National Institute of Economic and Social Research

THE INDETERMINACY AGENDA IN MACROECONOMICS

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Abstract

This article surveys a subset of literature in macroeconomics which embraces the existence of multiple equilibria. This indeterminacy agenda in macroeconomics uses multiple-equilibrium models to integrate economics with psychology. Economists have long argued that business cycles are driven by shocks to the productivity of labour and capital. According to the indeterminacy agenda, the self-fulfilling beliefs of financial market participants are additional fundamental factors that drive periods of prosperity and depression. The indeterminacy agenda provides a microeconomic foundation to Keynes' General Theory that does not rely on the assumption that prices and wages are costly to change.

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The Indeterminacy Agenda in Macroeconomics[1]

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Abstract

This article surveys a subset of literature in macroeconomics which embraces the existence of multiple equilibria. This ***indeterminacy agenda in macroeconomics*** uses multiple-equilibrium models to integrate economics with psychology. Economists have long argued that business cycles are driven by shocks to the productivity of labour and capital. According to the indeterminacy agenda, the *self-fulfilling beliefs* of financial market participants are additional fundamental factors that drive periods of prosperity and depression. The indeterminacy agenda provides a microeconomic foundation to Keynes' General Theory that does not rely on the assumption that prices and wages are costly to change.

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Introduction

This article is about a research agenda in macroeconomics which embraces the existence of multiple equilibria. This ***indeterminacy agenda in macroeconomics*** uses multiple-equilibrium models to integrate economics with psychology. Economists have long argued that business cycles are driven by shocks to the productivity of labour and capital. According to the indeterminacy agenda, the *self-fulfilling beliefs* of financial market participants are additional fundamental factors that drive periods of prosperity and depression.

The history of macroeconomics, as a theory distinct from microeconomics, began with the publication of *The General Theory of Employment Interest and Money*, (Keynes, 1936) a book by the English economist John Maynard Keynes. Keynes revolutionised the way economists think about the economy and he revolutionised the way politicians think about the role of economic policy. For the first time, with the publication of *The General Theory*, policy makers accepted that government has an obligation to maintain full employment. The publication of Keynes' master work led to two decades of research that attempted to integrate the ideas of *The General Theory*, with *general equilibrium (GE) theory*, the branch of microeconomics that deals with the working of the economy as a whole. That research led to the publication of *Money Interest and Prices*, a book by Don Patinkin, (Patinkin, 1956) which laid the foundation for much of the research that followed.

In *Money Interest and Prices*, Patinkin integrated general equilibrium theory, which explains the determination of the relative price of one good to another, with the quantity theory of money, which explains the average price of all goods measured in units of money. Patinkin's synthesis led to the development of *monetary general equilibrium* (MGE) theory, an approach that forms the basis for much of modern macroeconomics.

In MGE theory, as with non-monetary versions of GE theory, market outcomes result from the interactions of hundreds of millions of market participants, each of whom assumes that he has no influence over market prices. An equilibrium is a set of equilibrium trades and an equilibrium price vector such that, when confronted with equilibrium prices, no market participant would choose to make additional trades. Although it has been known for decades that there may be more than one equilibrium price vector, much of the literature in macroeconomics that developed from Patinkin's synthesis has made theoretical assumptions that render equilibrium unique.

Multiplicity and determinacy of equilibria

GE theory can be used to develop static GE models that explain the determination of prices and quantities traded at a single date. Or it can be used to develop dynamic GE models that explain the determination of prices and quantities traded at a sequence of dates. Static and dynamic general equilibrium models each have multiple equilibria. When the number of commodities and the number of people are finite, each equilibrium is locally isolated from every other equilibrium. In this case, each equilibrium is said to be *determinate.* When the number of commodities and the number of people is infinite, there may be a contiguous set of equilibria. In this case, each member of the set is said to be *indeterminate*.

Determinacy of equilibrium is an important property if one is interested in comparing how the equilibrium price vector changes in response to a change in economic fundamentals. For example, if the demand curve shifts to the right, what will happen to the equilibrium price of wheat? For this question to have a meaningful answer, the equilibrium price of wheat must be determinate.

Models of dynamic indeterminacy

The indeterminacy agenda in macroeconomics has gone through two phases. The initial phase, developed at the University of Pennsylvania in the United States and at CEPREMAP in France, consisted of dynamic models driven by the self-fulfilling beliefs of economic actors. Initially, these models were populated by overlapping generations of finitely-lived people.[2] The focus soon shifted to infinite horizon models inhabited by an infinitely-lived representative agent in which the technology exhibits increasing returns-to-scale. This literature, surveyed in Benhabib and Farmer (1999), generates equilibria that display *dynamic indeterminacy* and the promise of the literature on dynamic indeterminacy, was that it would provide a micro-foundation for Keynesian economics.[3]

The literature on dynamic indeterminacy made an important contribution to Keynesian economics by demonstrating that 'animal spirits' may be fully consistent with market clearing

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[2] Costas Azariadis (1981), used the term 'self-fulfilling prophecy' to describe this idea. Farmer and Woodford (1984) extended the concept to examples of the kind of equilibria discussed in this survey in which equilibria are randomizations across indeterminate sequences of perfect foresight equilibria. Shell (1977) and Cass and Shell (1983) refer to the phenomena of random allocations driven solely by beliefs as *sunspots*. In Paris, Jean Michel Grandmont (1985) was working on endogenous cycles and Roger Guesnerie collaborated with Azariadis to explore the relationship between sunspots and cycles (Azariadis & Guesnerie, 1986). Other early models that studied self-fulfilling beliefs in overlapping generations models include papers by Stephen Spear (1984) and Spear and Sanjay Srivistava (1986).

[3] The initial work on models with increasing-returns-to-scale, (Benhabib & Farmer, 1994) (Farmer & Guo, 1994) was criticized for assuming a degree of increasing returns that some considered unrealistic. In a response to the critics, Yi Wen (1998) showed that by assuming that capital utilization is variable over the business cycle, the increasing-returns explanation of endogenous fluctuations is fully consistent with empirical evidence.

and rational expectations. But it did not fulfill the promise of providing a micro-foundation to Keynes' *General Theory*.[4] Like the real business cycle (RBC) model (Kydland & Prescott, 1982) (Long & Plosser, 1983) (King, Plosser, & Rebelo, 1988), models of dynamic indeterminacy represent business cycles as stationary stochastic fluctuations around a non-stochastic steady state. In RBC models, the driving source of fluctuations is technology shocks. In models that display dynamic indeterminacy, the driving source of fluctuations is the self-fulfilling beliefs of agents. In both types of models, equilibria are *almost* Pareto efficient. These models cannot explain large persistent unemployment rates of the kind that occurred during the Great Depression or the recent financial crisis.[5]

Models of steady-state indeterminacy

A second branch of literature on indeterminacy departs from the assumption that the demand and supply of labour are always equal and assumes instead that labour is traded in a search market. This second branch of literature is able to generate large welfare losses and high persistent involuntary unemployment of the kind that Keynes discusses in the *General Theory*. The models developed in this literature possess equilibria that display *steady-state*

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[4] See Farmer (2014) for an elaboration of this point.

[5] It would be possible for models with increasing returns-to-scale to have very different welfare implications from RBC models, but in practice the welfare losses that occur in these models are small. The fact that welfare losses are small in many business cycle models was pointed out by Robert Lucas in (1987) in his book *Models of Business Cycles,* and updated in an article in (2003) in the *American Economic Review* to reflect a new generation of models that include incomplete markets and possibly sticky prices. Welfare losses are small in models of dynamic indeterminacy, as they are in RBC models, because both kinds of models generate fluctuations around a socially efficient steady state in which the demand and supply of labour are always equal.

indeterminacy.[6] These equilibria are characterized as non-stationary probability measures, driven by shocks to self-fulfilling beliefs, and they have very different empirical implications from either RBC models or models of dynamic indeterminacy. They imply that the unemployment rate is non-stationary and that it can wander a very long way from the social optimum unemployment rate. As a consequence, the welfare losses generated by self-fulfilling fluctuations in these models can be very large.

General equilibrium theory and macroeconomics

The initial formulation of General Equilibrium theory assumed that there are a finite number of goods and a finite number of people. There have been two extensions to deal with issues that arise naturally in macroeconomics of the passage of time and the fact that the future is uncertain. The first, due to Gérard Debreu (Debreu, 1959), redefines a commodity to be specific to the date, location and state of nature in which it is consumed. The second, due to Sir John Hicks (Hicks, 1939), explicitly recognizes the sequential nature of markets.

