An Empirical Investigation of Quasi-hyperbolic Discounting*

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Abstract

In contrast to the extensive experimental evidence that supports the hypothesis that many people exhibit myopia when planning for the future, very little work has been done to test the hypothesis on field data. This study reports estimates for a structural model of decision making calculated on survey data for a broad cross-section of the UK population. Myopia is identified by focussing upon behavioural margins over pensions and employment in context of an experience effect on wages. The results obtained support the conjecture that allowing for quasi-hyperbolic discounting obtains a better fit of the model to survey data; the short-run discount factor is estimated at 0.825, is significantly less than the long-run discount factor of 0.976.

1 Introduction

There is a growing consensus based on extensive experimental evidence that people are influenced by myopia when making decisions of an intertemporal nature. The view that people are myopic has now become sufficiently main-stream that it features in contemporary debate regarding public policy reform in the United Kingdom (e.g. Pensions Commission, 2005, pp. 68-69, and DWP, 2006b, p. 42). This is of material importance because the assumption that preferences are time inconsistent suggests a role for paternalistic policy intervention, which is absent from the classical rational agent model of behaviour. Despite its growing influence, however, very few studies have investigated the empirical evidence for myopia beyond controlled laboratory experiments, which leaves open the question of how important it is for decisions taken in the field. Here we consider how far behavioural myopia is supported by econometric estimates for a structural model of household savings and labour supply calculated on survey data for the United Kingdom.

One of the principal challenges in testing for myopia on field data is the difficulty in identifying associated margins for estimation.[1] Laboratory analyses of intertemporal preferences address this issue through careful experimental design. These studies commonly focus on present discounted value calculations of stated preferences in relation to discrete alternatives. An experimental study, for example, might ask a subject what payment, A made at time t_A they would require to be indifferent between that payment and another payment B made at time $t_B \neq t_A$. An estimate for the discount factor is

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[1] Laibson et al. (1998), pp. 122-123, for example, note that the rough similarity between the age-consumption profiles generated under the alternative assumptions of exponential and quasi-hyperbolic discounting complicates attempts to distinguish between the two models.

then obtained by calculating the rate of depreciation necessary to equate the present value of payments A and B. By testing over alternative time horizons, it is commonly observed that discount rates are higher in the short-run than the long-run (e.g. Green et al. (1994) and Kirby (1997), see Ainslie (1992) for a review). Unfortunately, the conceptual problems that arise when interpreting this type of evidence through the lens of the life-cycle framework are well known (see Frederick et al. (2002) for a review).

A promising avenue of empirical research on myopia considers structural models of decision making over margins that are distinguished by the timing of their associated welfare effects. This approach is based on the observation that sophisticated myopia gives rise to the potential for "conflict between the preferences of different intertemporal selves", Diamond & Köszegi (2003), p. 1840.[2] Sophisticatedly myopic agents will consequently attach a welfare benefit to commitment mechanisms that help to resolve their intra-personal conflict in favour of their present self, which agents with time-consistent preferences will not, and this disparity in preferences can be exploited for empirical identification. In the context of uncertainty and time-consistent preferences, for example, the illiquidity of a pension fund represents an unambiguous welfare cost because it exaggerates the influence of liquidity constraints prior to pension age. In contrast, a myopic individual may find the illiquidity of a pension to be welfare improving, if this helps to ease the worry that savings set aside for retirement may be spent prematurely.

Exploring decisions that are distinguished by the timing of their welfare effects places particular emphasis on the uncertainty associated with the decision making environment. In context of time-consistent preferences, for example, the bearing that investment illiquidity has on welfare depends crucially upon the uncertainty over future financial circumstances (as referred to above). This emphasis on uncertainty implies that standard econometric analysis of linearised Euler conditions is inappropriate.[3] The related literature has consequently focused on structural models of decision making in the context of uncertainty that are solved using dynamic programming methods.[4] Laibson et al. (2007), for example, estimate a life-cycle model of consumption and investment decisions that distinguishes between (net) liquid assets on the one hand, and a composite illiquid asset that is specified to reflect housing and pensions on the other. Laibson *et al.* estimate their model on US data for households with a high-school but not a college degree. They report that restricting their model to constant exponential discounting results in an estimate for the (per period) discount factor of 0.846 / 0.942 (depending on the weighting matrix applied). Allowing for quasi-hyperbolic discounting results in an estimate for the

[2] Sophisticated myopia refers to the condition where agents are aware of the time-inconsistency that is implied by their myopic preferences. The usual alternative is to assume that myopic agents are naïvely unaware of the time-inconsistency of their preferences. We do not explore the issue of naïve myopia here.

[3] See Browning & Lusardi (1996) for a review of the literature that focuses on linearised Euler conditions. Ludvigson & Paxson (2001) report on the potential biasses for regression estimates of linear approximations to the Euler equation in context of uncertainty.

[4] See, for example, Bellman (1957). The method of dynamic programming is closely related to Optimal Control theory, on which see Dorfman (1969) for a lucid description in an economic context. Zeldes (1989) was the first to study savings decisions in context of earnings uncertainty as a dynamic programming problem.

short-run discount factor of 0.674 / 0.687 and a long-run discount factor of 0.958 / 0.960. Almost all of the specifications that Laibson *et al.* consider reject the restriction that discount rates are equal across all time horizons, and suggest that myopia is of practical importance.

In a similar vein, Fang & Silverman (2007) estimate a model of labour supply and welfare programme participation for never-married mothers, again on US data. Like Laibson et al. (2007), Fang & Silverman (2007) allow for present biassed preferences in the form of quasi-hyperbolic discounting. They consider the hypothesis that people with myopic preferences fail to account fully for the experience effect on future wages of short-run labour supply decisions (an illiquid investment in human capital), thereby giving rise to a bias toward welfare dependency. The estimates that Fang and Silverman report reflect in exaggerated form those reported by Laibson *et al.*: the short-run discount factor at 0.296 / 0.308 (depending on assumed preferences) is significantly lower than the long-run discount factor at 0.875 / 0.868.

However, neither of these studies, nor others that have estimated time varying discount rates on survey data (e.g. DellaVigna and Paserman, 2005, Paserman, 2008, and Shui and Ausubel, 2004), take into account joint decisions over savings and labour supply.[5] These decisions are crucially important in relation to a life-cycle model because they are likely to be jointly determined. A household that holds less wealth than it would like to late in the working lifetime, for example, can choose to work a little longer to off-set their funding short-fall. To the extent that individuals vary both their labour supply and savings in response to unforeseen shocks, omitting either one of these decisions will consequently result in excessive behavioural responses over behavioural margins that are accounted for, with associated implications for estimated parameters.

