t}\left(P_t Y_t - W_t l_t - R_t^k K_t^s\right) \\
D_t^u \quad &= \quad \frac{W_t l_t}{P_t} - \frac{\overline{W}_t}{P_t}\left(\omega h_t^H + (1-\omega)\,h_t^R\right).
\end{aligned}
$$

Combining the two group-wise budget constraints, Equation (2.29) and (2.30), and the government's budget constraint, Equation (2.25), gives the following aggregate resource constraint:

$$
Y_t = C_t + I_t + G_t + a\,(z_t)\,K_{t-1}. \tag{2.31}
$$

Aggregating the demand schedules for the intermediate goods, Equation (2.18), gives the following goods market clearing condition:

$$
Y_t = \frac{\varepsilon_t^a\,(z_t K_{t-1})^\alpha \left(\gamma^t l_t\right)^{1-\alpha} - \gamma^t \Phi}{s_t^p}, \tag{2.32}
$$

where s_t^p is the price dispersion with the following law of motion:

$$
s_t^p = (1-\zeta_p)\left(\hat{\Pi}_t\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \zeta_p \left(\frac{\Pi_{t-1}^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} s_{t-1}^p, \tag{2.33}
$$

and similarly, the labour market clearing condition is given as follows:

$$
l_t s_t^w = \omega h_t^H + (1-\omega)\,h_t^R, \tag{2.34}
$$

where s_t^w is the wage dispersion with the following law of motion:

$$s_t^w = (1-\zeta_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \zeta_w \left(\frac{\Pi_{t-1}^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{W_{t-1}\gamma}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} s_{t-1}^w. \tag{2.35}$$

The full list of equilibrium conditions are attached in the Appendix A.

3 Parameters

To calibrate the parameters for our baseline model we assume that the government follows a flexible-debt-rule,

$$\log\left(\frac{t_t}{t_*}\right) = \alpha_1 \log\left(\frac{d_t/y_t}{\overline{D}}\right), \tag{3.1}$$

where t_t is government transfer and d_t denotes the real detrended value of new debt issuance at t:

$$d_t \equiv \left(1-\omega^G\right)\frac{b_t}{R_t} + \omega^G Q_t b_t^G,$$

instead of the fixed debt-to-GDP rule assumed in the previous section. This is simply because that the U.S. government had not been constrained by the fixed debt-to-GDP rule while the U.S. data we try to match was being produced. This flexible-debt-rule implies that the government tries to keep the debt-to-GDP ratio near its steady state, $\overline{D}$, by adjusting its government transfer. In Equation (3.1), $\alpha_1 < 0$ controls the volatility of debt-to-GDP ratio. The larger the absolute size of α_1, the more strongly the government tries to keep the debt-to-GDP ratio near its steady state. For example, when α_1 becomes an extremely large negative number, the model becomes similar to the baseline model with constant debt-to-GDP ratio. In this section, we set $\alpha_1 = -10$ in order to allow the debt-to-GDP ratio flexibly fluctuates[15].

Most of the parameters in our model are standard in the literature (see the list of parameters in Table 1 and 2). We set the curvature of period utility function with respect to relative consumption for constant labour, $\sigma_c = 2.0$. It is in the range of parameter values from most of the New Keynesian literature even though it is a bit larger than the estimates of Smets and Wouters (2003, 2007) of around 1.4. The inverse of the elasticity of labour, $\sigma_l = 1.9$ is borrowed from Smets and Wouters (2007). We set the degree of external habit, λ, at 0.7. The fraction of firms and unions which are not given the chance of re-optimising, ζ_p and ζ_w, are set to be 0.78 and 0.75 respectively, which imply an average period of around four quarters between

[15] This implies that 2% deviation of debt-to-GDP ratio leads to -20% deviation of government transfer. With $\alpha_1 = -10$, the highest level of simulated debt-to-GDP was around 30% higher than the steady state level in our simulation.

re-optimising, and the indexation parameter for the price and wage, ι_p and ι_w, are set to be 0.1 and 0.5, respectively. The steady state level of both price and wage markups are assumed to be 0.1. The monetary policy rule coefficients are also borrowed from Smets and Wouters (2007). $\alpha = 1/3$ implies a steady state share of labour income of 66%, and depreciation rate $\delta = 0.025$ means an annual depreciation of 10%. The discount factor, $\overline{\beta} \equiv \beta\gamma^{-\sigma_c} = 0.9905$ implies around 4% annual real interest rate in steady state. We assume the trend annual productivity growth rate slightly lower than 1%, $\gamma = 1.002$, and steady state gross inflation rate, $\pi_* = 1.008$, or around 3.2% annually. $\phi_p = 1.0$ implies that there is no fixed cost in the production of intermediate goods. The parameters for the investment adjustment cost ϕ and the elasticity of the capital utilisation cost ψ are assumed to be 5.5 and 0.5, respectively. All the parameters above are standard among New Keynesian literature (see Levin et al. 2006; Christiano et al. 2005; Smets and Wouters 2007).

The Epstein-Zin parameter σ_{EZ} is set to be -360 to match the term premium of 100 basis points on 10-year zero-coupon U.S government bonds. This is much larger (in absolute term) than the parameter value of -148 used in Rudebusch and Swanson (2012), but much smaller than that of Darracq Paries and Loublier (2010). There seems to be no consensus as to the size of this parameter. Darracq Paries and Loublier (2010) showed that the absolute size of Epstein-Zin parameter should be around 1,000 to generate term premium of 100 basis points if one uses the exactly same model as Smets and Wouters (2007). In general, to match a given size of term premium, the smaller σ_c, the larger σ_{EZ} is required. Rudebusch and Swanson (2012) was able to match the term premium of 100 basis points with a relatively small σ_{EZ} with the help of a very large σ_c (≈ 9)[16]. On the contrary, σ_c in Smets and Wouters (2007) is estimated to be only 1.39, and this is why an extremely larger σ_{EZ} is required. In this paper, we set σ_c to be 2, which is larger than that of Smets and Wouters (2007) but still in the range of the models in the literature.

The steady state ratio of government spending over output, $\varepsilon^g_* = 0.17$, is from Trabandt and Uhlig (2011) who calibrated the value using the historical U.S. data. We set the baseline value for the fixed ratio of debt-to-GDP, $\overline{D} = 2.56$, or the level of debt being 63% of annual GDP, using the post-war average U.S. data. The constant labour and capital rental income tax rates, $\tau_r = 0.36$ and $\tau_w = 0.28$, are also from Trabandt and Uhlig (2011). Lastly, we set the fraction of the hand-to-mouth households, ω, to 10% of the population. This is somewhat smaller than the fraction of the rule-of-thumb households assumed (or estimated) in the literature. For example, Campbell and Mankiw (1989) estimated that the fraction is around 50% of the population, and Galí et al. (2007) also used the same ratio. Recent papers in fiscal policy literature estimated that the fraction falls between 20% to 33% (see Coenen et al. 2012; Erceg and Lindé 2014;

[16] In fact, in their baseline model where $\sigma_c = 2$, the term premium is only a third of the best fit model where σ_c is around 9.

Table 1: **List of parameters**

Parameter	Value	Description
σ_c	2.0	IES in consumption
σ_l	1.9	labour supply elasticity
σ_{EZ}	-360	Epstein ann Zin parameter
λ	0.7	degree of consumption habit
ζ_p	0.78	price stickiness
ζ_w	0.75	wage stickiness
ι_p	0.1	price indexation
ι_w	0.5	wage indexation
λ_p	0.1	steady state price markup
λ_w	0.1	steady state wage markup
ρ_R	0.8	policy rate smoothing
ψ_1	2.0	inflation gap coefficient
ψ_2	0.1	output gap coefficient
ψ_3	0.2	output growth coefficient
α	1/3	share of capital
δ	0.025	depreciation
$\overline{\beta}$	0.9905	discount factor
γ	1.002	trend growth in productivity
π_*	1.008	steady state inflation
ϕ_p	1.0	parameter for fixed cost
ψ	0.5	utilisation adjustment cost
ϕ	5.5	investment adjustment cost
ε^g_*	0.17	share of government spending
$\overline{D}$	2.52	steady state debt to GDP ratio
ω	0.1	share of hand-to-mouth households
τ_r	0.36	labour income tax rate
τ_w	0.28	capital rental income tax rate

note: $\overline{\beta} \equiv \beta\gamma^{-\sigma_c}$, $\phi_p \equiv 1 + \Phi/y_t$ is 1 plus share of fixed cost in the production, and $\psi \equiv \delta_2/\delta_1$ is elasticity of the capital utilisation cost function.

Table 2: **List of shocks**

Parameter	**Value**	**Description**
ρ_a	0.95	AR(1) coefficient of productivity shock
ρ_b	0.20	AR(1) coefficient of preference shock
ρ_g	0.90	AR(1) coefficient of government spending shock
ρ_i	0.60	AR(1) coefficient of investment shock
ρ_r	0.20	AR(1) coefficient of monetary policy shock
ρ_p	0.80	AR(1) coefficient of price markup shock
ρ_w	0.89	AR(1) coefficient of wage markup shock
ρ_{ga}	0.52	correlation between a and g shocks
σ_a	0.45	standard deviation of productivity shock
σ_b	0.24	standard deviation of preference shock
σ_g	0.30	standard deviation of government spending shock
σ_i	0.45	standard deviation of investment shock
σ_r	0.24	standard deviation of monetary policy shock
σ_p	2.40	standard deviation of price markup shock
σ_w	2.40	standard deviation of wage markup shock

Cogan et al. 2010; Drautzburg and Uhlig 2015). However, we chose to use a smaller fraction of 10%. Our definition of the hand-to-mouth households are those who are fully rational and forward-looking but does not have any tool for saving or investing. It is hard to believe such households take up more than 20% of the population. Our assumption of 10% is slightly above around 7~8% of the fraction of the U.S. population who do not have a bank account (FDIC, 2015). We will see how the results are affected by ω in subsection 4.3. The autocorrelation coefficients and standard deviations of the exogenous shock processes are provided in Table 2.

Table 3 presents the standard deviations, autocorrelations, and cross correlations for key macroeconomic variables using the simulated data (for 10,000 periods) from the baseline model with flexible debt rule and conventional bonds. Note that the simulated model is not different from standard New Keynesian DSGE models except that we assume that 10% of the households are non-Ricardian and there exist distortionary taxes. As the table shows, our model well replicates the actual data of the U.S. despite the assumption of hand-to-mouth households. It replicates the negative correlation between inflation and output growth, and the highly persistent inflation process of the actual data, even though the inflation is a little bit more persistent than the actual data. The impulse responses in Figure 1 also show the similar patterns given from the standard New Keynesian DSGE models[17].