Finite General Equilibrium theory

General equilibrium theory deals with market exchange of ℓ commodities by m people and, as formulated by Kenneth Arrow and Debreu (1954), ℓ and m are finite numbers. By making assumptions about the structure of the economy one arrives at an *excess demand function*, $f(p)\colon \mathbb{R}_+^\ell \to \mathbb{R}^\ell$ which is a list of the differences between the aggregate quantities demanded and supplied of each of the ℓ commodities when the prices of each good are

[6] See (Farmer R. E., 2016) and the references therein. Papers that rely on static indeterminacy include (Farmer R. E., 2008) (2010) (2012a) (2013) (Farmer & Nicolò, 2018) (2019), (Plotnikov, 2019) and (Farmer & Platonov, 2019).

represented by the ℓ-element vector p. A vector p^* that satisfies the equation $f(p^*) = 0$ is called an equilibrium price vector.

In his initial formulation of GE theory, Walras assumed the existence of a fictitious character, the auctioneer, who stands on a platform in the centre of the marketplace and calls out prices at which trades will take place.[7] Given a vector of prices, each participant decides how much he would like to trade. The auctioneer adds up the desired trades of every person and if the aggregate quantity of every commodity demanded is equal to the aggregate quantity of every commodity supplied, the auctioneer declares success and trades are executed. Alternatively, if there is an excess demand or supply for one or more commodities, the auctioneer adjusts the vector of proposed prices and he tries again. The adjustment process by which the auctioneer homes in on an equilibrium price vector is called tâtonnement, a French word which means *groping*.

Debreu Chapter 7 as a paradigm for macroeconomics

In Debreu's (1959) extension of GE theory to an infinite dimensional space a commodity is not just an apple, a banana, or a loaf of bread; it is an apple, a banana or a loaf of bread traded on March 9th, 2024 in Mexico City if and only if it is raining in Caracas.

Infinite horizon general equilibrium models may be populated by two kinds of agents. If trades are made by a finite number of infinitely-lived families the model is said to be a Representative Agent (RA) model. If trades are made by an infinite number of finitely-lived people, it is said to be an Overlapping Generations (OLG) model. These two kinds of models

[7] Léon Walras (1899, English Translation 1954).

have very different properties.[8] In the RA model, there is a finite odd number of equilibria and, as in finite general equilibrium theory, every equilibrium is Pareto optimal.[9] In the OLG model, there may be *Pareto inefficient* equilibria that do not display this property as a consequence of the double infinity of goods and people.[10] The fact that market outcomes may be Pareto inefficient is important, because, when the competitive equilibrium is inefficient, there may a exist a government policy that would improve the welfare of everyone.

Temporary Equilibrium theory as a paradigm for macroeconomics

Debreu's extension of GE theory to infinite horizons does not explicitly require the passage of time. The date at which a commodity is consumed is simply one of many labels that index the good. Bread today is distinct from bread tomorrow in the same way that an apple is distinct from a banana. The tâtonnement process whereby the market achieves equilibrium takes place at the beginning of time, and once an equilibrium has been arrived at, the world begins and trades are executed. This is a rather unrealistic, and unsatisfactory description of the world we inhabit.

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[8] See Kehoe and Levine (1985) for a proof of this assertion and a discussion of the difference between these two classes of model.

[9] An allocation is Pareto optimal, named after the Italian scholar Vilfredo Pareto, if there is no way of reorganizing the social allocation of commodities to make at least one person better off without simultaneously making someone else worse off.

[10] Following the publication of Samuelson's paper on the OLG model (Samuelson, 1958) it was widely believed the difference between RA and OLG models was a consequence of the different timing assumptions. Karl Shell (1971) showed that even if everyone who will ever be born can participate in a market at the beginning of time, the OLG model *still* leads to inefficient equilibria.

A more promising alternative, *temporary equilibrium (TE) theory*, envisages the passage of time as a sequence of weeks.[11] Each week, market participants come to a market place to trade commodities with each other. Each person brings a bundle of commodities, his *endowment*, and he leaves with a different bundle of commodities, his *allocation*. Participants arrive at the marketplace with financial assets and liabilities contracted in previous weeks and they form beliefs about the prices of commodities they think will prevail in future weeks. TE theory allows for the beliefs of market participants about future prices to be different from prices that actually occur.

Asset markets, risk and uncertainty

This section discusses the connection between TE theory and Debreu Chapter 7 and demonstrates that, under some circumstances, the equilibria that occur in Debreu's formulation of an equilibrium are the same as the equilibria that occur in a TE model. To understand the connection of Debreu Chapter 7 with TE theory we turn first to an explanation of the way that GE theory accounts for the fact that the future is unknown.

The economist Frank Knight (Knight, 1921) distinguished *risk* from *uncertainty.* Risk refers to events that are quantifiable by a known probability distribution. Uncertainty refers to events that are unknown and unknowable. Almost all quantitative work in macroeconomics has been conducted in models where unknown future events fall into Knight's first category; they

[11] Hicks, (1939) provides the first developed account of TE theory. Later developments include Patinkin (1956) Roy Radner (1972) and Jean Michel Grandmont (1977). Although for Hicks, a period was a week, there is nothing special about that length of time and more commonly, the period of the of a TE model is identified with the period of data availability which is often a quarter or a year.

can be quantified by a known probability distribution. That approach will also be followed in the current survey.

Consider an environment where markets open each week but there is more than one possible future, characterized by a known set of N possible events. For example, if $N = 2$, nature flips a coin that comes up heads with probability χ_H and tails with probability $\chi_L = 1 - \chi_H$. To model this scenario, Arrow (1964) suggested people trade basic securities, called *Arrow securities*, that pay out a fixed dollar amount if and only an event occurs. When there are as many Arrow securities as events, the markets are said to be complete.

Complete markets in the case of a binary event such as a coin toss would require two securities. The *H* security is a promise to pay one dollar next week if and only if the outcome is heads. The T security is a promise to pay one dollar next week if and only if the outcome is tails. In week 1, person i faces the budget constraint

$$p_1^\top\left(x_1^i - w_1^i\right) + Q^H a_H^i + Q^T a_T^i \leq 0. \quad [1]$$

Here, p_1 is an $\ell \times 1$ vector of dollar prices in date 1, w_1^i is an $\ell \times 1$ vector that represents person i's endowment and x_1^i is an $\ell \times 1$ vector that represents her allocation. Q^H and Q^T are the dollar prices of the two arrow securities and a_H^i and a_T^i, which may be positive or negative, are the positions taken by the i'th person in the two securities.[12]

In week 2 one of two events may occur. If the outcome is heads, person i faces the constraint,

$$p_{2H}^\top\left(x_{2H}^i - w_{2H}^i\right) - a_H^i \leq 0. \quad [2]$$

[12] The notation $x^\top y$ for two $n \times 1$ vectors x and y represents vector multiplication where $x^\top$ is the transpose of x.

If the outcome is tails, she faces the constraint,

[3] $$p_{2T}^{\top}\left(x_{2T}^{i} - w_{2T}^{i}\right) - a_{T}^{i} \leq 0.$$

By substituting the expressions for a_H^i and a_T^i from the period 2 budget constraints, Equations [2] and [3], into the period 1 budget constraint, Equation [1], one arrives at the following consolidated budget constraint,

[4] $$p_1^{\top}\left(x_1^i - w_1^i\right) + Q^H p_{2H}^{\top}\left(x_{2H}^i - w_{2H}^i\right) + Q^T p_{2T}^{\top}\left(x_{2T}^i - w_{2T}^i\right) \leq 0.$$

In the GE interpretation of uncertainty, people maximize utility in period 1,

[5] $$\max_{\{x_1^i, x_{2H}^i, x_{2T}^i\}} U^i\left(x_1^i, x_{2H}^i, x_{2T}^i; \chi_H, \chi_T\right),$$

subject to the constraint defined by Inequality [4]. The dependence of utility on the probability of alternative outcomes is represented here by the appearance of the probabilities of heads or tails, χ_H and χ_L, in the utility function. In the TE interpretation of uncertainty, people solve two consecutive utility maximization problems. In period 1 they choose a vector of current consumptions x_1^i and a pair of asset positions a_H^i and a_T^i, subject to their beliefs about the prices that will occur in the future.