This paper consequently builds upon the two studies that are cited at length above by exploring the empirical evidence for myopia with reference to joint decisions over the two key marginal decisions of consumption / savings in context of an illiquid pension asset – closely related to Laibson et al. (2007) – and labour / leisure where wages reflect an experience effect – as in Fang & Silverman (2007). In this context, pensions are interesting because they represent one of the most important illiquid assets in which most people choose to invest during the course of their lives; indeed, this illiquidity is much greater than that of housing in the UK since pension wealth cannot normally be used as security for a loan. The focus upon decisions over a pension asset is supported by results reported in Laibson et al. (1998), which suggest that responses to an illiquid pension asset (modelled in their case on a 401(k) plan) are strongly influenced by present-biassed preferences. Decisions over employment where wages respond positively to past labour supply provides a nice contrast to decisions over pension saving, because some of the benefit of a high wage can be enjoyed prior to pensionable age.

[5] Paserman (2008) estimates a model of job search, subject to decisions over search effort and job acceptance.

The empirical support for myopia is assessed using a structural model of savings, labour supply, and pension scheme participation. Decisions are made in context of uncertain wages, employment opportunities, and demographics. The focus on labour supply motivated the explicit inclusion of relationship status, modelled as a stochastic process; this helps to improve the description of labour supply decisions at the household level, and to capture the impact of taxes and benefits. Individuals are also uncertain over their time of death, although they are certain about rates of age specific mortality. This model was estimated on data for a broad segment of the UK population, omitting public sector employees who are eligible to non-contributory pensions[6], and the self-employed whose circumstances upon reaching retirement often depend crucially upon the sale of their respective businesses. The omitted population subgroups accounted for just under 20 percent of the total work force in the UK in 2007/08.[7] The study is also the first to estimate a dynamic programming model of joint labour supply and savings decisions – with or without quasi-hyperbolic discounting – on UK data.[8]

The paper is organised as follows. Details regarding decisions over private pension arrangements in the UK are discussed in Section 2. The structural model is described in Section 3, and the estimation approach is discussed in Section 4. Parameter estimates are reported in Sections 5 and 6, with the preference parameters of interest reported in the second of these. Section 7 concludes.

2 Private Pension Decisions in the UK

Provisions for retirement in the UK are arranged in three tiers. The first tier is comprised of means-tested benefits that provide a welfare safety-net to insure against pensioner poverty. The second tier consists of a number of state administered pensions. And the third tier is represented by personal pensions administered through the private sector. The two tiers administered by the public sector were significantly amended under reforms set out in the 2006 Pensions White Paper (DWP, 2006a), and the resulting system of state retirement benefits is fully represented in the model. As these aspects of the pension system are tangential to the focus of analysis they are not discussed further here; the interested reader is referred to the Pensions White Paper for further details, or to O'Dea et al. (2007) for a detailed

[6] These include employees of the armed forces, national government, local government services, justice, police, fire, and social security departments.

[7] Calculated on 2007/08 FRS data, which indicates 12 percent of all workers self employed, and 7.6 percent employed in public sector (SIC code 75).

[8] Econometric estimates of dynamic programming models of saving have focussed exclusively on US data. These started with Gourinchas & Parker (2002), who report estimates for a constant exponential discount factor between 0.957 and 0.960 on nationally representative data, and find no evidence of significant differences in discount factors by education or occupation groups. In contrast Cagetti (2003) estimates constant exponential discount factors for people with no education of between 0.781 and 0.948, and between 0.977 and 1.14 for people with college qualifications. French (2005) was the first to estimate a model with endogenous consumption and labour decisions and obtained estimates for a constant exponential discount factor between 0.981 and 1.04. Chatterjee et al. (2007) focus on decisions over unsecured indebtedness and default, and report an estimate for the exponential discount factor of 0.917. Importantly the model considered by Chatterjee *et al.* also treats labour supply as exogenous. In an analysis of the savings decisions of retired people, Nardi et al. (2009) report estimates for an exponential discount factor of 0.97 under each of the four empirical specifications that they consider. They find little evidence of savings for a bequest motive, and suggest that most saving by the elderly in the US is undertaken to off-set uncertainty over longevity and medical costs.

summary of the contemporary system of state sponsored retirement income in the UK. The remainder of this section focusses upon the decisions over personal pensions that have an important bearing on the estimation results reported later in the paper.

The decision to participate in a private pension administered outside the public sector is a statutory right of any individual under age 75 and normally resident in the UK. Contributions to private pensions and associated investment returns are tax exempt, and an individual can take up to 25% of a pension fund's value as a tax-free lump-sum upon retirement (the remainder must be used to purchase a life annuity). Just over 60% of all full-time employees of working age chose to contribute to some form of private pension in 2007/08.[9]

An important factor in encouraging participation in private pensions in the UK is the extent of employer sponsorship; employers often make matching pension contributions on behalf of their employees, and have traditionally offered occupational pensions on a defined benefit (final salary) basis.[10] The study abstracts from the former of these issues by focussing upon a decision making environment in which the rate of return to all forms of wealth is assumed to be non-stochastic, and age specific mortality rates are known. Summary statistics regarding the latter are reported in Table 1.

Table 1 distinguishes between four types of private pension scheme: *occupational pensions (OP)*, which employers provide in-house to their employees; *group stakeholder pensions (GSHP)*, which are a form of private pension that is subject to statutory limits on administrative costs, and which employers negotiate on behalf of their employees with third-party pension providers; *group personal pensions (GPP)*, which are like GSHPs, but have no statutory limit applied to the costs of administration; and *privately arranged pensions (PAP)* that individuals have negotiated themselves with third-party providers. The top panel of the table indicates that the largest of these three schemes by active membership in 2007 were OPs, which covered 30% of all employees in the private sector. The next largest were GPPs, which covered 9% of all employees, followed by PAPs (7%), and GSHPs (3%). The statistics reveal that employers contributed to the vast majority of pensions that they arranged on behalf of their employees (OP, GSHP, GPP), but to very few that they did not (PAP). Employees, in contrast, contributed to almost all of the private pension schemes that are reported in the table, which is largely because the largest population subgroup that were eligible to non-contributory pensions (public sector employees) are excluded from the analysis.