[17] All the figures are in Appendix C.

Table 3: **Key moments of the benchmark model** $(\omega = 0)$

	$\sigma_{\Delta y}$	$\sigma_{\Delta c}$	σ_{π}	$\rho_{\Delta y,\pi}$	$\rho_{\Delta y,\Delta c}$	*AR1(Δy)*	*AR1(π)*
Model	0.79	0.64	0.64	-0.16	0.73	0.30	0.75
US data	0.80	0.65	0.61	-0.18	0.65	0.34	0.65

note: σ_x denotes the standard deviation of variable x, and Δx means the quarterly percentage growth of the variable. $\sigma_{x,y}$ denotes sample correlation coefficient between x and y. The U.S. actual data from 1971Q3 to 2016Q4 were used.

4 Results

4.1 Baseline results

Table 4 compares the simulation results from the benchmark model where there exist only Ricardian households (first four columns) and the baseline model where 10% of the population are hand-to-mouth households. Note that the fixed debt-to-GDP ratio is assumed in this subsection. Let us first compare the benchmark and the baseline results for the case where only the conventional bonds are used (first and second columns vs. fifth and sixth columns). The presence of the hand-to-mouth households only slightly changes the mean of the key variables, but its effect on volatility is substantial even though the fraction of hand-to-mouth households is only 10%. Especially, the consumption and labour supply of hand-to-mouth households are much more volatile than those of the Ricardian households in the benchmark model. Even the Ricardian households also experience more volatile labour supply in the baseline model than in the benchmark model.

The mechanism behind the larger volatility when there exist hand-to-mouth households is explained as follows. Under the assumption of constants debt-to-GDP ratio in this model, the net government transfer, NT_t, which is defined as the lump-sum government transfer minus distortionary taxes, can be expressed as below:[18]

$$NT_t \equiv t_t - \tau_t = -\varepsilon_t^g y_* + \overline{D}\left[y_t - \frac{y_{t-1}R_{t-1}}{\Pi_t \gamma}\right]. \tag{4.1}$$

This equation is given by substituting Equation (2.27) into the government budget constraint (Equation 2.25)[19]. τ_t denotes sum of the tax revenues from both labour and capital rental incomes. Equation (4.1) shows that a decrease in current output reduces the government's capacity of issuing new debts as the size of new debts should be proportional to the current

[18]This expression is for the case where only conventional bonds are used.
[19]Note that the lower case variables are detrended variables.

Table 4: **Baseline results**

	Benchmark ($\omega = 0$)				**Baseline** ($\omega = 0.1$)			
	Conv.		**NGDP**		**Conv.**		**NGDP**	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.54	2.87	3.54	2.87	3.53	3.12	3.54	2.98
consumption	2.21	2.42	2.21	2.42	2.20	2.85	2.21	2.68
(Ricardian)	2.21	2.42	2.21	2.42	2.25	2.59	2.25	2.48
(H2M)	n.a	n.a	n.a	n.a	1.76	7.54	1.77	6.30
inflation	0.68	0.63	0.68	0.63	0.70	0.64	0.71	0.63
labour supply	1.28	1.97	1.28	1.97	1.28	2.36	1.28	2.16
(Ricardian)	1.29	1.97	1.29	1.97	1.27	2.79	1.27	2.41
(H2m)	n.a	n.a	n.a	n.a	1.46	5.84	1.45	3.59
capital supply	27.08	3.76	27.08	3.76	27.12	4.01	27.15	3.91
wage	1.66	3.13	1.66	3.13	1.66	3.36	1.66	3.32
rent rate	0.04	1.91	0.04	1.91	0.04	2.08	0.04	2.01
interest rate	1.02	0.63	1.02	0.63	1.02	0.69	1.02	0.64
net transfer	-0.66	15.70	-0.66	12.36	-0.66	23.21	-0.66	14.78
corr(y,NT)	-0.00		-0.14		0.08		-0.10	
welfare cost, %								
(Ricardian)	2.75		2.75		2.92		2.89	
(H2m)	n.a		n.a		16.52		13.67	

Note: Conv. and NGDP denote the case where $\omega^G = 0$ and $\omega^G = 1$, respectively. The standard deviations are calculated using log of the variables except for the inflation, and they expressed in percent. The correlation coefficient between output and net transfer (= transfer minus distortionary taxes) is presented to show the cyclicality of fiscal policy. See Equation (4.3) for the definition of welfare cost of business cycle.

output. At the same time, as the decrease in current output lowers current inflation[20], the burden of debt repayment becomes larger. All in all, a negative shock on current output has a negative impact on NT_t. Up to this point, there is no difference between the benchmark and the baseline models. However, contrary to the benchmark model where changes in NT_t has no effect on the business cycle, it has substantial effects in the baseline model. As the hand-to-mouth households are lack of consumption smoothing tools, a substantial portion of the change in net transfer goes to their consumption via the change in their disposable income.

On top of the direct effect on the consumption of hand-to-mouth households, there also exist

[20] In fact, this is not the case in the benchmark model. In the benchmark model with no hand-to-mouth households, supply shock dominates, and thus output and inflation covary negatively. In the baseline model, however, demand shocks play more important roles for the business cycle. This is also explained by Equation (4.1). The difference between the baseline and the benchmark model is whether the changes in net transfer affect the business cycle, and the terms that determine the response of net transfer in the baseline model are the terms in parentheses in Equation (4.1). As the demand shocks move output and inflation in the opposite direction, the terms in the parentheses react much more strongly to the demand shocks then the supply shocks.

second round effects. Figure 3 summarises this. The decreased consumption in hand-to-mouth households means a reduced demand in aggregate output, and in turn reduced demands in labour and capital service as well. This further decreases the net transfer, and this cycle goes on and on. At the same time, the hand-to-mouth households try to smooth their consumption in response to the decrease in disposable income by supplying more labour. That means, the decreased demand in aggregate labour should be met by less labour supply from the Ricardian households. In equilibrium, the cycle ends up with lower C^H, higher h^H, and lower h^R. This is why the labour supply of not only the hand-to-mouth but also the Ricardian households become more volatile in the baseline model. Also, the opposite responses of the two labour supplies, h^H and h^R, explain why aggregate labour supply is less volatile than both of the group-wise labour supplies are. To sum up, when the government is forced to keep its debt-to-GDP ratio constant, a shock that changes output leads to a change in net transfer. If all the households are Ricardian, the business cycle is immune to this change, but when there exist hand-to-mouth households, the changes in net transfer can have significant impact on the business cycle through their consumption.

Let us then examine whether and how the use of NGDP-indexed bonds may stabilise the business cycle and improve the welfare of hand-to-mouth households in the baseline economy. The fifth to eighth columns of Table 4 contrast the simulation results from the baseline model under the two different financing structure: 100% conventional bonds vs. 100% NGDP-indexed bonds. The results show that using the NGDP-indexed bonds only slightly changes the mean of key variables, but it decreases their volatility significantly. When the government relies 100% on the NGDP-indexed bonds, the equation for the net transfer is given as follows:

$$NT_t \equiv t_t - \tau_t = -\varepsilon_t^g y_* + \overline{D}\left[y_t - \frac{y_t}{Q_{t-1}^G}\right]. \tag{4.2}$$

In this case, a decrease in y_t reduces not only the government's ability to issue new debts, but also the repayment burden of previously issued bonds. Therefore, given the same size of shock (e.g., a negative shock on output), the decrease in net transfer is smaller in size in the case of NGDP-indexed bonds than in the case of conventional bonds. This, in turn, reduces the decline in consumption of hand-to-mouth households, and thus, reduces the changes in output, labour and capital service as well following the cycle described in Figure 3. In short, NGDP-indexed bonds can be used as a kind of automatic stabiliser.

The impulse response functions in Figure 2 tell us more stories. Using NGDP-indexed bonds greatly reduces the responses to demand shocks (preference, government spending, investment and monetary policy shocks) of key variables, while the responses to supply shocks (productivity, price markup, and wage markup shocks) are not affected much. This can also be explained from Equation (4.1) and (4.2). When a demand shock comes, both output and inflation move to the same direction. This implies, from Equation (4.1) where only conventional government bonds are used, a positive demand shock increases the government's capacity of issuing new debts and reduces the burden of debt repayment in real terms. As a result, the shock significantly increases the lump-sum transfer, and through the increase in disposable income of hand-to-mouth households, destabilises the entire economy. When NGDP-indexed bonds are used instead, the positive demand shock increases both new and old debts as in Equation (4.2), and thus the response of the net transfer becomes a lot smaller, and so do the responses of the other variables. On the contrary, to a supply shock, output and inflation respond to the opposite directions. In case of the conventional bonds, a positive supply shock increases both new and old debts, and thus its impact on transfer cancel out each other. For this reason, the business cycle stabilising effect of issuing NGDP-indexed bonds is also reduced.

Meanwhile, the last two rows in Table 4 show the welfare costs of business cycle under various assumptions. In order to measure the changes in welfare in terms of final good consumption, we defined the welfare cost of business cycle, WC, as follows:

$$v_{mean} = \frac{\left[\frac{1}{1-\sigma_c}\left(\left(1-\frac{\lambda}{\gamma}\right)c_*\left(1-WC\right)\right)^{1-\sigma_c}\right]\exp\left[\frac{\sigma_c-1}{1+\sigma_l}\left(h_*\right)^{1+\sigma_l}\right]}{1-\overline{\beta}\gamma}, \tag{4.3}$$

where v_{mean} is the simulated mean of detrended welfare level, v_t[21]; and c_* and h_* are deterministic steady state levels of detrended consumption and working hours, respectively. This definition implies that the welfare cost, WC, shows the welfare loss incurred from the existence of business cycle in terms of deterministic steady state consumption level. In other words, WC shows the amount of the steady state consumption loss needed to lower the deterministic steady state welfare level down to the average welfare level when there exist business cycles.

Because the presence of hand-to-mouth households significantly destabilises the business cycle, even the Ricardian households face higher welfare cost in the baseline model than in the benchmark model (2.75%→2.92%)[22]. When the NGDP-indexed bonds are used in the baseline model, while there is no notable change in the welfare cost of the Ricardian households, the hand-to-mouth households can benefit substantially in terms of welfare cost (16.52%→13.67%). One thing to note is that the focus of this paper is not calculating the exact magnitude of welfare gain by the use of NGDP-indexed bonds, especially because of the simplistic assumptions on the government sector. Instead, we focus more on the mechanism how the use of NGDP-indexed bonds can affect the business cycle and welfare, and the condition under which the

[21] Note that the welfare levels of both households are directly captured by the value functions, v_t^R and v_t^H.