For the GE solution and the rational expectations TE solution to be the same, two conditions must hold. First, there must be as many Arrow securities as states of nature. This condition guarantees that the sequence of budget constraints can be reduced to a single budget constraint. Second, utility must be *time consistent*. This condition means that the way that people rank choices over the ℓ elements of x_2^i must be independent of the choices they made in period 1. And it is one reason that economists often assume that the problem in Equation [5] is linear in probabilities, that is

[6] $$U^i\left(x_1^i, x_{2H}^i, x_{2T}^i; \chi_H, \chi_T\right) \equiv \chi_H v^i\left(x_1^i, x_{2H}^i\right) + \chi_T v^i\left(x_1^i, x_{2T}^i\right).$$

The function $v^i(x_1^i, x_{2S}^i)$ for $s \in \{H, T\}$ is called a Von-Neumann Morgenstern utility function and when people maximize expression [6] they are said to be expected utility maximizers. Von-Neumann Morgenstern expected utility maximizers are time consistent.

The assumption of complete markets allows for a relatively straightforward extension of the perfect foresight assumption to a world with uncertainty. If there is one future state and people know all future prices, the agents in the model are said to possess *perfect foresight*. If there is more than one possible future state, and people know all future state-contingent prices, the agents in the model are said to possess *rational expectations*.

The translation of Debreu's version of GE theory into the language of TE theory exposes a problem with the assumption that people take prices as given. GE theory does not guarantee that equilibrium is unique. If there are multiple equilibria, how do participants in this week's market know which of the equilibrium price vectors will be attained in future markets? The following section turns to a description of the problems raised in GE models by the existence of multiple equilibria and it offers a solution to the problem of indeterminacy. Beliefs should be introduced as a separate fundamental in addition to preferences, endowments and technology.

Multiplicity and determinacy of equilibrium

This section compares finite and infinite horizon GE models and illustrates, by means of three simple figures, the meaning of determinacy of equilibrium. The section begins by demonstrating that there is a finite odd number of equilibria in a finite general equilibrium model, and it proceeds to explain how the equilibrium concept can be extended to deal with the passage of time.

Finite GE theory: Why equilibria are determinate

That equilibria are determinate is most easily understood in the case of a two-good model and is illustrated in Figure 1. Here, $f(p)$ is the aggregate excess demand for good 1 and $p = \frac{p_1}{p_1+p_2}$ is the money price of good 1, normalized by the sum of the two money prices. One can show that $f(0) > 0$, $f(1) < 0$ and $f(p)$ is continuous. It follows that the graph $f(p)$ is a continuous function $[0,1] \to \mathbb{R}$ that starts above the p-axis and ends below the p-axis. Hence $f(p)$ must cross the p-axis at least once and generically, the number of crossings is odd. Figure 1 illustrates the case of three equilibrium prices.

This figure also shows that equilibrium cannot, generically, be indeterminate. Indeterminacy, in the finite case, would require the excess demand function to be coincident with

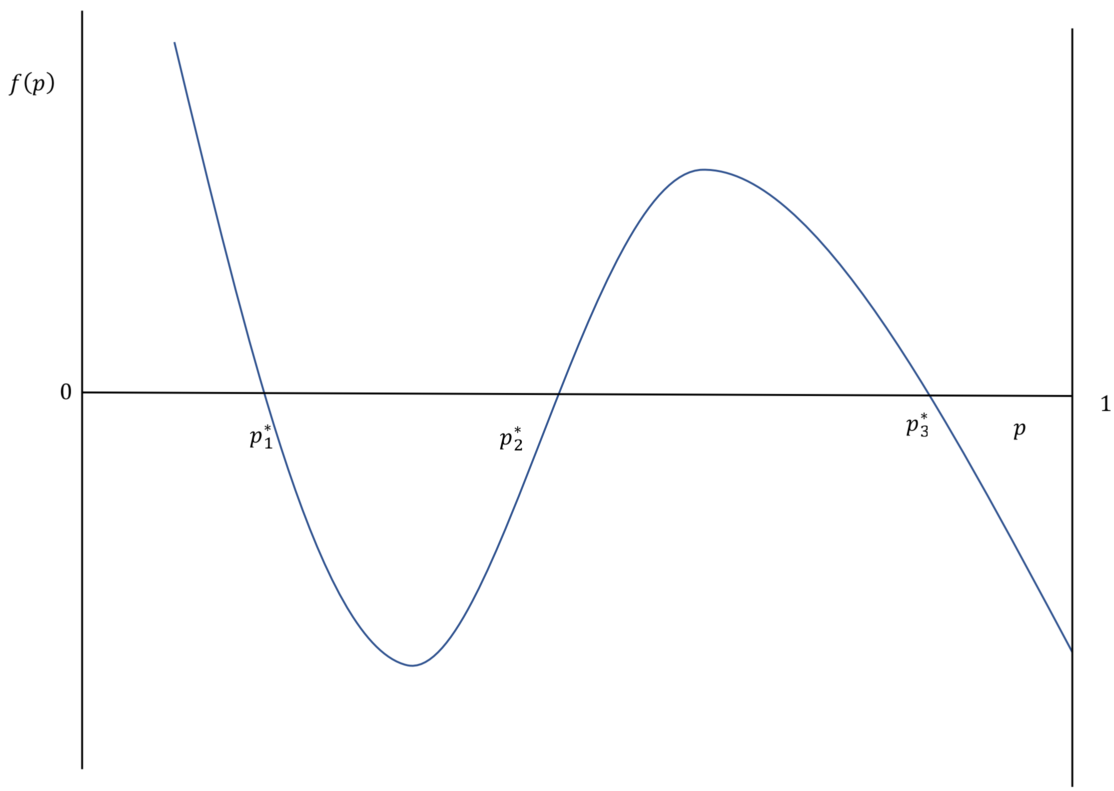

Figure 1: Three Equilibria in a Two-Good Model

the p-axis for an interval of p-values. That would be a very special case, as would a tangency of the excess demand function with the p-axis. Genericity means that, in the space of all parameterized 2-good GE models, models with indeterminate equilibria or models with an even number of equilibria occur vanishingly often. Although such a model could be constructed, a small perturbation of the parameters of the model would generate a different model where the indeterminacy or the tangency disappears.

Infinite horizon models with representative agents

When the number of commodities is infinite, an equilibrium price vector is an element of $\mathfrak{B}$, the space of non-negative bounded sequences.[13] If the number of people is finite, as in the RA model, there is an odd finite number of equilibria just as there is in the finite Arrow-Debreu model. These equilibria need not be stationary, but they cannot be indeterminate. If the number of people is infinite, as in the OLG model, there are always at least two stationary equilibrium price sequences and at least one of these stationary equilibrium price sequences is indeterminate.

Figure 2 illustrates the situation that typically occurs in RA models. The figure plots three sequences, p^*, p^{1*} and p^{2*} as functions of time. The elements of each sequence are indexed by subscripts that refer to weeks and the sequence p^* is, by assumption, a stationary perfect foresight equilibrium price sequence. The statement that p^* is an equilibrium price sequence means that the quantities of all goods demanded and supplied are equal in every week. The statement that p^* is a stationary sequence means that p_t^* is constant over time. And the qualifier, 'perfect foresight', means that when people form their demands and supplies at date t, they are fully aware of what the prices will be in all future periods.

[13] The space of non-negative bounded sequences is defined as $\mathfrak{B} \equiv \{\{p_t\}_{t=1}^{\infty} | \, p_t \in \mathbb{R}_+, p_t \leq 1, \text{for all } t\}$.