The bottom panel of Table 1 reveals substantial heterogeneity in employer pension contributions

[9] Based on data reported by the 2007/08 wave of the Family Resources Survey, focussing on population aged 20 to the year prior to state pension age (65 for men and 60 for women). Private pensions include both employer sponsored and privately arranged schemes.

[10] Most employers in the UK have a legal obligation to offer their employees access to some form of private pension, though they currently have no obligation to make matching pension contributions. The proportion of employer sponsored pensions specified as defined contribution increased from 17% in 1997 to 31% in 2008 (see Figure 7.4, *Pension Trends*, 2009, ONS).

Table 1: Private pension arrangements of employees in the private sector, 2007

	OP	GSHP	GPP	PAP
Membership rates (% of all private sector employees)				
all active members	30	3	9	7
only employee contributes	0	0	1	6
only employer contributes	2	0	1	0
both contribute	28	3	8	1
Distribution of employer pension contributions where these are made (%)				
Percentage of gross earnings (where known)				
less than 3%	0	9	3	6
3-3.9%	1	16	26	6
4-4.9%	3	7	11	6
5-5.9%	3	41	27	10
6-10%	22	16	26	32
more than 10%	70	10	8	39
not known*	10	32	10	68
mean percentage of earnings	16	6	6	16
median percentage of earnings	15	5	5	10

Source: Forth and Stokes (2008), and author's calculations on 2007/08 FRS data

Notes: All statistics weighted to account for sampling issues

OP = Occupational Pension; GSHP = Group Stakeholder Pension; GPP = Group Personal Pension

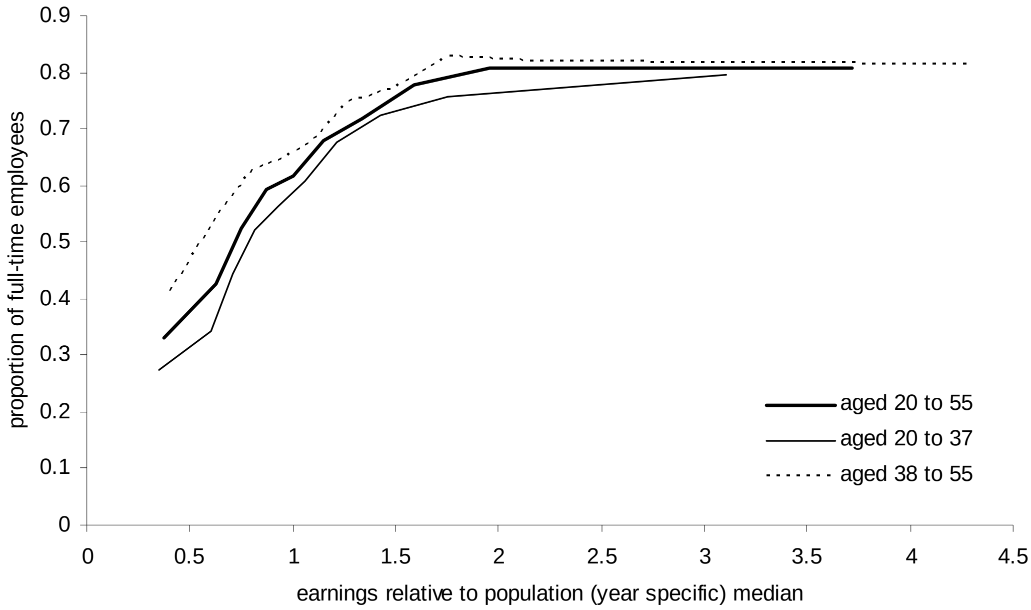
PAP = privately arranged Personal and Stakeholder Pensions

* includes both unidentifed rates of contribution, and contributions defined as fixed monetary sums

(where the employer contributes), both within and between scheme types. Ignoring PAPs – to which few employers make an active contribution – the table indicates that employer contributions to OPs, specified as a proportion of employee gross earnings, were on average almost three times the contributions made to either GSHPs or GPPs.

The above data highlight the importance of employer pension provisions in determining rates of private pension participation. But not all employees are eligible for an employer sponsored pension in the UK. Data on pension scheme eligibility are reported in Figure 1. This figure reveals that employee eligibility to a employer sponsored pension bears a stronger correlation with income than it does with age. Eligibility to an employer sponsored pension increases from a low of between 27 and 41 percent among individuals on less than half of median earnings (increasing by age group), to between 76 and 83 percent among individuals on more than one and a half times median earnings. At very high earnings, the proportion of employees eligible to an employer sponsored pension peaks at just over 80 per cent.

Data reported by the FRS indicate that of the 65 percent of full-time employees who believed that they were eligible to belong to a pension scheme run by their employer in 2006/07 and 2007/08, only 50 percent chose to participate in their employer's sponsored pension. The vast majority of the remainder chose not to contribute to a private pension at all (2.5 percent were identified as substituting a privately arranged personal pension for their employer sponsored scheme). Hence, more than one in every six employees who believed that they were eligible to participate in their employer's sponsored pension chose not to take up their option. Similarly, although any individual under age 75 and normally resident in the



Source: Author's calculations on individual level data from the 2006/07 and 2007/08 waves of the FRS

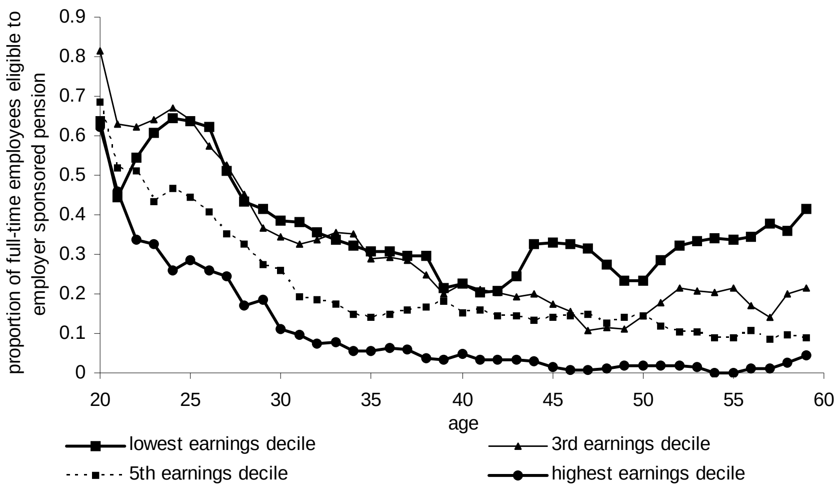
Notes: Earnings deciles defined within survey waves, and averaged across waves

Excludes employees in the public sector and the self-employed

Figure 1: Eligibility rates of full-time employees to employer sponsored pensions by age and earnings

UK can choose to contribute to a private pension, approximately 40 percent of all full-time employees chose not to participate in any form of private pension in 2006/07 and 2007/08.