[22] The welfare cost of around 3% in the benchmark model may seem a lot lager compared with the literature. For example, Lucas (1987) showed that the welfare loss from fluctuations in consumption is less than 0.01% under the assumption of logarithmic preference. However, it is known that the welfare cost of business cycle can be much larger for models with recursive preferences (see Dolmas, 1998; Tallarini, 2000; Barrillas et al., 2006).

Table 5: **Sensitive analysis (share of H2M households)**

	($\omega = 0.07$)				($\omega = 0.15$)			
	Conv.		**NGDP**		**Conv.**		**NGDP**	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.53	3.03	3.54	2.94	3.52	3.48	3.54	3.13
consumption	2.20	2.69	2.21	2.59	2.19	3.44	2.20	2.96
(Ricardian)	2.24	2.52	2.24	2.46	2.27	2.85	2.28	2.58
(H2M)	1.76	7.00	1.77	6.05	1.75	9.44	1.77	7.08
inflation	0.70	0.64	0.70	0.63	0.72	0.65	0.72	0.62
labour supply	1.28	2.22	1.28	2.09	1.28	2.88	1.28	2.37
(Ricardian)	1.28	2.47	1.28	2.26	1.26	4.21	1.26	2.96
(H2m)	1.45	5.10	1.45	3.31	1.47	9.13	1.45	4.85
capital supply	27.11	3.92	27.13	3.86	27.11	4.32	27.19	4.08
wage	1.66	3.29	1.66	3.26	1.66	3.54	1.67	3.47
rent rate	0.04	2.02	0.04	1.98	0.04	2.27	0.04	2.12
interest rate	1.02	0.67	1.02	0.64	1.02	0.76	1.02	0.66
net transfer	-0.66	20.20	-0.66	13.86	-0.66	42.67	-0.66	19.02
corr(y,NT)	0.05		-0.12		0.20		-0.02	
welfare cost, %								
(Ricardian)	2.85		2.84		3.07		3.06	
(H2m)	14.90		13.28		25.73		15.87	

note: See the note in Table 4

government and the households can benefit. We will discuss this with the sensitivity analysis in the next subsection.

4.2 Sensitivity analysis

In the baseline model, we assumed that the fraction of hand-to-mouth households, ω, is only 10% of the population. As mentioned already in section 3, this is somewhat lower than the fractions used in the literature. Therefore, it would be worthwhile to examine how different fractions of hand-to-mouth households change the results. Table 5 presents the simulation results when the fraction is changed to 7% and to 15%, leaving all the other parameters unchanged from the baseline. Obviously, the results show that the business cycle becomes more volatile when the share of hand-to-mouth households grows. In this model, the key channel through which the changes in net transfer can affect the business cycle is the consumption of hand-to-mouth households. Therefore, given the same change in net transfer, it is natural the larger the fraction of hand-to-mouth households, the more volatile the economy becomes.

Table 6: **Sensitive analysis (debt-to-GDP ratio)**

	$(\overline{D}=0)$				$(\overline{D}=3.6)$			
	Conv.		**NGDP**		**Conv.**		**NGDP**	
	Mean	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**	**Mean**	**Std.**
output	3.54	2.87	3.54	2.87	3.53	3.30	3.54	3.05
consumption	2.21	2.56	2.21	2.56	2.20	3.09	2.21	2.77
(rational)	2.25	2.41	2.25	2.41	2.25	2.72	2.26	2.53
(H2M)	1.81	5.13	1.81	5.13	1.75	9.83	1.75	7.38
inflation	0.66	0.64	0.66	0.64	0.72	0.64	0.72	0.62
labour supply	1.28	2.02	1.28	2.02	1.28	2.61	1.28	2.25
(rational)	1.27	2.21	1.27	2.21	1.27	3.40	1.27	2.59
(H2m)	1.43	1.55	1.43	1.55	1.48	10.37	1.46	5.55
capital supply	27.22	3.78	27.22	3.78	27.08	4.15	27.13	3.98
wage	1.66	3.25	1.66	3.25	1.66	3.41	1.66	3.35
rent rate	0.04	1.94	0.04	1.94	0.04	2.16	0.04	2.06
interest rate	1.02	0.67	1.02	0.67	1.02	0.70	1.02	0.63
net transfer	-0.60	4.88	-0.60	4.88	-0.69	53.49	-0.69	22.74
corr(y,NT)	-0.40		-0.40		0.17		-0.03	
welfare cost, %								
(rational)	2.76		2.76		2.94		2.93	
(H2m)	9.08		9.08		28.50		18.96	

note: See the note in Table 4

This also explains why the welfare gain from the use of NGDP-indexed bond gets larger as ω grows.[23]

Another key assumption in our model is that the government should keep its debt-to-GDP ratio at a constant level, and we assumed that this ratio is 252% of quarterly GDP (or 63% of annual GDP) from the U.S. data. Table 6 shows how the baseline results are altered when we apply different debt-to-GDP ratios. In the first four columns, we assume that the government keeps no debt at all times ($\overline{D}=0.0$), and the next four columns show the simulation results when the ratio is 90% of annual output ($\overline{D}=3.6$).

When the government keeps no outstanding debt, most of the variables become more stable than the baseline model. This is because the existence of positive debt plays a role in making fiscal policy more pro-cyclical as seen from Equation (4.1) and (4.2). When $\overline{D}=0.0$, the two equations collapse into $NT_t=-\varepsilon_t^g y_*$, and net transfer becomes strongly negatively correlated with output. This allows the hand-to-mouth households to have more stable consumption and

[23]Even though we did not mention in this paper, the assumption on how the government transfer is distributed between the two groups can also affect the results. Cogan et al. (2010) have showed that the government spending multiplier gets larger when the rule-of thumb households get more fraction of government transfer.

Table 7: **More flexible debt rule**

	Baseline				Flexible-debt-rule			
	Conv.		NGDP		Conv.		NGDP	
	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
output	3.53	3.12	3.54	2.98	3.54	2.88	3.54	2.87
consumption	2.20	2.85	2.21	2.68	2.21	2.52	2.21	2.51
(Ricardian)	2.25	2.59	2.25	2.48	2.26	2.40	2.26	2.39
(H2M)	1.76	7.54	1.77	6.30	1.76	4.73	1.77	4.62
inflation	0.70	0.64	0.71	0.63	0.69	0.64	0.70	0.64
labour supply	1.28	2.36	1.28	2.16	1.29	2.01	1.29	1.99
(Ricardian)	1.27	2.79	1.27	2.41	1.27	2.18	1.27	2.15
(H2m)	1.46	5.84	1.45	3.59	1.45	1.41	1.45	1.51
capital supply	27.12	4.01	27.15	3.91	27.19	3.77	27.18	3.76
wage	1.66	3.36	1.66	3.32	1.66	3.22	1.66	3.21
rent rate	0.04	2.08	0.04	2.01	0.04	1.93	0.04	1.92
interest rate	1.02	0.69	1.02	0.64	1.02	0.67	1.02	0.66
net transfer	-0.66	23.21	-0.66	14.78	-0.67	5.80	-0.67	6.96
corr(y,NT)	0.08		-0.10		-0.50		-0.50	
welfare cost, %								
(Ricardian)	2.92		2.89		2.77		2.80	
(H2m)	16.52		13.67		10.60		11.81	

labour path. For the same reason, as $\overline{D}$ becomes higher, it puts more pressure of pro-cyclical fiscal policy, and thus makes the consumption of hand-to-mouth households more volatile. As the use of NGDP-indexed bonds mediates the pressure of conducting pro-cyclical fiscal policy, we may expect more welfare gain when $\overline{D}$ becomes higher.

Lastly, we examined how the baseline results may be affected if we relax the assumption of constant debt-to-GDP ratio. To see this, we replaced the constant debt-to-GDP rule in the baseline model with the flexible-debt-rule of Equation (3.1) in Section 3.

Table 7 compares the two cases: flexible-debt-rule model and the baseline model. We can see that the flexible-debt-rule significantly stabilises the consumption and labour of hand-to-mouth households even with conventional bonds. Under the flexible-debt-rule, the government still needs to adjust its transfer in response to the shocks that affect debt-to-GDP ratio, but as there is a leeway allowed in the debt-to-GDP ratio, the pressure of pro-cyclical fiscal policy can be much smaller than the constant debt-to-GDP case. This directly leads to more stable consumption path for hand-to-mouth households. In the meantime, as there is much smaller pressure of pro-cyclical fiscal policy under the flexible-debt-rule, the business cycle stabilising effects of NGDP-indexed bonds also becomes smaller (or disappear), and so does the welfare gain. This shows that our results rely highly on the assumption of the constant debt-to-GDP ratio.

5 Conclusion and Summary

In this paper, we examined how a government can use NGDP-indexed bonds as an alternative fiscal policy tool when it is constrained to keep a constant debt-to-GDP within the New Keynesian framework. As Ricardian equivalence holds in the standard New Keynesian DSGE models, the assumption of constant debt-to-GDP is irrelevant to the business cycle in such models. However, when a fraction of the population is non-Ricardian, the constant debt-to-GDP assumption plays a role of making fiscal policy more pro-cyclical, and this makes the disposable income of non-Ricardian households very volatile. Since they are not able to smooth consumption intertemporally, their consumption becomes very volatile as well. Under this situation, NGDP-indexed bonds can play a role of an automatic stabiliser. That is to say, the use of NGDP-indexed bonds mitigates the pressure of pro-cyclical fiscal policy and helps stabilise the consumption of non-Ricardian households. This may increase the welfare of non-Ricardian households as well.

In addition, in contrast to the previous papers with the presence of non-Ricardian households, we assume that the hand-to-mouth households in our model have a desire for consumption smoothing and do it at least intratemporally. For this reason, the use of NGDP-indexed bonds stabilises not only the consumption of hand-to-mouth households, but also their supply of labour. Moreover, as the labour supply of the two group of households are closely interconnected through the labour market, the labour supply from the Ricardian households is stabilised as well. To sum up, the government with restricted fiscal policy tools can rely on NGDP-indexed bonds to stabilise business cycle and improve the welfare of at least a part of the households without damaging the others. We also showed that the larger benefits can be obtained in an economy with a larger share of hand-to-mouth households, a higher level of debt-to-GDP ratio, and when the business cycle is mainly driven by demand shocks.