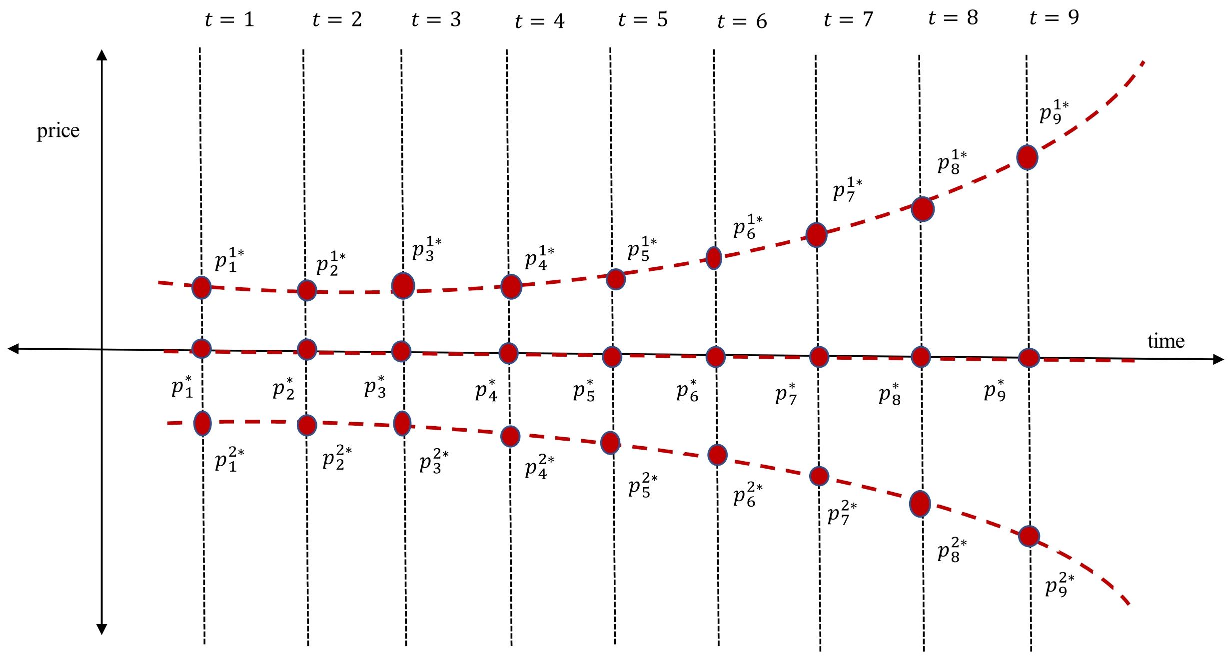

Figure 2: A Unique Determinate Equilibrium in an Infinite Horizon Model

In general, p_t^* could be an element of $\mathbb{R}_+^\ell$, that is, there may be multiple goods traded in each period. For the purposes of exposition, it will be assumed here that $\ell = 1$, and that p_t^* is the dollar price of the unique date t commodity which will be referred to as 'wheat'. Using this convention, the notation p_3^*, for example, refers to the dollar price of wheat in week 3. In contrast to the stationary equilibrium price sequence, p^*, the sequences p^{1*} and p^{2*}, are non-stationary. A sequence that begins at p_1^{1*} or p_1^{*2} grows without bound and heads off either to plus infinity, in the case of p^{1*} or to negative infinity, in the case of p^{2*}. The sequences p^{1*} and p^{2*} start close to the stationary equilibrium sequence, p^*, but they eventually diverge from it.

To measure the distance between two elements of $\mathfrak{B}$, economists use the *sup norm*, which records the distance between two sequences as the largest distance between any two elements of the sequence. If p^* is a stationary equilibrium price sequence of an RA model, often it will be possible to find perfect foresight price sequences, p^{1*} and p^{2*} that obey the market clearing conditions for some finite number of periods. But, however close these sequences are initially to the steady-state equilibrium price sequence p^*, they will eventually move away from it. In the example depicted in Figure 2, the sequences p^{1*} and p^{2*} diverge to plus or minus infinity.

One can show using a result first proved by Negishi (1960), that every equilibrium price sequence of an RA model is bounded away from every other equilibrium price sequence by a positive number. An implication of Negishi's theorem, is that stationary equilibrium price sequences must always display the local instability property depicted in Figure 2.

Infinite horizon models with overlapping generations

OLG models are different from RA models. In OLG models it is no longer true that equilibrium price sequences must be isolated from each other, and instead, sets of indeterminate equilibria are common. Figure 3 illustrates a situation that occurs generically in OLG models. This figure plots three price sequences, p^*, p^{1*} and p^{2*} as functions of time. p^* is a stationary-equilibrium price sequence and p^{1*} and p^{2*} are non-stationary equilibrium price sequences.

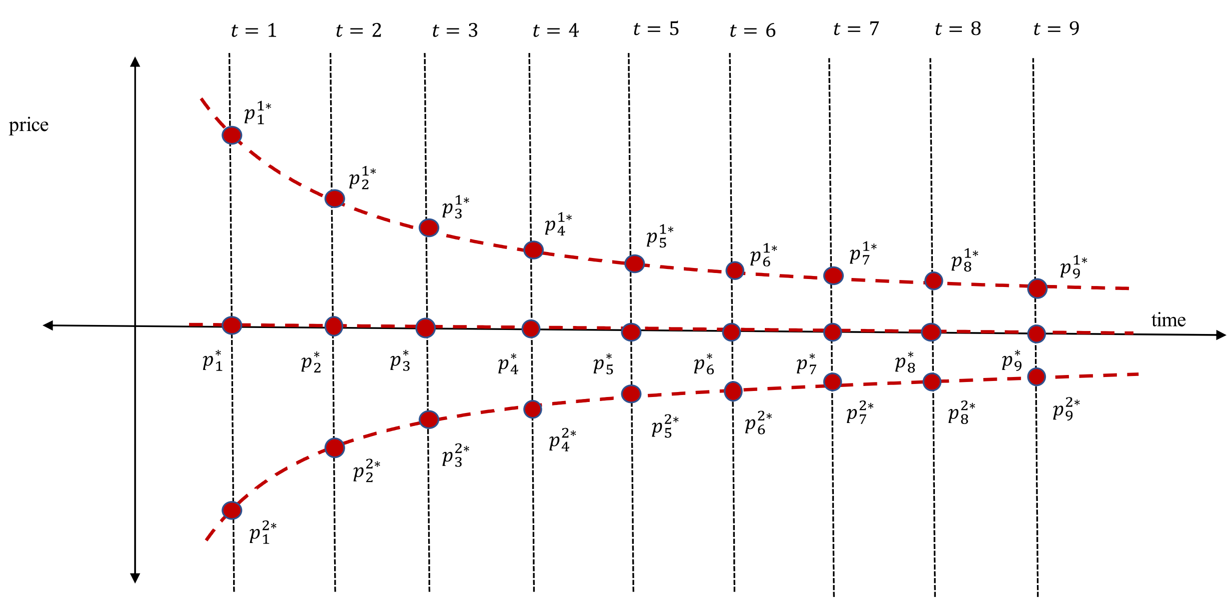

Figure 3: A Set of Indeterminate Equilibria in an Infinite Horizon Model

Unlike the example in Figure 2, a sequence that begins at p_1^{1*} or p_1^{*2} converges to the steady-state equilibrium price sequence p^*. It is easy to find examples of OLG models where there are non-stationary equilibrium prices sequences, like p^{1*} and p^{2*}, that obey the market clearing conditions and remain bounded as $t \to \infty$. All of these sequences are perfect-foresight equilibrium price sequences and all of them are indeterminate. For any one of these equilibrium price sequences, there is another one that is arbitrarily close, where closeness of one sequence to another is measured by the sup norm.

Indeterminacy and rational expectations

The existence of indeterminate equilibrium price sequences in overlapping generations models is curious; but it might be thought uninteresting. In the examples depicted in Figure 3, almost all of the equilibria are non-stationary and all of these non-stationary equilibria converge to a stationary perfect foresight equilibrium. They appear to explain transitory phenomena that would never be observed in practice. They are however of considerable practical importance once one moves beyond the assumption of perfect foresight.

In rational expectations models, even with a complete set of Arrow securities, there exist multiple stationary rational expectations equilibria (Farmer & Woodford, 1984). In these equilibria, prices and allocations fluctuate from one week to the next purely because people believe that they will. They are examples of equilibria driven by *self-fulfilling prophecies*, a phenomenon that can occur in RA models where money is used as a medium of exchange as well as in OLG models with or without money.

Indeterminacy in rational expectations models with representative agents

The research agenda in macroeconomics that developed in the 1980s was initially restricted to purely real models. This Real Business Cycle (RBC) agenda added production to the pure exchange model but preserved the RA assumption. Real RA models with production, like the RA pure-exchange real models discussed here, retain the property that equilibria are both determinate and Pareto optimal. In the RBC model money plays no role, a feature that is hard to square with empirical evidence which suggests that monetary policy is an important driver of economic fluctuations.

Some of the earliest empirical evidence for these effects came from the work of Christopher Sims (1980) (1989) who found big real effects of shocks to the nominal interest rate. These effects cannot easily be explained in cash-in-advance models with a unique determinate equilibrium. They can, however, be explained easily in models with indeterminate equilibria as demonstrated by Farmer (1991), Kenneth Matheny (1998) and Benhabib and Farmer (2000).