Figure 2 disaggregates these statistics by reporting the proportion of full-time employees who were eligible to an employer sponsored pension, but were not participating in any type of private pension, by age and earnings decile. This figure reveals that high proportions of young employees who were eligible for employer sponsored pensions chose not to take up any form of private pension, regardless of their position in the earnings distribution. Rates of participation in private pensions climbed throughout the earnings distribution to age 40, with the highest rates of increase observed among individuals toward the top of the earnings distribution. From age 40, there is some evidence of falling participation in private pensions among individuals in the lowest income decile, which is not evident among higher earning subgroups. By age 45, very few full-time employees in the top earnings decile who were eligible for an employer sponsored pension did not contribute to some form of private pension, in contrast to the bottom earnings decile for which just over 30 percent chose not to contribute. These statistics indicate a substantial degree of behavioural heterogeneity, which appears to be consistent with life-cycle savings motives that are weaker at younger ages and among individuals in the lowest earnings deciles.



Source: Author's calculations on individual level data from the 2006/07 and 2007/08 waves of the FRS

Notes: Earnings deciles defined within survey waves, and averaged across waves

Five year moving averages reported

Excludes employees in the public sector and the self-employed

Figure 2: The proportion of full-time employees who are eligible to an employer sponsored pension, but do not contribute to a private pension scheme, by age and earnings decile

3 A Structural Model of Savings and Labour Supply

We are concerned with how far data regarding decisions over labour supply and pension scheme participation provide empirical support for quasi-hyperbolic discounting. Particular care has consequently been taken to limit the structural model to treat only those characteristics that are important in determining private sector attitudes toward pensions. The household – defined as a single adult or partner couple and their dependant children – is the unit of analysis, and was selected in response to the prevalence of pooled saving and consumption. The model divides the life course into annual increments, and projects household decisions regarding consumption, labour supply, and pension scheme participation. These decisions are considered to be made to maximise expected lifetime utility, given a household's prevailing circumstances, their preferences, and beliefs regarding the future. A household's circumstances are described by their age, number of adults, number of children, earnings, net liquid worth, pension rights, and time of death. The belief structure is rational in the sense that expectations are consistent with the intertemporal decision making environment, and the model is partial equilibrium in that the distribution of wages and returns to saving are independent of agent decisions.

Incorporating an appreciation of uncertainty into individual expectations regarding future circumstances increases the complexity of the utility maximisation problem. Of the seven characteristics that define the circumstances of a household, four are stochastic (earnings, relationship status, number of children, and time of death), and three are deterministic (age, pension rights, and net liquid worth).

In the terminology of the dynamic programming literature, consumption, labour supply, and pension membership are control variables, that are selected to maximise expected lifetime utility, subject to seven state variables, four of which are stochastic. This section gives an abbreviated description of the structural model; for a more detailed description, see van de Ven (2009).

3.1 The utility function

Expected lifetime utility of household i at age t is described by the time separable function:

$$U_{i,t} = \frac{1}{1-\gamma} \left\{ u\left(\frac{c_{i,t}}{\theta_{i,t}}, l_{i,t}\right)^{1-\gamma} + \beta E_t \left[\sum_{j=t+1}^{t_{death}} \delta^{j-t} u\left(\frac{c_{i,j}}{\theta_{i,j}}, l_{i,j}\right)^{1-\gamma} \right] \right\} \quad (1)$$

where intratemporal utility u_t is combined in the intertemporal specification by taking an isoelastic transformation. E_t is the expectations operator; β and δ are discount factors (assumed to be the same for all households); t_{death} is the age at death; $c_{i,t} \in R^+$ is discretionary composite consumption; $l_{i,t} \in [0,1]$ is the proportion of household time spent in leisure; and $\theta_{i,t} \in R^+$ is adult equivalent size based on the "revised" or "modified" OECD scale. t_{death} is considered to be uncertain, subject to known age specific probabilities of survival. $\phi_{j-t,t}$ describes the probability of surviving to age j given survival to age t.

The labour supply decision is chosen from a set of discrete alternatives, which reflects the view that this provides a closer approximation to reality than if it is defined as a continuous decision variable for given wage rates.[11] To the extent that the focus on discrete labour options limits employment decisions relative to reality, it will dampen the responsiveness of labour supply behaviour implied by the simulation model, and dampen variation in employment incomes. The former of these effects implies that the parametrisation of the model may require a labour elasticity that overstates the practical reality, while the latter suggests that excessive variation in labour incomes may be required to reflect the wage dispersion described by survey data. The assumption that leisure decisions are made between discrete alternatives also implies that the preferred consumption to leisure mix will not be independent of the wealth endowment, which might otherwise be the case given the homothetic form that is assumed for preferences (described below).

The modified OECD scale assigns a value of 1.0 to the household reference person, 0.5 to each additional adult member and 0.3 to each child, and is currently the standard scale for adjusting before housing costs incomes in European Union countries. Its inclusion in the preference relation reflects the fact that household size has been found to have an important influence on the timing of consumption (e.g. Attanasio & Weber (1995), Blundell et al. (1994), and Fernandez-Villaverde & Krueger (2006)).

[11] Although working time arrangements have become increasingly flexible since the 1970s, substantial labour market rigidities continue to affect employment decisions. Fagan (2003), for example, reports that approximately 1 in 5 employed people in Europe work full-time when they would prefer to work part-time.

The model assumes that the discount parameters are the same for all individuals, and are time invariant. This is in contrast to the approach that is adopted by Gustman & Steinmeier (2005), who allow variation in the rate of time preference to be an important factor in reflecting heterogeneity in household retirement behaviour. We have not allowed for heterogeneity in this form to ensure that variation in household behaviour implied by the model is driven by heterogeneity in observable household characteristics.