One may point out several shortcomings of the analysis in this paper. One of them is the fact that the conclusion of this paper is strongly dependent on the assumption of constant debt-to-GDP ratio. In fact, we also showed that the benefits have disappeared in the model with more relaxed fiscal policy rule. Therefore, our results should not be interpreted that the government can benefit from the use of NGDP-indexed bonds unconditionally. Nevertheless, as many advanced countries are actually approaching their debt limits as Ostry et al. (2010) shows, it may be reasonable to consider NGDP-indexed bonds as part of their fiscal policy tools.

We want to close this paper by discussing a few model extensions for the future. In this paper, our model does not explicitly include the possibility of default. If there exists an endogenous mechanism through which a rise in debt-to-GDP ratio raises the probability of default and related risk premium, we can have a vicious cycle in which a positive shock to debt-to-GDP ratio raises the government's overall borrowing costs and further increases its debt-to-GDP ratio. When such a mechanism is included to the model, we may expect a lot larger benefits from the use of NGDP-indexed bonds as suggested by the previous papers (Chamon and Mauro, 2006; Ostry et al., 2010; Barr et al., 2014; Kim and Ostry, 2018).

Another shortcoming we acknowledge is that our model is a closed economy model and calibrated with the U.S. macroeconomic data which is believed to have little or no possibility of government default. Therefore, the analyses and results presented can be extended only to a set of advanced economies. By extending the model to a small open economy model and explicitly incorporating foreign currency denominated debts, we may be able to discuss the benefits of issuing NGDP-indexed bonds to the emerging market countries as well.

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Appendix

A List of detrended non-linear equilibrium conditions

- Production sector[24]

$$\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_t}{r_t^k}\right)=\left(\frac{z_t k_{t-1}/\gamma}{l_t}\right) \tag{A.1}$$

$$mc_t=\frac{1}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\varepsilon_t^a}\left[\left(w_t\right)^{1-\alpha}\left(r_t^k\right)^{\alpha}\right] \tag{A.2}$$

- Price setting[25]

$$g_t^1 = \hat{\Pi}_t^{-\frac{1}{\lambda_p}} y_t+\zeta_p E_t\left[M_{t,t+1}^R\gamma\left(\frac{\hat{\Pi}_t}{\hat{\Pi}_{t+1}}\right)^{-\frac{1}{\lambda_p}}\left(\frac{\Pi_t^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1}{\lambda_p}} g_{t+1}^1\right] \tag{A.3}$$

$$g_t^2 = \varepsilon_t^p\hat{\Pi}_t^{-\frac{1+\lambda_p}{\lambda_p}} y_t\cdot mc_t+\zeta_p E_t\left[M_{t,t+1}^R\gamma\left(\frac{\hat{\Pi}_t}{\hat{\Pi}_{t+1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}}\left(\frac{\Pi_t^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_{t+1}}\right)^{-\frac{1+\lambda_p}{\lambda_p}} g_{t+1}^2\right] \tag{A.4}$$

$$g_t^1 = \left(1+\lambda_p\right)g_t^2 \tag{A.5}$$

- Law of motion: price

$$1=\left(1-\zeta_p\right)\hat{\Pi}_t^{-\frac{1}{\lambda_p}}+\zeta_p\left(\frac{\Pi_{t-1}^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1}{\lambda_p}} \tag{A.6}$$

- Law of motion: capital[26]

$$k_t=\frac{(1-\delta)}{\gamma}k_{t-1}+\varepsilon_t^I\left[1-\frac{\phi}{2}\left(\frac{i_t\gamma}{i_{t-1}}-\gamma\right)^2\right]i_t \tag{A.7}$$

[24] $w_t \equiv \frac{W_t}{P_t\gamma^t}$, $r_t^k \equiv \frac{R_t^k}{P_t}$, $k_t \equiv \frac{K_t}{\gamma^t}$, $mc_t \equiv \frac{MC_t}{P_t}$

[25] $y_t \equiv \frac{Y_t}{\gamma^t}$, $\hat{\Pi}_t \equiv \frac{\hat{P}_t}{P_t}$

[26] $i_t \equiv \frac{I_t}{\gamma^t}$

- Value function and period utility[27]

$$u_t^R = \varepsilon_t^b \left[\frac{1}{1-\sigma_c}\left(c_t^R - \frac{\lambda}{\gamma}c_{t-1}^R\right)^{1-\sigma_c}\right]\exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^R\right)^{1+\sigma_l}\right] \tag{A.8}$$

$$u_t^H = \varepsilon_t^b \left[\frac{1}{1-\sigma_c}\left(c_t^H - \frac{\lambda}{\gamma}c_{t-1}^H\right)^{1-\sigma_c}\right]\exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^H\right)^{1+\sigma_l}\right] \tag{A.9}$$

$$v_t^R = u_t^R + \overline{\beta}\gamma E_t\left[\left(v_{t+1}^R\right)^{1-\sigma_{EZ}}\right]^{\frac{1}{1-\sigma_{EZ}}} \tag{A.10}$$

$$v_t^H = u_t^H + \overline{\beta}\gamma E_t\left[\left(v_{t+1}^H\right)^{1-\sigma_{EZ}}\right]^{\frac{1}{1-\sigma_{EZ}}} \tag{A.11}$$

- First order conditions: Ricardian households[28]

$$\lambda_t^R = \varepsilon_t^b \left(c_t^R - \frac{\lambda}{\gamma}c_{t-1}^R\right)^{-\sigma_c}\exp\left[\frac{\sigma_c - 1}{1+\sigma_l}\left(h_t^R\right)^{1+\sigma_l}\right] \tag{A.12}$$

$$\frac{1}{R_t} = E_t\left[M_{t,t+1}^R \Pi_{t+1}^{-1}\right] \tag{A.13}$$

$$Q_t^G = E_t\left[M_{t,t+1}^R \frac{y_{t+1}\gamma}{y_t}\right] \tag{A.14}$$

$$\left(1-\tau_w\right)\overline{w}_t = \left(c_t^R - \frac{\lambda}{\gamma}c_{t-1}^R\right)\left(h_t^R\right)^{\sigma_l} \tag{A.15}$$

$$\begin{aligned} 1 &= q_t\varepsilon_t^I\left[1 - \frac{\phi}{2}\left(\frac{i_t\gamma}{i_{t-1}} - \gamma\right)^2 - \phi\left(\frac{i_t\gamma}{i_{t-1}} - \gamma\right)\frac{i_t\gamma}{i_{t-1}}\right] + \\ &\quad E_t\left[M_{t,t+1}^R q_{t+1}\varepsilon_{t+1}^I \phi\left(\frac{i_{t+1}\gamma}{i_t} - \gamma\right)\left(\frac{i_{t+1}\gamma}{i_t}\right)^2\right] \end{aligned} \tag{A.16}$$

$$q_t = E_t\left[M_{t,t+1}^R\left\{\begin{array}{c}\left(1-\tau_r\right)r_t^k z_t + \delta\tau_r - \delta_1\left(z_{t+1}-1\right) \\ -\frac{\delta_2}{2}\left(z_{t+1}-1\right)^2 + q_{t+1}\left(1-\delta\right)\end{array}\right\}\right] \tag{A.17}$$

$$\left(1-\tau_r\right)r_t^k = \delta_1 + \delta_2\left(z_t - 1\right) \tag{A.18}$$

[27] $u_t \equiv \frac{U_t}{\gamma^{t(1-\sigma_c)}}$, $v_t \equiv \frac{V_t}{\gamma^{t(1-\sigma_c)}}$, $c_t \equiv \frac{C_t}{\gamma^t}$, $\overline{\beta} \equiv \beta\gamma^{-\sigma_c}$

[28] $\lambda_t^R \equiv \Xi_t^R \gamma^{\sigma_c t}$, $\overline{w}_t \equiv \frac{\overline{W}_t}{P_t\gamma^t}$

- First order conditions: hand-to-mouth households

$$\lambda_t^H = \varepsilon_t^b \left(c_t^H - \frac{\lambda}{\gamma} c_{t-1}^H\right)^{-\sigma_c} \exp\left[\frac{\sigma_c - 1}{1+\sigma_l} \left(h_t^H\right)^{1+\sigma_l}\right] \tag{A.19}$$

$$(1-\tau_w)\,\overline{w}_t = \left(c_t^H - \frac{\lambda}{\gamma} c_{t-1}^H\right) \left(h_t^H\right)^{\sigma_l} \tag{A.20}$$

- Budget constraint of hand-to-mouth households[29]

$$\left(c_t^H - t_t\right) = (1-\tau_w)\left(\overline{w}_t h_t^H + \underbrace{w_t l_t - \omega \overline{w}_t h_t^H - (1-\omega)\,\overline{w}_t h_t^R}_{D_t^u:\text{ union profit}}\right) \tag{A.21}$$

- Wage setting[30]

$$f_t^1 = l_t \left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \hat{w}_t + \zeta_w E_t \left\{ M_{t,t+1}^R \gamma \left(\frac{\hat{w}_t}{\hat{w}_{t+1}}\right)^{-\frac{1}{\lambda_w}} \left(\frac{\Pi_t^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1}{\lambda_w}} f_{t+1}^1 \right\} \tag{A.22}$$

$$f_t^2 = \varepsilon_t^w l_t \left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \overline{w}_t + \zeta_w E_t \left\{ M_{t,t+1}^R \gamma \left(\frac{\hat{w}_t}{\hat{w}_{t+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} \left(\frac{\Pi_t^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_{t+1}}\right)^{-\frac{1+\lambda_w}{\lambda_w}} f_{t+1}^2 \right\} \tag{A.23}$$

$$f_t^1 = (1+\lambda_w)\, f_t^2 \tag{A.24}$$

- Law of motion: wage

$$(w_t)^{-\frac{1}{\lambda_w}} = (1-\zeta_w)\,(\hat{w}_t)^{-\frac{1}{\lambda_w}} + \zeta_w \left(\frac{\Pi_{t-1}^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_t} w_{t-1}\right)^{-\frac{1}{\lambda_w}} \tag{A.25}$$

- Monetary policy rule

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_*}\right)^{\psi_1} \left(\frac{y_t}{y_*}\right)^{\psi_2}\right]^{1-\rho_R} \left(\frac{y_t}{y_{t-1}}\right)^{\psi_3} \varepsilon_t^r \tag{A.26}$$