In their (1987) paper Robert Lucas and Nancy Stokey added the assumption that cash must be held to purchase goods. To understand the real effects of monetary shocks, a large part of the profession adopted monetary versions of the RBC model, but retained the assumptions which guarantee that equilibrium is locally determinate.[14] To understand the real effects of money that had been identified by Sims in his empirical work on small-scale time series models (1980), economists assumed that money prices are 'sticky' as a consequence of small costs of price adjustment. The rise of this agenda, which Mankiw and Romer dubbed 'New-Keynesian Economics' (Mankiw & Romer, 1998), has dominated the field for the past thirty years.

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[14] The assumptions required to generate this result are strong; for example, the uniqueness result does not survive the introduction of inflationary finance to pay for budget deficits. It has been known at least since the work of William Brock (1974) that MGE models have at least two steady states and one of these steady states is generically indeterminate.

Initially, the NK agenda dealt with indeterminacy through ruling it out by assumption. There have been attempts to deal with dynamic indeterminacy in estimated NK-models, but the core model still adopts a menu-cost approach to sticky prices and, whenever possible, introduces assumptions to render a single stationary equilibrium locally unique.[15]

The flagship New-Keynesian model

The flagship *New-Keynesian* (NK) model consists of the following three equations,

$$y_t = \mathbb{E}_t[y_{t+1}] - a(i_t - \mathbb{E}_t[p_{t+1} - p_t]) + \rho + u_t^D, \tag{6}$$

$$i_t = \eta_\pi(p_t - p_{t-1}) + u_t^P, \tag{7}$$

$$\mathbb{E}_t[p_{t+1} - p_t] = (p_t - p_{t-1}) + \kappa(y_t - \bar{y}_t) + u_t^S. \tag{8}$$

Here, y_t is the log of GDP, p_t is the log of the price level, $\bar{y}_t$ is the log of potential GDP, i_t is the short-term money interest rate and $p_t - p_{t-1}$, is the log difference of the price level. The log difference of the price level is also, by definition, the date t inflation rate. $\mathbb{E}_t[\cdot]$ is the conditional expectations operator and the symbols a, ρ, η_π and κ are parameters derived from assumptions about private-sector and government behaviour. Equation [6] called an *optimizing IS curve*, is derived from the first order intertemporal condition of an optimizing infinitely-lived representative consumer, Equation [7] is a central-bank reaction function, also referred to as a *Taylor Rule* (Taylor, 1999), and Equation [8] is a *NK Phillips curve*. The terms u_t^D, u_t^P and u_t^S are respectively a demand shock, a policy shock and a supply shock.

The NK Keynesian model can be amended to include the following equation

[15] Farmer, Vadim Khramov and Giovanni Nicolò (2015) and Francesco Bianchi and Nicolò (2017) provide methods to solve and estimate indeterminate GE models with indeterminate equilibria using standard software packages.

[9] $$\frac{B_t}{P_t} = \sum_{\tau=t}^{\infty} \mathfrak{Q}_t^{\tau} s_{\tau},$$

where B_t is the dollar value of government debt, $P_t = \exp(p_t)$ is the price level, $\mathfrak{Q}_t^{\tau}$ is the present value at date t, measured in units of date t goods, of a claim to goods at date τ and s_{τ} is the budget surplus, equal to the real value at date τ of government tax revenues net of expenditure. The present value price, $\mathfrak{Q}_t^{\tau}$, is determined by the interest rates and inflation rates that hold between periods t and τ.[16]

Active and passive fiscal and monetary policies in the flagship NK-model

Eric Leeper (Leeper, 1991) has suggested the following classification of policies in the NK-model. If the coefficient η_{π} in the Taylor rule is greater than one, monetary policy is said to be *active*. If it is less than one, monetary policy is *passive*. This classification is useful because one can show that if monetary policy is active, the equilibrium of the NK-model is locally determinate. The classification of fiscal policies as active or passive requires a little more explanation.

If the government were to be treated in the same way as any other actor in a general equilibrium model, the Treasury would need to ensure that Equation [9] holds for every possible price level P_t and every sequence of present value prices $\{\mathfrak{Q}_t^{\tau}\}_{\tau=t}^{\infty}$. Under this interpretation of the constraints on feasible fiscal policies, Equation [9] is a government budget constraint. Taking prices and interest rates as given, the government would need to ensure that it raises enough revenue to eventually pay off its outstanding debt. If the Treasury does indeed adjust taxes or expenditure plans, or both, to ensure that it remains solvent for all possible prices and interest

[16] It is defined as $\mathfrak{Q}_t^{\tau} = \prod_{s=t}^{\tau-1} \frac{(1+i_s)P_s}{P_{s+1}}$ where P_t is the price level and i_t is the money interest rate.

rates, the fiscal policy is said to be $passive$. If instead, the government sets the sequence of surpluses $\{s_\tau\}_{\tau=t}^{\infty}$ independently of P_t, B_t or $\{\mathfrak{Q}_t^\tau\}_{\tau=t}^{\infty}$, fiscal policy is said to be *active*.

Equilibrium determinacy when monetary policy is active and fiscal policy is passive

When the NK-model was first developed in the 1980s, little attention was paid to fiscal constraints and it was assumed that fiscal policy is always passive. Like any other actor in a general equilibrium model, the government was assumed to be a price taker that cannot spend more than it receives in income. Attention during this period was focused on the possibility that active monetary policy can select a locally determinate equilibrium by influencing the stability properties of the steady state equilibrium of a log-linear NK-model.[17]

If one defines $X_t \equiv [i_t, p_t - p_{t-1}, y_t]^\top$, the NK-model is an example of a linear rational expectations model of the form,

[10] $$X_t = A\mathbb{E}_t[X_{t+1}] + C + U_t,$$

where C is a 3×1 vector of constants, A is a 3×3 matrix of coefficients and $U_t = [u_t^D, u_t^P, u_t^S]^\top$ is a vector of random variables which have zero expected value and are independently and

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[17] Monetary general equilibrium models assumed initially that the Central Bank adopts a money supply rule in which the quantity of money grows at a fixed rate. This, for example, is the assumption in Brock (1974). In part, this assumption was motivated by the fact that monetary models where the Central Bank pegs the money interest rate lead to price level indeterminacy (Sargent & Wallace, 1975). Since central banks appear to use the interest rate as their main instrument to influence the economy, the assumption that the Central Bank sets a money growth target was problematic for attempts to build a realistic monetary theory. McCallum (1981) showed that determinacy of the price level is restored if the Central Bank adjusts the interest rate aggressively enough in response to inflation where 'aggressively enough' is defined as an interest rate response coefficient, η_π, greater than 1.

identically distributed.[18] As long as monetary policy is active, all of the eigenvalues of the matrix A are inside the unit circle and, in this case, the NK-model has the following reduced form

$$X_t = (I - A)^{-1} C + U_t. \quad [11]$$

The inflation rate, GDP and the interest rate are all functions only of the shocks, U_t, and the price level is pinned down by the definition of inflation in the initial period.

Equilibrium determinacy when monetary policy is passive and fiscal policy is active

When monetary policy is passive, as it was in the United States prior to 1979, and again from 2009 – 2017, the price level is no longer determined by the equations of the NK-model.[19] Instead, the price level may fluctuate randomly, driven by the self-fulfilling beliefs of market participants. To handle this apparent 'problem' with NK economics, a number of economists (Leeper, 1991) (Sims C. A., 1994) (Woodford, 1995) have suggested that Equation [9] should be interpreted not as a budget constraint, but as a debt valuation equation. When monetary policy is active, the attempt to run an active fiscal policy by the Treasury would cause government debt to explode. When monetary policy is passive, advocates of this idea, called the Fiscal Theory of the Price Level (FTPL), claim that government debt remains bounded even if fiscal policy is active.

When fiscal policy is active, the Treasury no longer adjusts its tax and spending plan to ensure budget balance; instead, Equation [9] determines the price level as a function of the

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[18] It will be assumed, for the purpose of exposition, that $\bar{y}_t$ is constant. The model is easily adapted to allow for growth in potential output.

[19] Thomas Lubik and Frank Schorfheide (2004) show in an estimated GE model that policy was passive before 1979 and active afterwards. The period from 2009 to 2017 is characterized by an interest rate peg at, effectively, zero.

expected present value of all future surpluses. Equation [9], in this interpretation, is not a budget constraint, it is a debt valuation equation. According to Leeper's classification, the price level is locally determinate if monetary policy is active and fiscal policy is passive, or if monetary policy is passive and fiscal policy is active.[20]

The indeterminacy of the price level in the RA model is a problem, but there is at least a potential resolution. One can restrict attention to the Central Bank's preferred steady state and assume that the policy mix is always a combination of one active and one passive policy. The following section demonstrates that no such resolution is possible in the OLG model where indeterminacy of the equilibrium prices and quantities is more pervasive. In an OLG model, calibrated to U.S. data, Farmer and Zabczyk (2019) show that monetary and fiscal policy can both be active at the same time and yet economic fundamentals are insufficient to completely determine either absolute or relative prices.