A Constant Elasticity of Substitution function was selected for within period utility,

$$u\left(\frac{c_{i,j}}{\theta_{i,j}}, l_{i,t}\right) = \left(\left(\frac{c_{i,j}}{\theta_{i,j}}\right)^{(1-1/\varepsilon)} + \alpha^{1/\varepsilon} l_{i,t}^{(1-1/\varepsilon)}\right)^{\frac{1}{1-1/\varepsilon}} \tag{2}$$

where $\alpha > 0$ is referred to as the utility price of leisure, and $\varepsilon > 0$ is the (period specific) elasticity of substitution between equivalised consumption $(c_{i,t}/\theta_{i,t})$ and leisure $(l_{i,t})$. This specification of preferences implicitly assumes that characteristics which affect utility, but are not explicitly stated, enter the utility function in an additive way. Although not explicitly included in the preference relation, accidental bequests do occur due to the uncertainty assumed over the time of death. Where a household dies with positive wealth balances, these are assumed to accrue to the state in the form of a 100% inheritance tax.

We are principally concerned with how far the short-run discount factor, β departs from 1. If $\beta = 1$, then the preferences described by equations (1) and (2) are time-consistent, and take a form that is standard in the associated literature (despite the contention over the assumption of time separability; see Deaton & Muellbauer (1980), pp. 124-125, or Hicks (1939), p. 261). If $\beta < 1$, then preferences exhibit a present bias, and are time inconsistent.

3.2 The wealth constraint

Equation (1) is maximised, subject to the credit constraint on liquid net worth, $w_{i,t} \geq D_t$ for all households i at all ages t. Liquid net worth is a composite asset that reflects the aggregate value of housing wealth and net financial securities. Intertemporal variation of $w_{i,t}$ is described by:

$$w_{i,t} = \begin{cases} \hat{w}_{i,t} & t \neq t_{SPA} \\ \hat{w}_{i,t} + 0.25 w_{i,t}^{p} & t = t_{SPA} \end{cases} \tag{3a}$$

$$\hat{w}_{i,t} = \begin{cases} \pi_{div}\left(w_{i,t-1} - c_{i,t-1} + \tau_{i,t-1}\right) & n_t^a < n_{t-1}^a, t < t_{SPA} \\ w_{i,t-1} - c_{i,t-1} + \tau_{i,t-1} & \text{otherwise} \end{cases} \tag{3b}$$

$$\tau_{i,t} = \tau(l_{i,t}, x_{i,t}, n_{i,t}^a, n_{i,t}^c, r_{i,t} w_{i,t}, pc_{i,t}, t) \tag{3c}$$

where $w_{i,t}^{p}$ denotes wealth held in personal pensions; t_{SPA} is state pension age; π_{div} is the proportion of net liquid worth that is assumed to be lost upon marital dissolution prior to t_{SPA} (to capture the impact of divorce); and $\tau\left(.\right)$ denotes disposable income net of non-discretionary expenditure. At t_{SPA}, 75% of personal pension wealth is converted into an inflation adjusted life annuity, and the remainder is received as a tax free lump-sum, reflecting the current statutory maximum lump sum that can be drawn from a retirement pension in the UK (discussed in Section 5.9). We assume that pension assets can only be accessed from age t_{SPA}, which implies that pension contributions can be used by sophisticatedly myopic agents to lock savings away to fund consumption in retirement. This is the first of two principal dimensions that are taken into account in evaluating the empirical evidence for (sophisticated) myopia.

Particular care was taken in formulating the model to reflect the effects of taxes and benefits on household disposable incomes. Equation (3c) indicates that taxes and benefits are calculated with respect to labour supply, $l_{i,t}$; private non-property income, $x_{i,t}$; the numbers of adults, $n^a_{i,t}$, and children, $n^c_{i,t}$; the return to liquid assets, $r_{i,t}w_{i,t}$ (which is negative when $w_{i,t} < 0$); private contributions to pensions, $pc_{i,t}$; and age, t.

3.3 Disposable income, τ

The lifetime is divided into two periods when calculating disposable income: the working lifetime $t < t_{SPA}$, and pension receipt $t_{SPA} \leq t$. Throughout the lifetime, household disposable income is calculated by:

1. evaluating aggregate *take-home pay* from the taxable incomes of each adult member of a household – this reflects the taxation of individual incomes in the UK

2. calculating *benefits* receipt (excluding adjustments for childcare and housing costs) from aggregate household take-home pay – this reflects the fact that benefits tend to be provided at the level of the family unit

3. calculating non-discretionary *net childcare costs* (after adjusting for childcare related benefits) from aggregate take-home pay – of separate importance because of their bearing upon labour supply decisions

4. calculating non-discretionary *net housing costs* (after adjusting for relevant benefits receipt) from aggregate take-home pay plus benefits less childcare costs – this reflects the fact that 'Housing Benefit' and 'Council Tax Benefit' in the UK are means tested with respect to income net of most other elements of the tax and benefits system

5. household *disposable income* is then equal to aggregate take-home pay, plus benefits, less net childcare costs, less net housing costs.

Calculation of taxable income for each adult in a household depends on the household's age, with property and non-property income treated separately. Prior to state pension age, $t < t_{SPA}$, household non-property income $x_{i,t}$ considered for tax purposes is equal to labour income $g_{i,t}$ less pension contributions. From state pension age, $x_{i,t}$ is equal to labour income plus pension annuity income:

$$x_{i,t} = \begin{cases} g_{i,t} - pc_{i,t} & t < t_{SPA} \\ g_{i,t} + pp_{i,t} + sp_t & t \geq t_{SPA} \end{cases} \tag{4}$$

$$\text{where} \quad : \quad pp_{i,t} = \begin{cases} \chi 0.75 w^p_{i,t} & t = t_{SPA} \\ \left(\frac{\pi^s + (1-\pi^s).(n^a_{i,t}-1)}{\pi^s + (1-\pi^s).(n^a_{i,t-1}-1)} \right) pp_{i,t-1} & t > t_{SPA} \end{cases} \tag{5}$$

$pp_{i,t}$ denotes private pension annuity, sp_t denotes state pension income, and χ is the annuity rate. The annuity purchased at age t_{SPA} is inflation linked, and reduces to a fraction π^s of its (real) value in the preceding year if one member of a couple departs the household (to reflect spousal mortality). This treatment of pension contributions and pension annuity income reflects the EET form of taxation that is applied in the UK, in common with most other OECD countries.