- Government budget constraint

$$\varepsilon_t^g y_* + t_t + \frac{b_{t-1}}{\Pi_t \gamma} + \frac{b_{t-1}^G y_t}{y_{t-1}} \tag{A.27}$$

$$= \frac{b_t}{R_t} + Q_t^G b_t^G + \tau_w w_t l_t + \tau_r z_t r_t^k \frac{k_{t-1}}{\gamma} - \tau_r \delta \frac{k_{t-1}}{\gamma}$$

[29] $t_t \equiv \frac{T_t}{\gamma^t}$

[30] $\hat{w}_t \equiv \frac{\hat{W}_t}{P_t \gamma^t}$

- Debt rules[31]

$$
\begin{aligned}
b_t/R_t &= \left(1-\omega^G\right)\overline{D}y_t && \text{(A.28)}\\
Q_t^G b_t^G &= \omega^G \overline{D} y_t && \text{(A.29)}
\end{aligned}
$$

- Aggregate consumption

$$
c_t = (1-\omega)\, c_t^R + \omega c_t^H \tag{A.30}
$$

- Aggregate resource constraint

$$
y_t = c_t + i_t + \varepsilon_t^g y_* + \underbrace{\left\{\delta_1\left(z_t-1\right)+\frac{\delta_2}{2}\left(z_t-1\right)^2\right\}}_{=a(z_t)}\frac{k_{t-1}}{\gamma} \tag{A.31}
$$

- Market clearing condition: final goods

$$
y_t = \frac{\varepsilon_t^a\left(\frac{z_t k_{t-1}}{\gamma}\right)^{\alpha}\left(l_t\right)^{1-\alpha} - y_*\left(\phi_p - 1\right)}{s_t^p} \tag{A.32}
$$

- Law of motion: price dispersion

$$
s_t^p = \left(1-\zeta_p\right)\left(\hat{\Pi}_t\right)^{-\frac{1+\lambda_p}{\lambda_p}} + \zeta_p\left(\frac{\Pi_{t-1}^{\iota_p}\Pi_*^{1-\iota_p}}{\Pi_t}\right)^{-\frac{1+\lambda_p}{\lambda_p}} s_{t-1}^p \tag{A.33}
$$

- Market clearing condition: labour

$$
\omega h_t^H + (1-\omega)\, h_t^R = s_t^w l_t \tag{A.34}
$$

- Law of motion: wage dispersion

$$
s_t^w = \left(1-\zeta_w\right)\left(\frac{\hat{w}_t}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} + \zeta_w\left(\frac{\Pi_{t-1}^{\iota_w}\Pi_*^{1-\iota_w}}{\Pi_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}}\left(\frac{w_{t-1}}{w_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} s_{t-1}^w \tag{A.35}
$$

[31] $b_t \equiv \frac{B_t}{P_t\gamma^t}$, $b_t^G \equiv \frac{B_t^G}{P_t\gamma^t}$

- Shock processes

$$\begin{aligned}
\log \varepsilon_t^a &= \rho_a \log \varepsilon_t^a + \eta_t^a && \text{(A.36)}\\
\log \varepsilon_t^b &= \rho_b \log \varepsilon_t^b + \eta_t^b && \text{(A.37)}\\
\log \left(\frac{\varepsilon_t^g}{\varepsilon_*^g}\right) &= \rho_g \log \left(\frac{\varepsilon_{t-1}^g}{\varepsilon_*^g}\right) + \eta_t^g + \rho_{ga}\eta_t^a && \text{(A.38)}\\
\log \varepsilon_t^i &= \rho_i \log \varepsilon_t^i + \eta_t^i && \text{(A.39)}\\
\log \varepsilon_t^r &= \rho_r \log \varepsilon_t^r + \eta_t^r && \text{(A.40)}\\
\log \varepsilon_t^p &= \rho_p \log \varepsilon_t^p + \eta_t^p && \text{(A.41)}\\
\log \varepsilon_t^w &= \rho_w \log \varepsilon_t^w + \eta_t^w && \text{(A.42)}
\end{aligned}$$

B Steady states

- $z_* = 1$ is assumed and Π_* is an exogenously given parameter.
- The following steady state conditions are analytically given with pencil and paper:

$\hat{\Pi}_* = q_* = s_*^p = s_*^w = 1$

$r_*^k = \frac{\left(\overline{\beta}\right)^{-1} - \delta\tau_r - (1-\delta)}{1-\tau_r}$

$mc_* = 1/\left(1+\lambda_p\right)$

$w_* = (1-\alpha)\left(mc_*\left(\frac{\alpha}{r_*^k}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}$

$\overline{w}_* = w_*/\left(1+\lambda_w\right)$

$\hat{w}_* = w_*$

$\left(\frac{k_*}{l_*}\right) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_*}{r_*^k}\right)\gamma$

$\frac{i_*}{k_*} = \frac{\gamma - 1 + \delta}{\gamma}$

$R_* = \frac{\Pi_*}{\overline{\beta}}$

$Q_*^G = \overline{\beta}\gamma$

$\frac{y_*}{k_*} = \left(\frac{k_*}{l_*}\right)^{\alpha-1}\gamma^{-\alpha}\phi_p^{-1}$

$\frac{c_*}{y_*} = 1 - \frac{i_*}{k_*}\frac{k_*}{y_*} - \varepsilon_*^g$

$\frac{b_*}{y_*} = \left(1-\omega^G\right)\overline{D}R_*$

$\frac{b_*^G}{y_*} = \omega^G\overline{D}/Q_*^G$

$\frac{t_*}{y_*} = \frac{b_*}{y_*}\left(\frac{1}{R_*} - \frac{1}{\Pi_*\gamma}\right) + \frac{b_*^G}{y_*}\left(Q_*^G - 1\right) + \tau_w\left(\frac{k_* r_*^k}{y_*\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right) + \tau_r\frac{\left(r_*^k - \delta\right)}{\gamma}\frac{k_*}{y_*} - \varepsilon_*^g$

- We get $\frac{c_*^H}{y_*}, \frac{c_*^R}{y_*}, h_*^H, h_*^R, l_*$ numerically from the following five equations:

  - From Equation (A.15) and (A.20):

$$\left(\frac{1-\tau_w}{1+\lambda_w}\right)\frac{w_* l_*}{y_*} = \left(1-\frac{\lambda}{\gamma}\right)\frac{c_*^R}{y_*}\left(h_*^R\right)^{\sigma_l} l_*$$

  and

$$\left(\frac{1-\tau_w}{1+\lambda_w}\right)\frac{w_* l_*}{y_*} \quad = \quad \left(1-\frac{\lambda}{\gamma}\right)\frac{c_*^H}{y_*}\left(h_*^H\right)^{\sigma_l} l_*.$$

  - From Equation (A.30):

$$\frac{c_*}{y_*} = \omega\frac{c_*^H}{y_*} + (1-\omega)\frac{c_*^R}{y_*}.$$

  - From Equation (A.21):

$$\left(\frac{c_*^H}{y_*} - \frac{t_*}{y_*}\right) = (1-\tau_w)\left(\frac{w_* l_*}{y_*}\right)\left\{1 + \frac{(1-\omega)\left(h_*^H - h_*^R\right)}{(1+\lambda_w)\, l_*}\right\}.$$

  - From Equation (A.34):

$$l_* = \omega h_*^H + (1-\omega)\, h_*^R$$

- Then, we can find the rest of the steady state conditions analytically as well:

  $y_* = \frac{w_* l_*}{\left(\frac{w_* l_*}{y_*}\right)}$

  $k_* = y_*\left(\frac{k_*}{y_*}\right)$

  $i_* = k_*\left(\frac{i_*}{k_*}\right)$

  $t_* = y_*\left(\frac{t_*}{y_*}\right)$

  $b_* = y_*\left(\frac{b_*}{y_*}\right)$

  $b_*^G = y_*\left(\frac{b_*^G}{y_*}\right)$

  $c_* = y_*\left(\frac{c_*}{y_*}\right)$

  $c_*^R = y_*\left(\frac{c_*^R}{y_*}\right)$

  $c_*^H = y_*\left(\frac{c_*^H}{y_*}\right)$

  $g_*^1 = \frac{y_*}{\left(1-\zeta_p\overline{\beta}\gamma\right)}$

  $g_*^2 = \frac{y_* mc_*}{\left(1-\zeta_p\overline{\beta}\gamma\right)}$

  $f_*^1 = \frac{l_* w_*}{\left(1-\zeta_w\overline{\beta}\gamma\right)}$

$$f_*^2 = \frac{l_* \overline{w}_*}{\left(1-\zeta_w \overline{\beta}\gamma\right)}$$

$$u_*^R = \left[\frac{1}{1-\sigma_c}\left(c_*^R - \frac{\lambda}{\gamma}c_*^R\right)^{1-\sigma_c}\right]\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\left(h_*^R\right)^{1+\sigma_l}\right)$$

$$u_*^H = \left[\frac{1}{1-\sigma_c}\left(c_*^H - \frac{\lambda}{\gamma}c_*^H\right)^{1-\sigma_c}\right]\exp\left(\frac{\sigma_c-1}{1+\sigma_l}\left(h_*^H\right)^{1+\sigma_l}\right)$$

$$v_*^R = \frac{u_*^R}{1-\overline{\beta}^R\gamma}$$

$$v_*^H = \frac{u_*^H}{1-\overline{\beta}^H\gamma}$$

$$\lambda_*^R = \left(c_*^R - \frac{\lambda}{\gamma}c_*^R\right)^{-\sigma_c}\exp\left[\frac{\sigma_c-1}{1+\sigma_l}\left(h_*^R\right)^{1+\sigma_l}\right]$$

$$\lambda_*^H = \left(c_*^H - \frac{\lambda}{\gamma}c_*^H\right)^{-\sigma_c}\exp\left[\frac{\sigma_c-1}{1+\sigma_l}\left(h_*^H\right)^{1+\sigma_l}\right]$$