Indeterminacy in rational expectations models with overlapping generations

Timothy Kehoe and David Levine (Kehoe & Levine, 1985) compared the differences in the determinacy properties of RA models and OLG models. They demonstrated that in the OLG

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[20] To some, the classification into active and passive rules may appear natural; to others, it may appear artificial. Whatever one's view of the elegance of the theory, it is not sufficient to determine the price level globally, even in the NK-model with the correct combination of active and passive policies. Jess Benhabib, Stephanie Schmidt-Grohé and Martín Uribe (2001) pointed out that a linear Taylor Rule is inconsistent with the existence of the fact that the nominal interest rate cannot be negative. When the model is amended to allow the Taylor Rule to respect the zero-lower bound, the model always has at least two steady-state equilibria. If the Taylor Rule is active at the steady state that the Central Bank prefers, there will always exist a second steady state with a low and possibly negative real interest rate at which the Taylor Rule is passive and the initial price level is indeterminate.

model, equilibria may be generically indeterminate of arbitrary degree.[21] It was widely believed in the 1980s that the Kehoe-Levine result that equilibria in the OLG model are generically indeterminate had little relevance to practical models of the macroeconomy. As shown in recent research by Farmer and Zabczyk (2019), that belief was premature.

A calibrated example of an OLG model with indeterminacy

Farmer and Zabczyk (2019) provide an example of an OLG model in which people live for 62 periods. In the FZ-model, people begin economic life at age 18, retire at age 66 and live

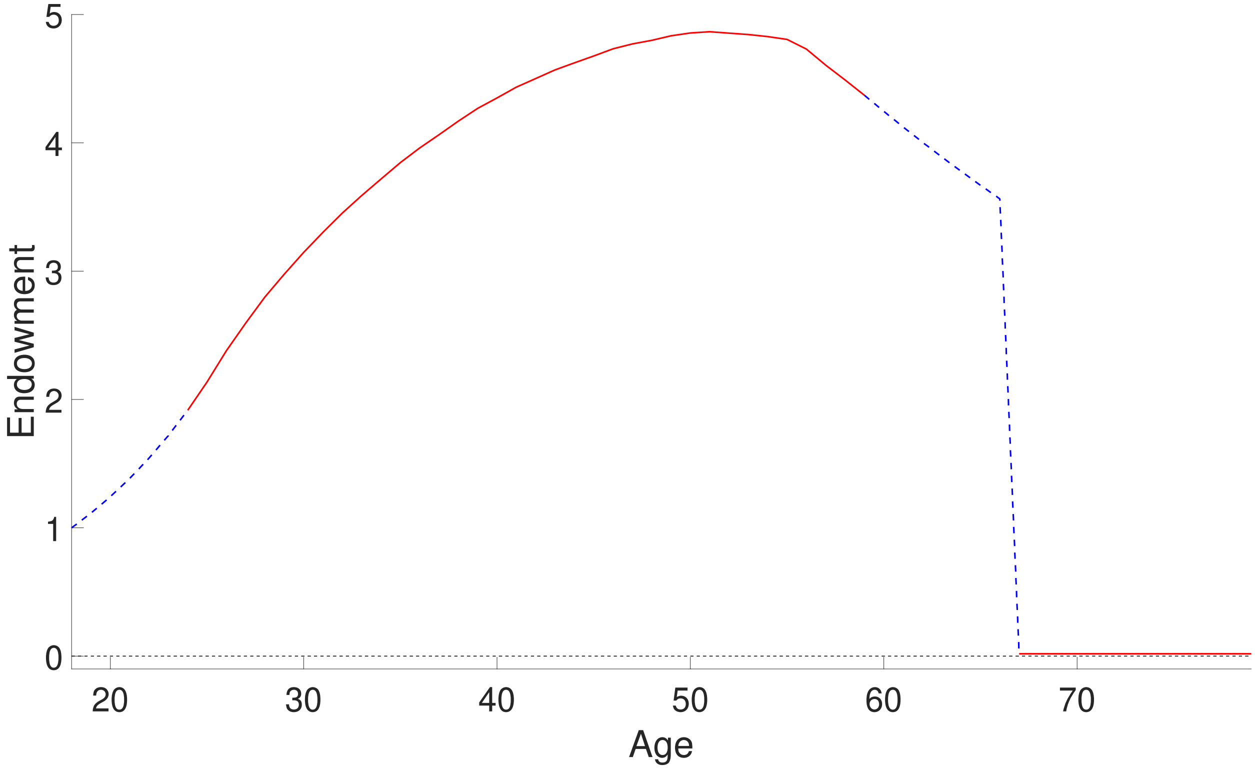

Figure 4: The endowment profile for U.S. males

[21] Kehoe-Levine's work was largely ignored by macroeconomists. This was due, in part, to resistance from leading figures in the development of the rational expectations school who advocated for a research agenda in which expectations are endogenously determined by preferences, endowments and technology (Cherrier & Saïdi, 2018).

until age 79. The retirement age was chosen to accord with the age at which U.S. males qualify for retirement benefits and the date of death was chosen to coincide with U.S. male life expectancy at birth. Farmer and Zabczyk assumed that people are endowed with the income profile depicted in Figure 4. The solid line for ages 24 to 60 is calibrated to U.S. data using estimates of Guvenen et. al (2015), and the solid line for ages 66 through 79 is calibrated to U.S. Supplementary Social Security Income. The dashed lines are log-linear interpolations to fill in periods for which the Guvenen et. al. data are missing. The important feature of this profile is its hump-shape.

Farmer and Zabczyk construct an overlapping generations model in which people have perfect foresight of future prices and choose life-cycle consumption to maximize a Constant Relative Risk Aversion (CARA) utility function with a coefficient of risk aversion of 6 and a time preference rate of 0.953. They show that the FZ-model has four steady state equilibrium price sequences and they focus on one of these sequences in which money has positive value. Importantly, this steady-state equilibrium displays two degrees of indeterminacy. This property is important because it implies that a very standard macroeconomic model, when calibrated to actual data, is incapable of uniquely determining prices and quantities.

Farmer and Zabczyk assumed initially that monetary policy is passive and fiscal policy is active. In the NK-model, that combination of policies would result in local determinacy of the monetary steady-state equilibrium. Instead, they found that their calibrated model displays two-degrees of indeterminacy. They conclude that the FTPL cannot be used to determine the price level as a function of economic fundamentals alone.

The rational expectations research agenda envisioned by Robert Lucas and Thomas Sargent was an attempt to explain expectations in terms of a narrowly defined set of fundamentals.[22] If the FZ-model is accepted as an accurate description of the world we inhabit, the rational expectations agenda is doomed from the outset. Preferences, endowments and technologies are insufficient to explain how prices and quantities are determined in a market economy.

The implications of the Farmer-Zabczyk results for theories of inflation

Milton Friedman coined the famous dictum that "inflation is always and everywhere a monetary phenomenon". Although there is a strong correlation between cross-country inflation rates and cross-country money growth rates in high inflation countries; that correlation was never in evidence in low-inflation countries and it has broken down completely in the past two decades where we have observed low or even negative inflation rates in conjunction with very high rates of money growth. This lack of correlation is explained in models with price-level indeterminacy by introducing an alternative theory of inflation that appeals to the 'animal spirits' of market participants. To resolve the indeterminacy problem in MGE models, Farmer (1993) introduced a new independent equation that characterizes the way people form their beliefs about future variables. That approach has been shown empirically to outperform the NK-model. Farmer and Nicolò (2018) (2019) used Bayesian statistics to compare posterior odds ratios for the NK-model and an alternative Farmer Monetary (FM) model that replaces the NK Phillips curve with a belief function. They found that the FM-model decisively outperforms the NK-model in U.S., Canadian and U.K. data.

[22] See the agenda described by Robert Lucas and Thomas J. Sargent in their essay, "After Keynesian Macroeconomics", (Lucas, Jr. & Sargent, 1981).