Non-property income is only allocated between spouses, for households under state pension age and where both members of the couple work; otherwise all non-property income is allocated to a single spouse. Where non-property income is allocated between spouses, a separate ratio is used to split household employment income for each feasible labour decision. Similarly, property income is only allocated between spouses for households below state pension age, and who supply some labour. In this case, property income is allocated evenly between working couples.

Property income, $y_{i,t}$, is equal to the return from positive balances of liquid net worth:

$$y_{i,t} = \begin{cases} r_{i,t} w_{i,t} & \text{if } w_{i,t} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The interest rate on liquid net worth is assumed to be non-stochastic, and to depend upon whether $w_{i,t}$ indicates net investment assets or net debts:

$$r_{i,t} = \begin{cases} r^I & \text{if } w_{i,t} > 0 \\ r^D_l + \left(r^D_u - r^D_l\right) \min\left\{ \frac{-w_{i,t}}{\max\left[g_{i,t}, 0.7 g(h_{i,t}, l^{ft}_{i,t})\right]}, 1 \right\}, r^D_l < r^D_u & \text{if } w_{i,t} \leq 0 \end{cases}$$

where $l^{ft}_{i,t}$ is household leisure when one adult in household i at age t is full-time employed. This specification for the interest rate implies that the interest charge on debt increases from a minimum of r^D_l when the debt to income ratio is low, up to a maximum rate of r^D_u, when the ratio is high. The specification also implies that households that are in debt are treated less punitively if they have at least one adult earning a full-time wage than if they do not.

Disposable income is given by:

$$\tau_{i,t} = \begin{cases} \hat{\tau}_{i,t} & \text{if } w_{i,t} \geq 0 \\ \hat{\tau}_{i,t} + r_t w_{i,t} & \text{otherwise} \end{cases} \tag{7}$$

$$\hat{\tau}_{i,t} = \begin{cases} x_{i,t} + y_{i,t} - tax_{i,t} + benefits_{i,t} - cc_{i,t} - hsg_{i,t} & \text{if } t < t_{SPA} \\ x_{i,t} + y_{i,t} - tax_{i,t} + benefits_{i,t} - hsg_{i,t} & \text{if } t \geq t_{SPA} \end{cases} \tag{8}$$

where $tax_{i,t}$ denotes the simulated tax burden, $benefits_{i,t}$ welfare benefits received, $cc_{i,t}$ non-discretional childcare costs net of associated benefits, and $hsg_{i,t}$ non-discretional housing costs net of associated benefits. Equations (6) and (7) indicate the interest cost on loans (when $w_{i,t} < 0$) cannot be written off against labour income for tax purposes.

3.4 Pension savings

As is implicit in Subsection ??, pensions are modelled at the household level, and are defined contribution in the sense that every household is assigned an account into which their respective pension contributions are notionally deposited. The notional pension account accrues a (post-tax) rate of return, r^p, which is assumed to be certain. Prior to age t_{SPA}, households with some employment in the prevailing year choose whether to contribute to their pension. Households that choose to contribute to their pension in any given year are referred to as participating. Participating households contribute a fixed proportion, π^{pc}, of their (total) labour income to their notional pension account. The pension accounts of participating households also benefit from matching contributions, paid at the rate π^p_{ec} on labour income, which are made on their behalf by their respective employers. The balance of household i's (notional) pension account at any age, $t < t_{SPA}$, is given by:

$$\hat{w}^p_{i,t} = \begin{cases} (1+r^p)\, w^p_{i,t-1} + (\pi^{pc} + \pi^p_{ec})\, g_{i,t} & \text{if household } i \text{ participates at age } t \\ (1+r^p)\, w^p_{i,t-1} & \text{otherwise} \end{cases}$$

$$w^p_{i,t} = \begin{cases} \pi_{div} \hat{w}^p_{i,t} & n^a_t < n^a_{t-1} \\ \hat{w}^p_{i,t} & \text{otherwise} \end{cases} \tag{9}$$

where $g_{i,t}$ defines aggregate household labour income.

This specification of personal pensions is designed to reflect employer sponsored pensions in the UK, which usually offer very little flexibility over important margins, including contribution rates, investment allocations, dispersions, and so on.

3.5 Labour income dynamics

Three household characteristics influence labour income: the household's labour supply decision, its latent wage, $h_{i,t}$, and whether a wage offer $wo_{i,t}$ is received.[12] A wage offer is received with a relationship specific (exogenous) probability, $p^{wo}\left(n^a_{i,t}\right)$. This is included to capture the incidence of (involuntary) unemployment. If a household receives a wage offer, then its labour income is equal to a fraction of its latent wage, with the fraction defined as an increasing function of its labour supply. A household that receives a wage offer and chooses to supply the maximum amount of labour receives its full latent wage, in which case $g_{i,t} = h_{i,t}$. A household that does not receive a wage offer is assumed to receive $g_{i,t} = 0$ regardless of its labour supply decision (implying no labour supply where employment incurs a leisure penalty).

[12] Defining wage potential at the household level rather than at the level of the individual significantly simplifies the analytical problem by omitting the need to take account of a range of issues including the sex of employees, imperfect correlation of temporal innovations experienced by spouses, and so on.

Latent wages are assumed to evolve as a random walk with drift:

$$\begin{aligned} \log\left(h_{i,t+1}\right) - \log\left(h_{i,t}\right) &= f_h\left(n^a_{i,t}, t\right) + \kappa\left(n^a_{i,t}, l_{i,t}\right) + \omega_{i,t} \\ g_{i,t} &= \mu^{rel\_trans}\left(emp_{i,t}\right) h_{i,t} \end{aligned} \tag{10}$$

where $\kappa(.)$ is an experience effect, and $\omega_{i,t} \sim N\left(0, \sigma^2_{\omega,n^a_{i,t}}\right)$ is a household specific disturbance term. $\mu^{rel\_trans}\left(emp_{i,t}\right)$ is a multiplicative adjustment factor that defines the proportion of the latent wage h obtained as an observed wage g, varying by employment status *emp*.