C Figures

Figure 1: **IRFs for benchmark model (I)**

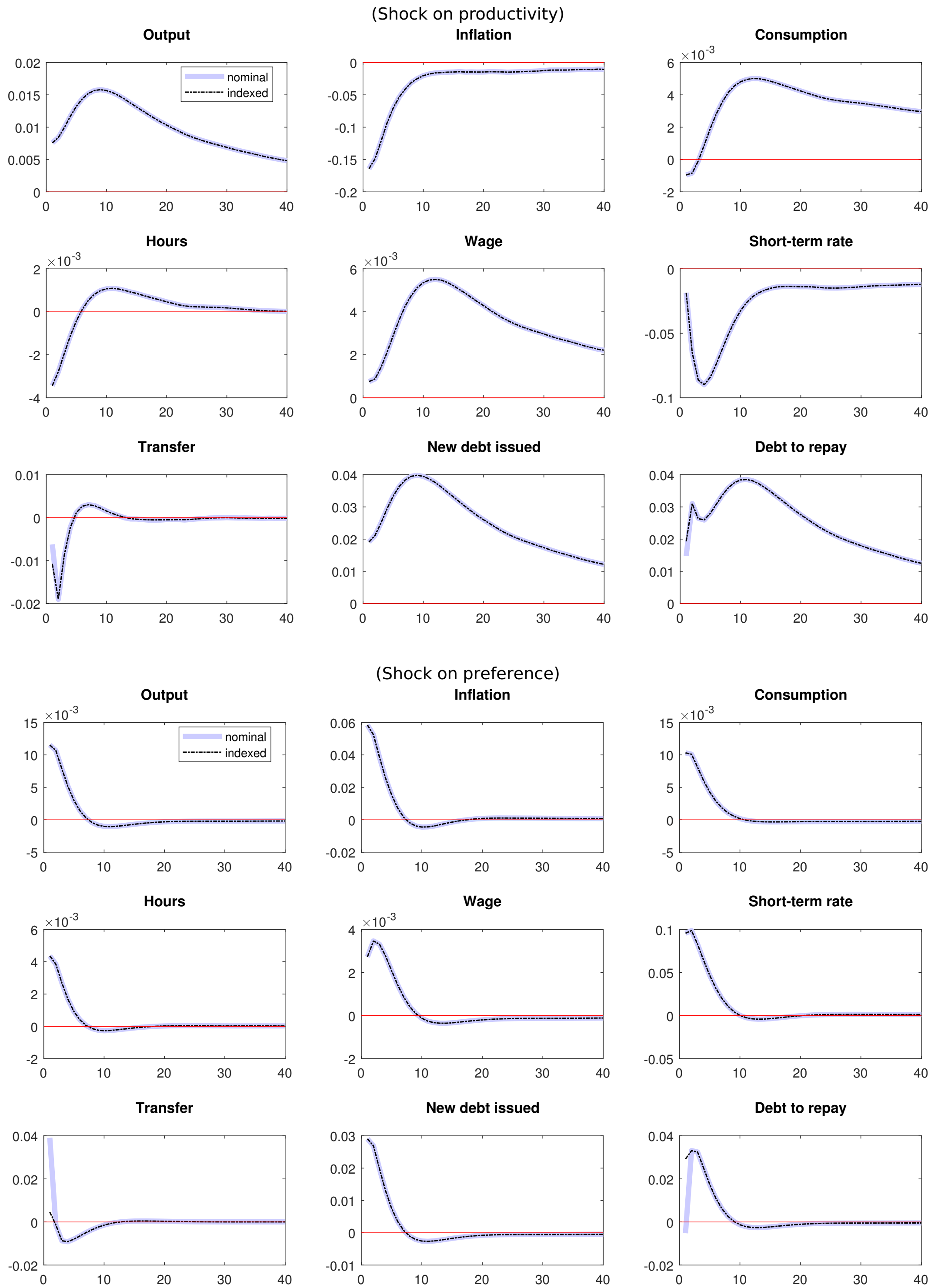
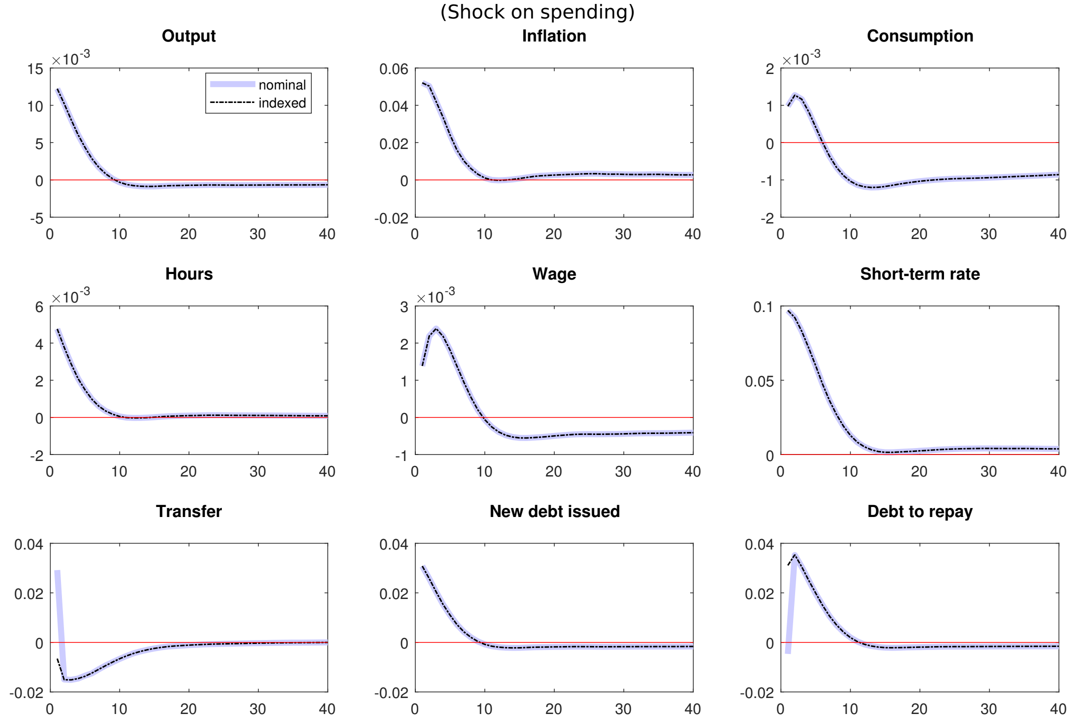


Figure 1: **IRFs for benchmark model (II)**

(Shock on spending)



(Shock on investment)

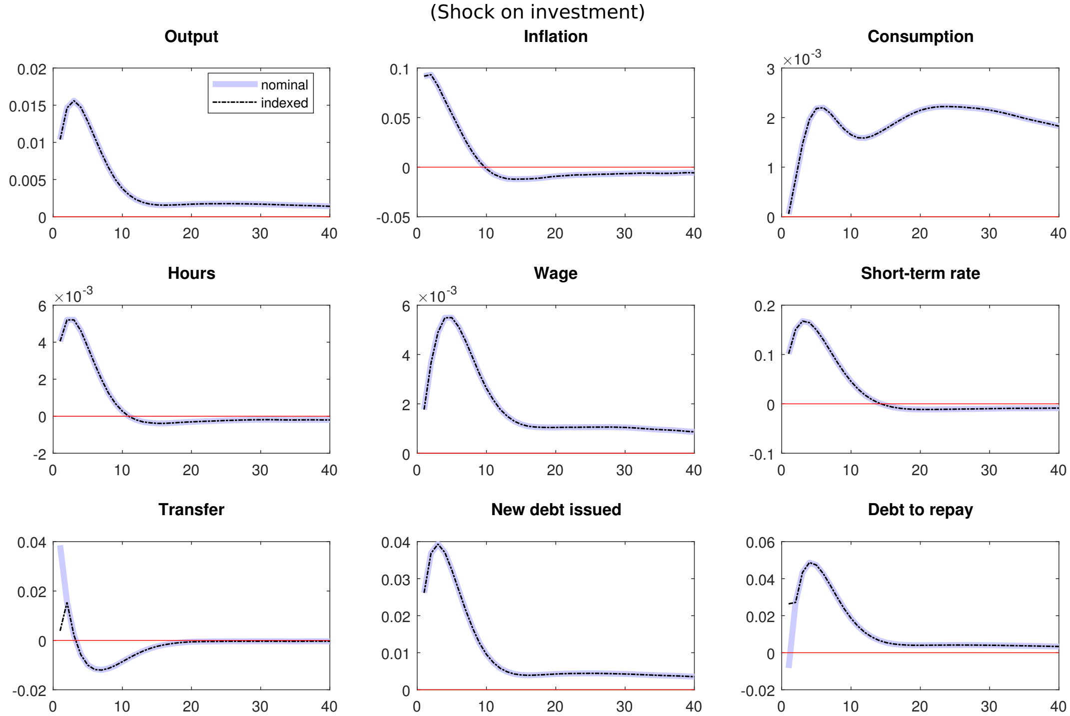
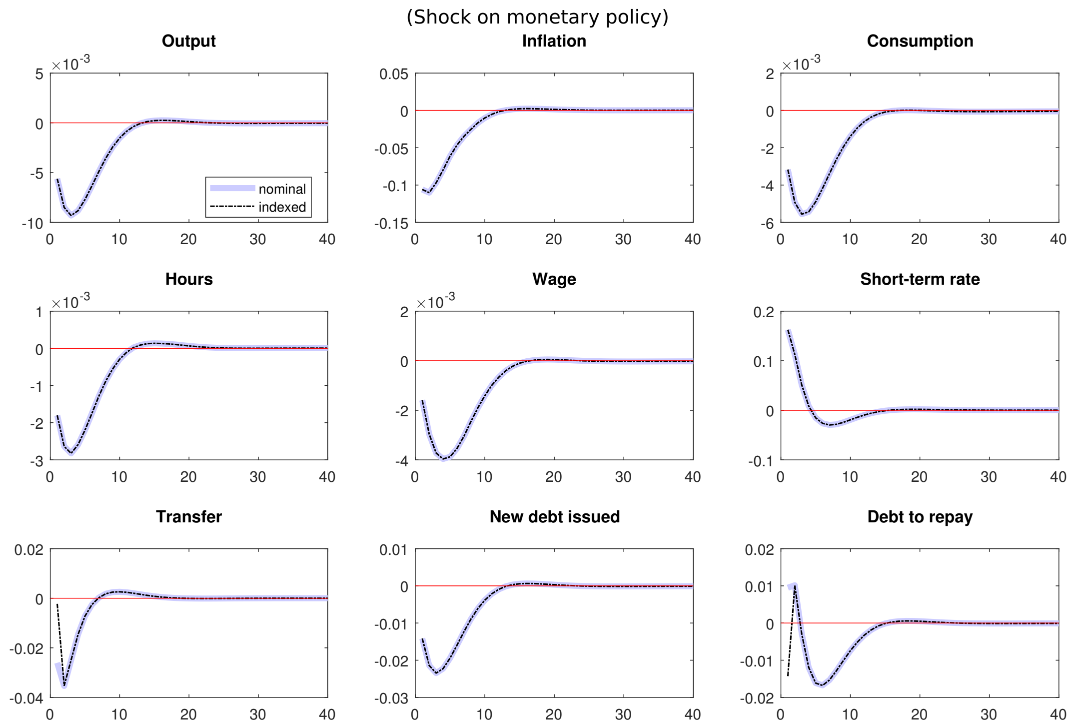