Price level indeterminacy is a problem for a research agenda that purports to explain prices and quantities as functions of a set of economic fundamentals that includes only preferences, technologies and endowments. But it is not an unprecedented problem. When the equilibrium of an OLG model displays one degree of indeterminacy, it is possible to adopt the assumptions used by Leeper to determine prices. In the case of the FZ-model that avenue is closed. Even if monetary and fiscal policy are both active, a possibility that would lead to instability in the NK-model, the FZ-model still displays one degree of indeterminacy. It is not only the price level that is indeterminate. Real interest rates and asset prices are also indeterminate if one includes only preferences, technology and endowments in the set of fundamentals that determine economic outcomes. To resolve this indeterminacy, Farmer argued in his 1993 book on the *Macroeconomics of Self-Fulfilling Prophecies*, that *beliefs* should be given the same fundamental status as preferences.

The implications of the Farmer-Zabczyk results for theories of efficient asset markets

A large literature, beginning with Robert Shiller (1981) and Stephen Leroy and Robert Porter (1981) has found that asset prices fluctuate too much to be explained by subsequent fluctuations in dividend payments. There must be instead, a substantial movement in the price of risk.[23] Fluctuations in the price of risk are measured, in a rational expectations model, by variations in the Arrow security prices Q^H and Q^T in Equation 1. In the RA model, fluctuations in these prices are sometimes attributed to shocks to constraints on how much agents are allowed

[23] This was the message that John Cochrane pushed home in his Presidential Address to the American Finance Association (Cochrane, 2011).

to borrow. In the OLG model they can be fully explained as self-fulfilling prophecies even in a model where there is a complete set of Arrow securities (Farmer R. E., 2018).

The finance literature explains asset price movements as efficient responses of financial markets to fluctuations in preferences, technologies and endowments. Eugene Fama (1970) coined the term “efficient markets hypothesis” (EMH) to refer to the idea that the capital markets reflect all available information and it is not possible, according to the EMH, to make money by buying and selling securities unless one has insider information. A large body of empirical evidence suggests that this hypothesis is at least approximately true. But it has nothing to do with the claim of general equilibrium theorists that markets are Pareto efficient.

The efficient markets hypothesis refers to *informational efficiency*. This is the statement that there are no riskless arbitrage opportunities and it is a property of any economic model with a complete set of Arrow securities. The assumption there are complete financial markets may or may not be a close approximation to the real world.[24] Whatever position one takes in the debate over complete versus incomplete markets, indeterminacy in the FZ-model has serious implications for the proposition that the financial markets efficiently allocate capital to competing ends. Assume, for the sake of argument, that the financial markets are complete and that it is not possible to discover riskless arbitrage opportunities. The existence of indeterminate

[24] Some economists point to the costs of establishing contingent markets. They argue that it is pure fiction to think that real world financial markets are complete, and they study economic models with fewer Arrow securities than states of nature. Others argue that the world is well approximated by the complete-markets assumption. In their view, a large divergence from complete markets would create a big incentive for an arbitrager to create a new security.

relative prices implies that, nevertheless, the capital markets do not allocate capital efficiently to competing ends. Why might that be?

Recall that the FZ model provides a calibrated example of a model where money has value and the relative price of goods today for goods tomorrow is indeterminate, even when the Central Bank and the Treasury each follow active policies. The individuals that inhabit this model are able to trade with each other and to write insurance contracts against every event that may occur in subsequent weeks. But each week, a new set of people arrives in the market. These people are unable to participate in insurance markets that open before they were born and their actions may differ across states of nature. Complete markets does not guarantee *complete participation* in the financial markets and the fact that people have finite lives vitiates the possibility of complete participation (Cass & Shell, 1983). As a direct consequence of the indeterminacy of equilibrium, Farmer (2018) has shown that simple MGE models generate equilibria where there are substantial asset price fluctuations, and a large risk premium, even when there is no underlying uncertainty of any kind. Because people are assumed to be risk averse and these fluctuations are avoidable, they are necessarily Pareto inefficient. The paper by Farmer (2018) takes the Cass-Shell concept of sunspots and demonstrates its relevance to the real world.

Models of steady-state indeterminacy

The models of dynamic indeterminacy that arise from the OLG structure suffer from the same deficiencies as the models of dynamic indeterminacy that arise from models of increasing-returns-to to scale. If the labour market is Walrasian, these models will generate business cycles as small fluctuations around a socially efficient steady state. That insight suggests that, if one seeks a micro-foundation for the Keynesian concept of involuntary unemployment, one must consider temporary equilibrium models where the quantity of labour traded each period is determined by some mechanism other than Walrasian market clearing. The following section elaborates on this theme.

Classical search theory as an alternative equilibrium concept

The Walrasian auctioneer has been widely criticized, not just by non-economists or economists from outside the field, but also by Kenneth Arrow, a leading general equilibrium theorist who, along with Gerard Debreu and Lionel McKenzie, was the first to provide a rigorous mathematical proof of the existence of equilibrium.[25] Criticisms of Walrasian equilibrium as a theory of market prices and quantities led to the development of a variety of alternative equilibrium concepts including *quantity-constrained equilibrium*, *asymmetric information equilibrium*, *contract theory* and *search equilibrium*. This article is limited by space considerations to a discussion of just one of these alternative concepts, search equilibrium.

In 1973 Edmund Phelps co-edited an influential volume of articles *The Microeconomic Foundations of Employment and Inflation Theory* (Phelps, 1972) which contained two of the first articles on the economics of *search unemployment* (Alchian, 1969) (Mortensen, 1972). These papers were precursors to the development of a large literature that Farmer (2016) has referred to as *classical search theory*. Classical search theory was further developed by Peter Diamond (1982) and Christopher Pissarides (1979) and in 2010, Diamond, Mortensen and Pissarides (DMP) were awarded the Nobel Prize in Economics "for their analysis of markets with search frictions".

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[25] (Arrow & Debreu, 1954), (McKenzie, 1954).

Classical search theory is an alternative equilibrium concept, distinct from Walrasian equilibrium. It sees the labour market as a dynamic changing environment where people are constantly transitioning among three states: They may be out of the labour force; they may be in the labour force and employed or they may be in the labour force and unemployed. One could envisage a Walrasian model where unemployed individuals must allocate time between searching for a job or enjoying leisure and where firms must allocate the time of their workers between filling vacancies or producing goods. In a Walrasian equilibrium, the auctioneer would steer the economy towards an outcome in which unemployed and employed members of the labour force were each optimally allocating their time between these alternative activities.

Diamond Mortensen and Pissarides envisage a different mechanism from Walrasian equilibrium to decide what happens each market week. Instead of an auctioneer who sets prices, unemployed workers bump randomly into firms with vacant jobs. In a Walrasian market, search by an unemployed worker for a job, and search by a corporate recruiter for a worker, are distinct activities that would be associated with different prices. Instead, in search theory, there are not enough relative prices and, as a consequence, the equilibrium unemployment rate is indeterminate. To resolve this indeterminacy, classical search theory introduces a new assumption. Firms and workers do not take the market wage from the auctioneer as they would in Walrasian theory; instead, the wage is set through a bargaining process.

If an unemployed worker meets a firm with a vacancy, the firm is willing to pay any wage less than or equal to the worker's marginal product. The worker is willing to accept any wage greater than or equal to her reservation wage. To resolve this indeterminacy in the bargaining process, Diamond Mortensen and Pissarides introduce a parameter, *the bargaining* *weight*, which picks a wage somewhere between the worker's marginal product and the firm's reservation wage.

Keynesian search theory as an alternative equilibrium concept

Farmer (2008) (2010) (2012a) (2013) has proposed a different approach to the one adopted by Diamond Mortensen and Pissarides to resolve the indeterminacy of search equilibrium. He calls this alternative *Keynesian search theory*. Instead of assuming that firms bargain with workers, Farmer assumes that firms employ enough workers to produce the quantity of goods demanded by consumers. This quantity depends on consumers' wealth which is itself determined by the value of their assets. He posits that asset market participants form beliefs about the price of shares in the stock market and that, in equilibrium, these beliefs are validated by the actions of future asset market participants.

In the classical Diamond-Mortensen-Pissarides approach to search theory the wage and the unemployment rate are determined in the labour market. In the absence of shocks to labour demand or supply, the unemployment rate may or may not coincide with the rate that would be chosen by a benevolent social planner. Whether or not the unemployment rate is Pareto efficient depends on the bargaining weight. If the bargaining weight just happens to equal a particular number that is related to the match technology, the classical search equilibrium is Pareto Efficient.[26] If the bargaining weight does not fortuitously equal this number, the classical search equilibrium will deliver either too much or too little unemployment.

[26] Specifically, the bargaining weight must equal the elasticity of unemployment in the matching technology (Hosios, 1990).