Most of the associated literature omits an experience effect from the wage process as this complicates solution of the utility maximisation problem by invalidating two-stage budgeting. Related studies have, however, found it difficult to match the high rates of labour market participation that are reported in survey data among the young relative to the old in context of the strong wage growth that is typically observed with age. French (2005) suggests that this consideration was behind the high estimated values that he reports for the discount factor. Career building appears to be a plausible explanation for the high rates of employment participation that are observed among people early in the working lifetime, and an experience effect is included to capture this. Furthermore, when wages depend upon accrued labour market experience, then labour supply can be used by sophisticatedly myopic consumers to commit savings for the longer term (as in Fang & Silverman (2007)). This is the second of two principal dimensions that are taken into account in evaluating the empirical evidence for (sophisticated) myopia.

3.6 Household demographics

Household relationship status is exogenous and stochastic. The probability of a relationship transition at any age is described by the reduced form logit equation:

$$s_{i,t+1} = f_s(t) + \alpha^A s_{i,t} \tag{11}$$

where $s_{i,t}$ is a dummy variable, that takes the value 1 if household i is comprised of a single adult at age t and zero otherwise. The number of children in a household is considered to evolve as a deterministic function of household age and relationship status, so that: $n^c_{i,t} = n^c\left(n^a_{i,t}, t\right)$.

3.7 Model solution

The allowance that is made for stochastic income implies that an analytical solution to the utility maximisation problem does not exist, and numerical solution routines need to be employed. Starting in the last possible period of a household's life, T, uncertainty plays no further role and the optimisation problem is simple to solve for given numbers of adults n^a_t, liquid net worth w_T, and annuity income p_T, omitting the household index i for brevity. We can then denote the maximum achievable utility in period T, the value function, by $V_T(n_T^a, w_T, p_T)$:

$$V_T(n_T^a, w_T, p_T) = u\left(\frac{\widehat{c}_T\left(n_T^a, w_T, p_T\right)}{\theta_T}, 1\right) \tag{12}$$

$$W_T(n_T^a, w_T, p_T) = V_T(n_T^a, w_T, p_T) \tag{13}$$

where $\widehat{c}_T$ denotes the optimised measure of consumption, and $\hat{l}_T$ optimised leisure. V_T is solved at each node of the three dimensional grid over the permissable state space (n_T^a, w_T, p_T). W_T is an intermediate term that is stored to evaluate utility maximising solutions in period $T-1$; it is necessarily equal to V_T (as indicated above) in the final period, but may differ from V_T in earlier periods as described below.

At time $T-1$, the maximisation problem reduces to solving the Bellman equation:

$$V_{T-1}(n_{T-1}^a, w_{T-1}, h_{T-1}, wo_{T-1}, p_{T-1}) = \max_{c_{T-1}, l_{T-1}} \left\{ \frac{1}{1-\gamma} u\left(\frac{c_{T-1}}{\theta_{T-1}}, l_{T-1}\right)^{1-\gamma} + \right.$$
$$\left. + \frac{\beta\delta}{1-\gamma}\phi_{1,T-1}E_{T-1}\left[W_T(n_T^a, w_T, p_T)^{1-\gamma}\right]\right\} \tag{14}$$
$$W_{T-1}(n_{T-1}^a, w_{T-1}, h_{T-1}, wo_{T-1}, p_{T-1}) = \frac{1}{1-\gamma} u\left(\frac{\widehat{c}_{T-1}}{\theta_{T-1}}, \widehat{l}_{T-1}\right)^{1-\gamma} +$$
$$+ \frac{\delta}{1-\gamma}\phi_{1,T-1}E_{T-1}\left[W_T(n_T^a, w_T, p_T)^{1-\gamma}\right] \tag{15}$$

subject to the intertemporal dynamics that are described above. Note that, $W_{T-1} \neq V_{T-1}$, if $\beta \neq 1$, which indicates the influence of time inconsistency in the context of myopic preferences. This optimisation problem is solved for the $T-1$ value function V_{T-1} and intermediate term W_{T-1} at each node of the five dimensional grid over the permissable state-space. The expectations operator is evaluated in the context of the log-normal distribution that is assumed for wages using the Gauss-Hermite quadrature, which permits evaluation at a discrete number of abscissae (five abscissae are used). Linear interpolation methods are used to evaluate the value function at points between the assumed grid nodes throughout the simulated lifetime.

Solutions to the optimisation problem are identified by searching over the value function, using Powell's method in multiple dimensions and Brent's method in a single dimension (see Press et al. (1986)). Although these routines are efficient when the objective function is reasonably well behaved, they are not designed to distinguish between local and global optima. A supplementary search routine is consequently used, which tests over a localised grid above and below an identified optimum for a preferred decision set. If a preferred decision set is identified, then the supplementary routine searches recursively for any further solutions. This process is repeated until no further solutions are found. Of all feasible solutions, the one that maximises the value function is selected.

Solutions for ages less than $T-1$ then proceed via backward induction, based upon the solutions obtained for later ages. For ages under t_{SPA}, solutions are also required for the pension participation decision, and pension wealth replaces annuity income in the state space. A more complete description of the analytical problem, including the treatment of boundary conditions, is reported in van de Ven (2009).

Having solved for utility maximising behavioural responses at grid nodes (described above), the life-courses of individual households are simulated by running households forward through the grids. This is done by first populating a simulated sample by taking random draws from a joint distribution of all potential state variables at the youngest age considered for analysis. The behaviour of each simulated household, i, at the youngest age is then identified by reading the decisions stored at their respective grid co-ordinates. Given household i's characteristics (state variables) and behaviour, its characteristics are aged one year following the processes that are considered to govern their intertemporal variation. Where these processes depend upon stochastic terms, new random draws are taken from their respective distributions (commonly referred to as Monte Carlo simulation). This process is repeated for the entire simulated life of each household. The data generated for the simulated cohort are then used as the basis for estimation, as is described in the following section.

4 Estimation Approach

4.1 Methodology

The model that is described in Section 3 – hereafter referred to as *the (structural) model* – is designed to identify household decisions regarding labour supply, consumption, and pension participation as simultaneous non-parametric functions of a set of observable characteristics that include age, relationship status, human capital, and accrued wealth. Suppose that detailed panel data exist regarding all of the characteristics that are treated by the model, collected from a period when the tax and benefit regime was stable. Then estimation would proceed by adjusting the model's parameters to minimise the disparity – as measured by some assumed loss function – between the household decisions projected by the model and those described by the associated survey data. Unfortunately this approach to estimation is made impractical by the limitations of available survey data, and the computational burden associated with testing over alternative parameter combinations. These two difficulties are addressed here by estimating the model parameters using the Method of Simulated Moments (MSM), following Gourinchas & Parker (2002).