Figure 1: **IRFs for benchmark model (III)**



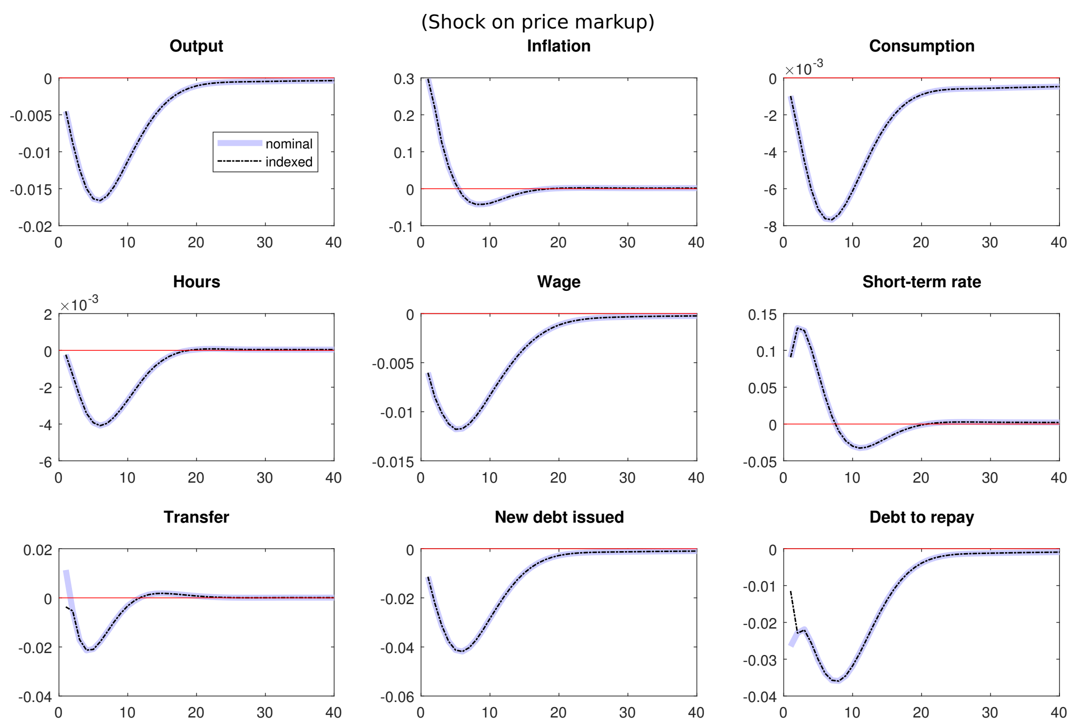


Figure 1: **IRFs for benchmark model (IV)**

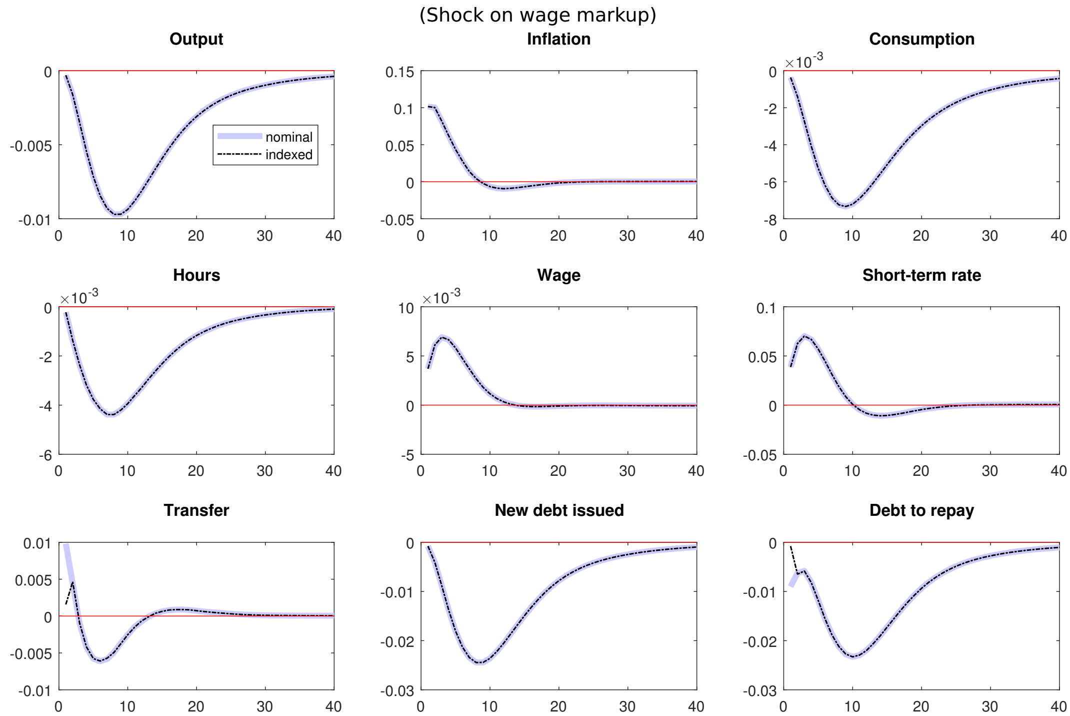


Figure 2: **IRFs for baseline model (I)**

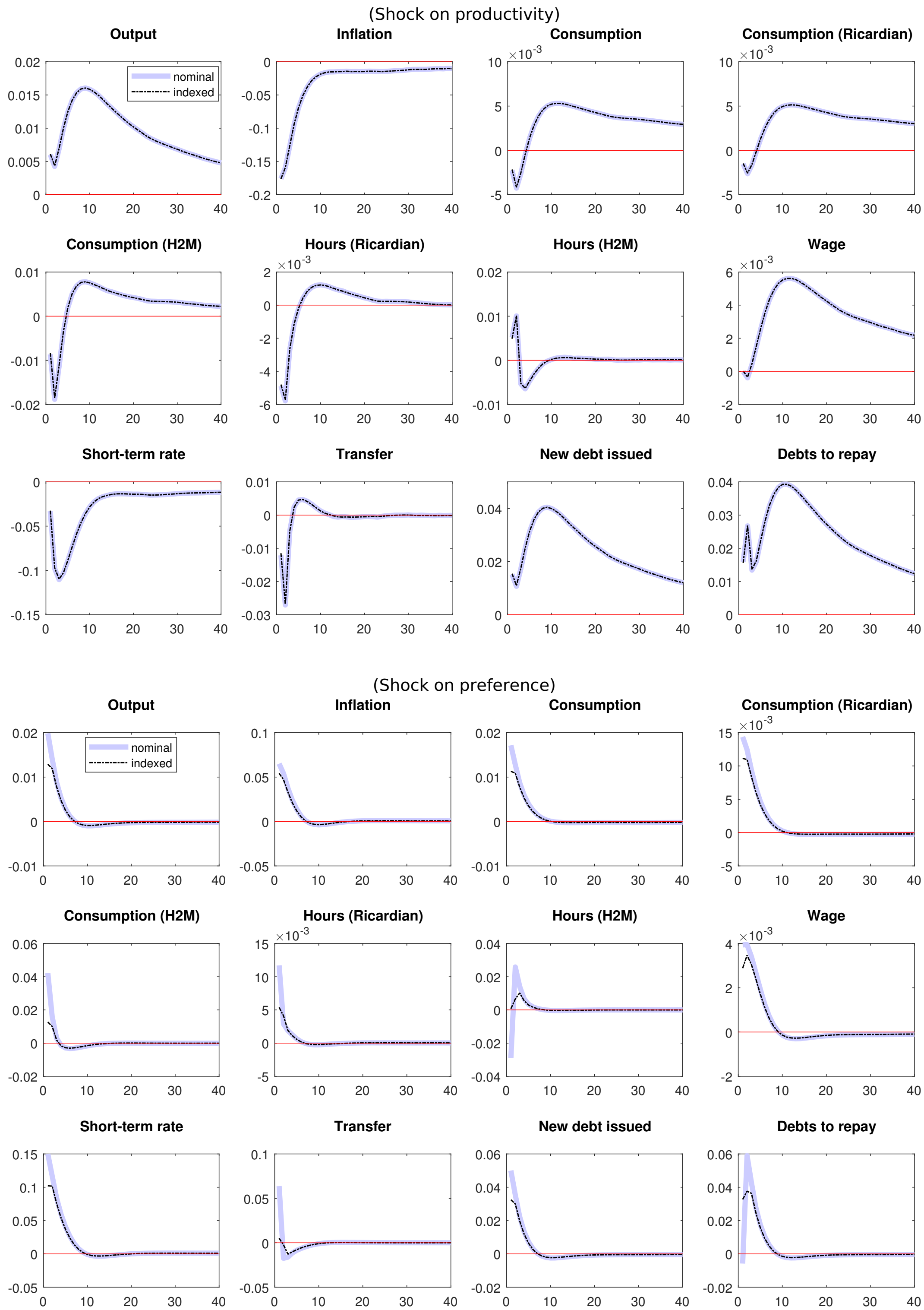


Figure 2: **IRFs for baseline model (II)**

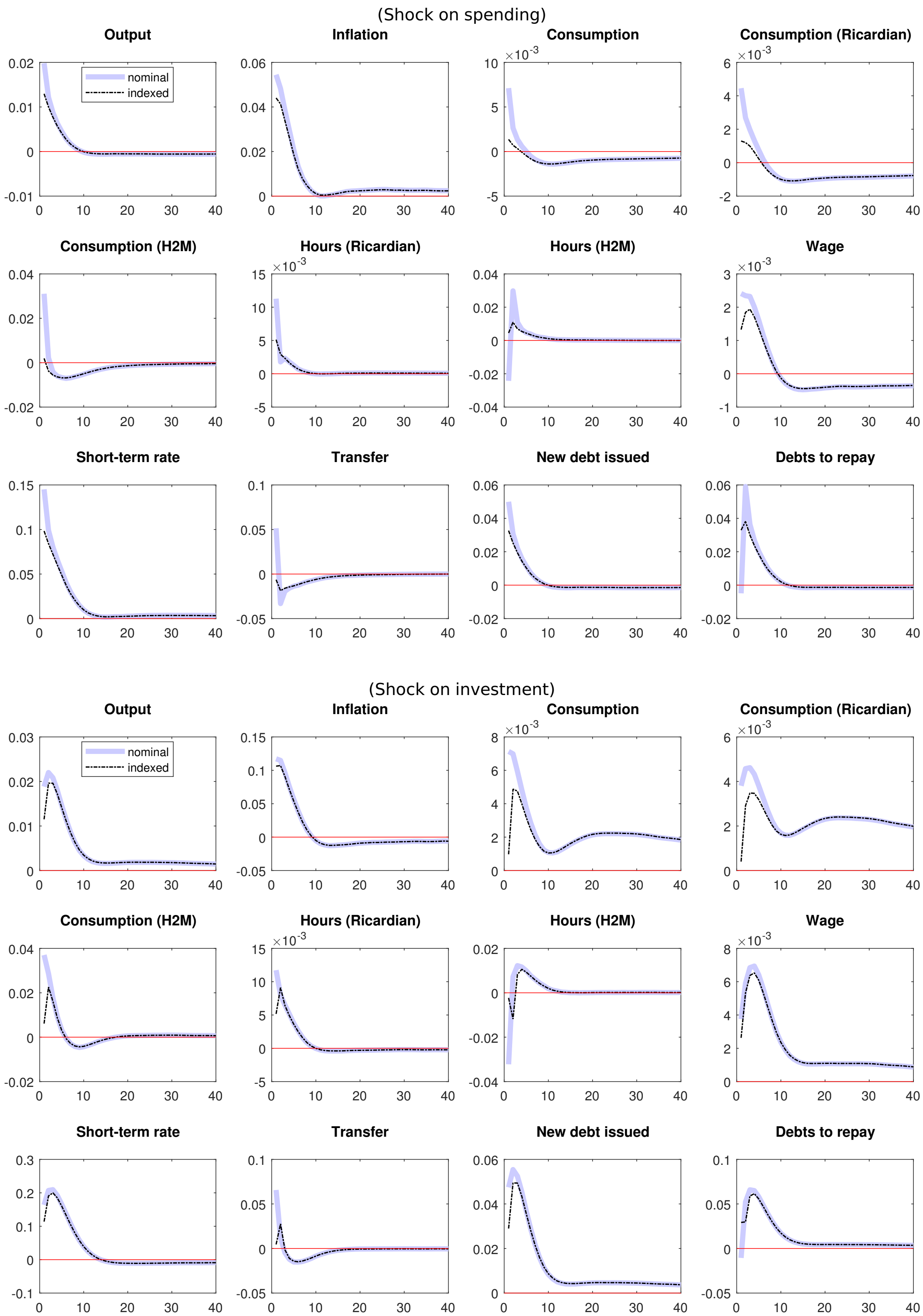