Too much unemployment is socially inefficient because resources are wasted when workers willing to produce goods are unable to find jobs. Too little unemployment is socially inefficient because society is spending valuable resources matching people to jobs and too little effort in the act of production. In extreme examples of this phenomenon, an economy that has a zero-unemployment rate will produce zero goods because everyone is employed in temp agencies finding jobs for other people in temp agencies and no-one is producing valued commodities.

One could imagine that the bargaining weight is not a constant; instead it fluctuates randomly from week to week as a function of market conditions. If the bargaining weight fluctuates, so will the unemployment rate. And if the unemployment rate fluctuates, these fluctuations will be reflected in the stock market. In the data, persistent low-frequency movements in the unemployment rate are associated with persistent low-frequency movements in the stock market. If classical search theory is correct, movements in the stock market are caused by the rational expectations of market participants that there will be future movements in fundamentals. For example, a future court decision might give unions more power and increase the bargaining power of workers.

If Farmer's Keynesian search theory is correct, the direction of causation is reversed. According to this theory, it is not movements in the bargaining weight that cause movements in the unemployment rate and consequent movements in asset prices. It is movements in self-fulfilling beliefs about asset prices that cause variations in demand and subsequent variations in employment. The bargaining weight, in this interpretation of the facts, is endogenous.

Keynesian search theory and indeterminacy in macroeconomics

OLG models and models with increasing-returns to scale both display dynamic indeterminacy. If they are situated in a temporary equilibrium model where the equilibrium concept is Walrasian, neither of them leads to revolutionary implications for stabilization policy. Markets, left to themselves, may misallocate resources; but these misallocations are not of the orders of magnitude that characterize real-world financial crises. In contrast, the shift from Walrasian equilibrium to Keynesian search equilibrium is a far more plausible candidate for a micro-founded theory of major recessions.

In Keynesian search theory, if asset market participants stubbornly persist in believing that the stock market has low value, those beliefs will be associated with a permanently elevated unemployment rate. The indeterminacy of equilibrium is not confined to many dynamic paths converging on an approximately efficient steady state. Instead, in Keynesian search theory pessimistic expectations can steer the economy into one of many low-level equilibrium traps in which there is a permanently elevated rate of unemployment.

The theoretical difference between models with an isolated determinate steady-state equilibrium, and models with a continuum of contiguous steady state equilibria, has important implications for the time series properties of data. If beliefs about the value of the stock market can wander randomly, as they do in real world data, so can the unemployment rate. Keynesian search theory predicts that persistent low-frequency movements in the stock market *cause* persistent low-frequency movements in the unemployment rate.

The stock market and the unemployment rate

The theoretical possibilities thrown up by Keynesian search theory are consistent with the behaviour of unemployment and asset prices that we see in data. Figure 5 shows that there was a close connection between the stock market and the unemployment rate during the Great

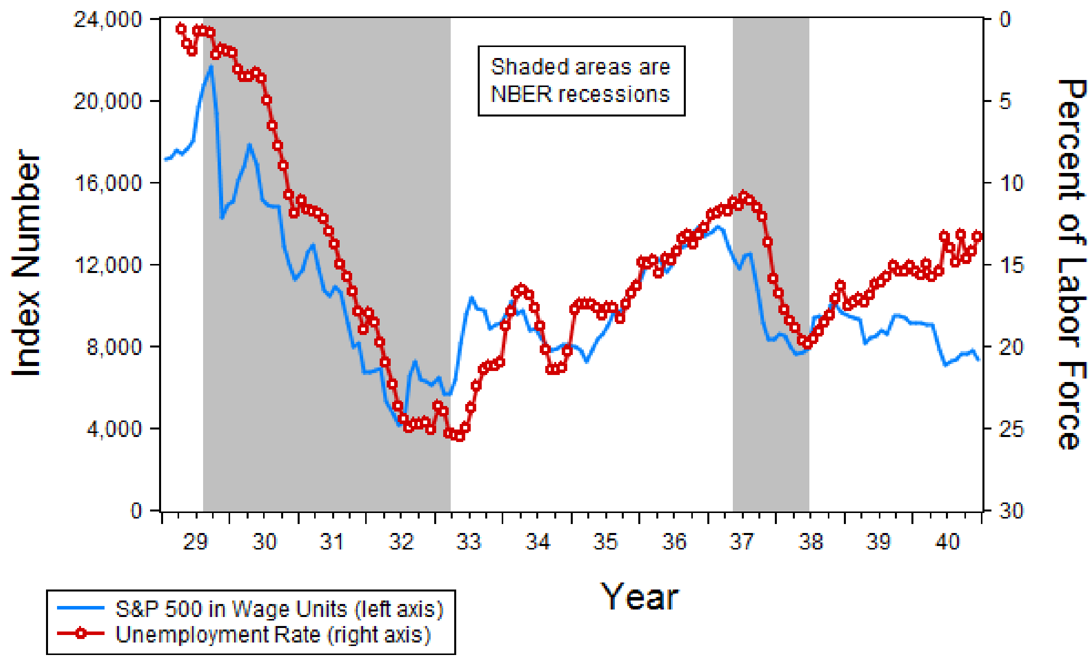

Figure 5: Unemployment and the Stock Market in the U.S., 1928 – 1942

Depression and in the years up to the start of WWII.[27] If the labour market is Walrasian, the mass unemployment associated with the Great Depression must be explained by a shift in the preference for leisure. Or as Modigliani quipped, the Great Depression was a 'sudden attack of contagious laziness'.[28] This seems implausible.

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[27] Source: *Prosperity for All* (2016).

[28] Franco Modigliani, (1977, p. 6).

The connection of the unemployment rate to the stock market documented in Figure 5 is not an experience confined to the Great Depression in the U.S. It is a universal connection that holds in post WWII U.S. data, (Farmer R. E., 2012b), (2015) German data, (Fritsche & Pierdzioch, 2016), and in a panel of industrialized and non-industrialized countries (Pan, 2018). In all of these instances, researchers have provided confirmation of Farmer's initial findings that that the unemployment rate and the stock market are, approximately, co-integrated random walks. Unemployment and the real value of assets each wander randomly but they do not wander too far from each other. The Keynesian version of search theory explains these facts as movements among a continuum of possible steady-state equilibria, caused by self-fulfilling shifts in beliefs.

Keynesian Economics without the Phillips Curve

This article has reviewed models that display dynamic and steady-state indeterminacy but as macroeconomists are fond of saying 'it takes a model to beat a model'.[29] The three equation NK-model is a popular vehicle for understanding how the interest rate, the inflation rate and real GDP are related to each other. How might that model be amended if one accepts the indeterminacy agenda in macroeconomics? In two recent papers, Farmer and Nicolò (2018) (2019) answer that question. They run a horse race of the three-equation NK-model, closed with the Phillips curve, against an alternative *Farmer Monetary* FM-model, that replaces the New-Keynesian Phillips Curve with a parameterized belief function. This alternative model retains equations [6] and [7] but replaces equation [8] with

$$[8a] \qquad \mathbb{E}_t[x_{t+1}] = x_t + u_t^S,$$

[29] Sargent (2011, p. 198).

where $x_t = (y_t + p_t) - (y_{t-1} + p_{t-1})$ is the growth rate of nominal GDP. This equation is an example of beliefs as a new fundamental. The *belief function*, modelled in Equation [8a], captures the idea that people expect nominal income growth next year to equal nominal income growth this year. The model, closed in this way, allows beliefs to wander randomly and its reduced form representation is a system of random walks in which inflation, the money interest rate and the deviation of GDP from potential exhibit non-stationary but cointegrated behaviour. The Farmer Monetary Model displays dynamic ndeterminacy. This feature allows it to capture the fact that prices are sticky in the data. And it displays steady-state indeterminacy. This feature allows it to capture the fact that the unemployment rate is highly persistent and cointegrated with nominal GDP growth, a proxy for movements in real wealth.

The NK-model assumes that the steady-state equilibrium is determinate. The FM-model, allows the steady-state equilibrium to be indeterminate. To understand which model better explains the data, Farmer and Nicolò use Bayesian statistics to compare the posterior odds ratios. The FM-model has a reduced form that allows the data to be non-stationary but cointegrated. The econometric model that captures this reduced form is a Vector Error Correction Model. The NK-model has a reduced form that restricts the data to be stationary. The econometric model that captures this reduced form is a Vector Autoregression. Farmer and Nicolò show that in U.S., U.K. and Canadian data, the posterior odds ratio favour the FM-model by a large margin. It takes a model to beat a model. And the indeterminacy agenda wins the day by a decisive margin.

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