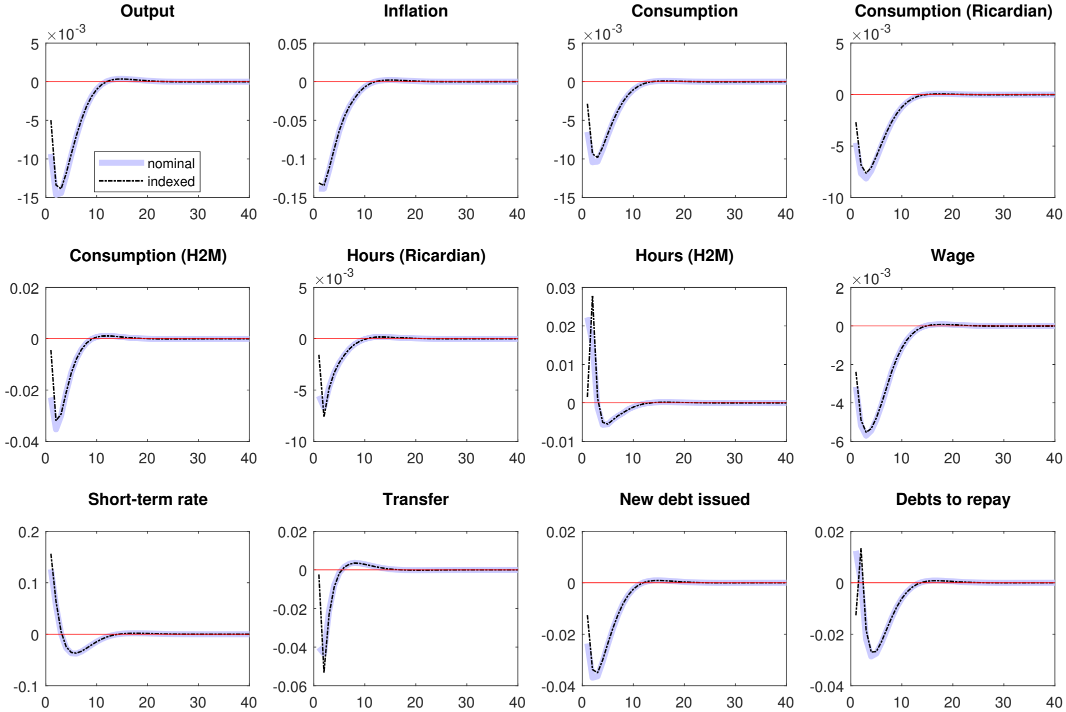


Figure 2: **IRFs for baseline model (III)**

(Shock on monetary policy)



(Shock on price markup)

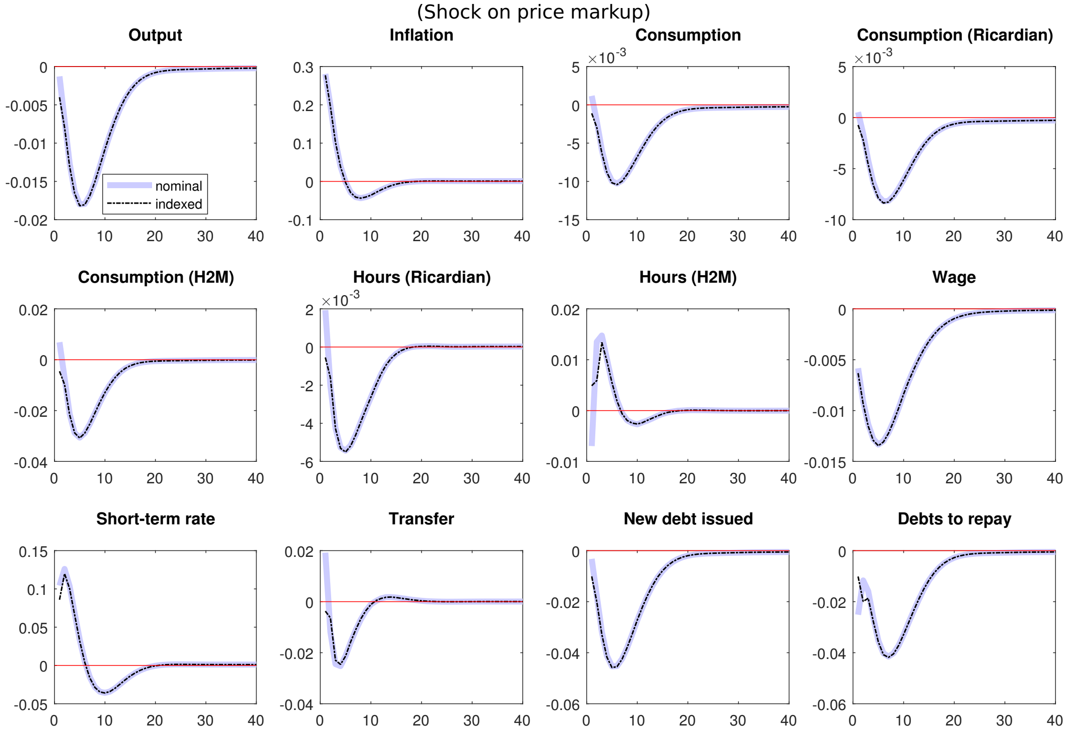


Figure 2: **IRFs for baseline model (IV)**

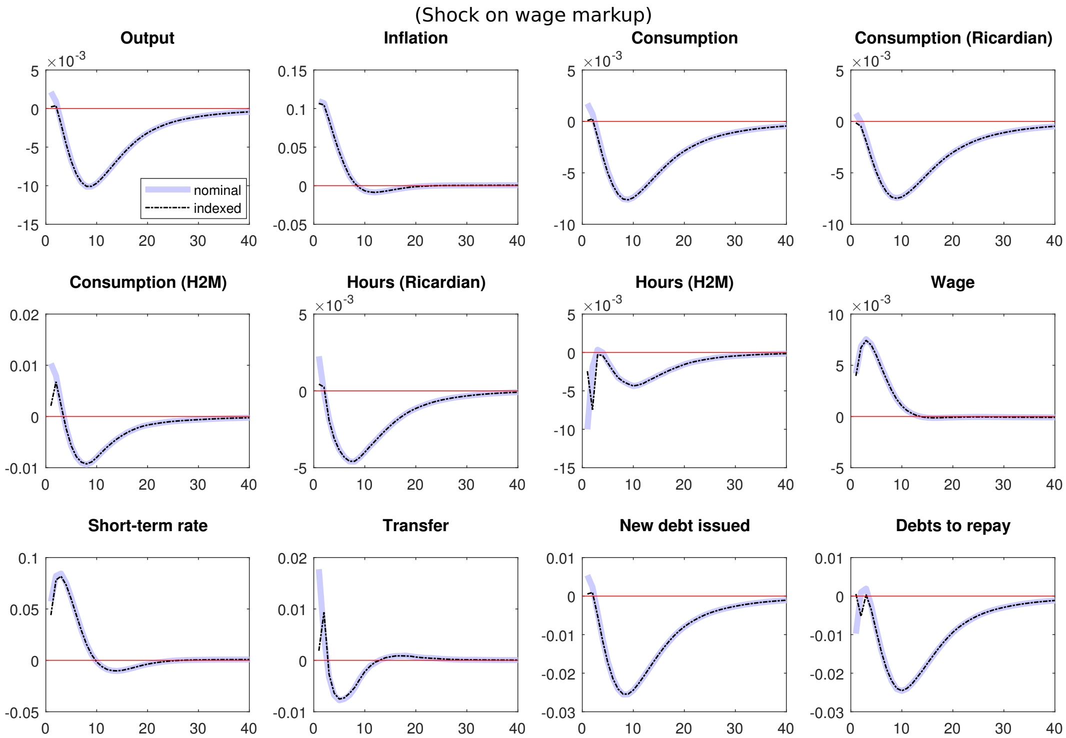


Figure 3: **Destabilising effect from the presence of hand-to-mouth households**