The MSM, in the tradition of the Generalised Method of Moments, adapts to the limitations of the available survey data by adjusting model parameters to match moments for a selected set of population characteristics. The computational burden of the study is limited by partitioning the parameters into two sub-groups, which are estimated in separate stages. Parameters that are independently observable are estimated exogenously from the structural model. Given these exogenous estimates, the remaining parameters of the model are then adjusted to minimise a weighted loss function.

Define the moment conditions for the second stage estimation, $(m_1, ..., m_k)$, and the associated loss function:

$$F = g(\theta, \hat{\chi})' W g(\theta, \hat{\chi}) \tag{16}$$

where W is a $k \times k$ weighting matrix, and $g(\theta, \hat{\chi})$ is a $k \times 1$ vector, the ith element of which describes the simulated moment less the associated sample moment, $g_i(\theta, \hat{\chi}) = \bar{m}_i(\theta, \hat{\chi}) - \hat{m}_i$ for all $i = (1, ..., k)$. The sample moments, $\hat{m}_i$, are estimated from observed survey data. The simulated moments, $\bar{m}_i(\theta, \hat{\chi})$, are calculated from data that are generated by the model for a simulated population cohort (as described in subsection 3.7), given the parameter vectors θ and $\hat{\chi}$.

θ is the vector of parameters that are adjusted in the second stage of the estimation, and χ the vector of parameters that are estimated in the first stage and held fixed in the second stage. W is commonly specified with reference to the optimal weighting matrix in the region of the (hypothetically) true population parameters $\theta \rightarrow \theta_0$ and $\hat{\chi} \rightarrow \chi_0$:[13]

$$W_{opt} = \left[\hat{I} E\left\{g(\theta_0, \chi_0) g(\theta_0, \chi_0)'\right\} + \hat{I} G_\chi \hat{J}^{-1} V_\chi G'_\chi\right]^{-1} \tag{17}$$

Here $\hat{I}$ is a diagonal matrix with elements $\hat{I}_{i,i} = I_i$, the sample size used to estimate the ith moment from survey data. The analogue to $\hat{I}$ for parameters χ is denoted $\hat{J}$ (so that the number of observations used to estimate the ith parameter of the vector χ is equal to $\hat{J}_{i,i}$). $E\{.\}$ is the expectations operator, and G_χ is the $k \times l_1$ matrix of first derivatives of $g(.)$ with respect to the parameters χ (l_1 being the number of parameters included in the vector χ). V_χ is the covariance matrix of the parameter vector χ.

The second term within the square brackets of (17) is the impact of the χ estimates on the estimates for θ. The specification of this second term indicates that the estimates for χ will have a negligible impact on the second stage estimation where the χ estimates are based on substantially larger sample sizes relative to the second stage, or where the parameters estimated in the first stage have little impact on $g(\theta_0, \chi_0)$, or where the variance of the first stage estimates is small.

A number of studies that report estimates based on the MSM omit the influence of the first stage estimates on the weighting matrix. This removes the need to address the computational burden of numerically evaluating G_χ, and results in the 'robust' weighting matrix:

$$W_{robust} = \left[\hat{I} E\left\{g(\theta_0, \chi_0) g(\theta_0, \chi_0)'\right\}\right]^{-1} \tag{18}$$

Although the optimal weighting matrix W_{opt} is more efficient than W_{robust}, use of W_{robust} in the estimation is usually justified on the basis that W_{opt} can result in more biassed estimates in context

[13]See Appendix A for derivation.

of small samples (see Altonji & Segal (1996)).[14] Studies that report estimates obtained using both the optimal and robust weighting matrices commonly find that the choice between the two has little bearing upon the qualitative results obtained. This study consequently reports estimates obtained using the robust weighting matrix only.

Following Gourinchas & Parker (2002), it is common practice to assume zero covariance terms when evaluating W_{robust}. This is a reasonable assumption when the alternative moments considered for the second stage estimation are calculated on survey data for mutually exclusive population sub-samples. Similarly, it is reasonable to assume zero cross-correlation between the simulated and sample moments. The same does not, however, tend to hold for the simulated moments of the structural model, which are commonly calculated on data for a single set of Monte Carlo draws, or where the survey data used to estimate alternative sample moments describe the same population sub-sample. In some contexts it seems reasonable to conjecture that cross-correlation terms in the covariance matrix will have little bearing on the weights attached to alternative moments in the second stage estimation. This is not the case in here, however, and so care was taken to take covariance terms properly into account.

4.2 Data

The focus on endogenous labour supply decisions requires taxes and benefits to be explicitly accounted for in the structural model.[15] As a result of this, the changing nature of taxes and benefits over time complicates the use of (pseudo) panel data for estimation: even if the evolving tax and benefits system faced by a given birth cohort were faithfully reproduced by the model, an accurate reflection of the practical reality would require the state space of the model to include uncertainty over the evolving policy environment. A simplifying abstraction is consequently required.

To focus on behaviour in the context of a single policy environment (controlling for *time effects*), the study reports estimates that are predominantly based upon data for a population cross-section. Cross-sectional data, however, fail to describe the circumstances of any given cohort in practice, due

[14] For studies that focus exclusively on estimates derived on W_{robust}, see Cagetti (2003), French (2005), and Nardi et al. (2009). Bucciol (2009) addresses the computational burden associated with W_{opt} by first calculating estimates on W_{robust}. Bucciol then evaluates G_χ once on the preferred parameter combination obtained with W_{robust}, for use in evaluating W_{opt}. Notably, there is very little difference between the estimates that Bucciol (2009) reports based on the two alternative weighting matrices.

[15] To appreciate this point, consider the following. Labour supply decisions affect labour income, and savings decisions affect property income. The aggregate of income from labour and property gives gross private income, which is transformed into disposable income by the tax and benefits schedule. In the UK, as in many countries, the tax and benefits schedule can reasonably be represented by a linear spline in its arguments, which include gross private income, demographics, labour market status and so on. If behaviour in the region of the optimum is unlikely to imply a shift between the linear segments of the tax schedule, then the effective influence of the tax structure may be taken as approximately linear in its arguments. In this case, the post tax real rate of return to capital will be a fixed ratio of the pre tax return to capital and the disposable wage rate will be a fixed ratio of the pre-tax wage rate, with the ratio defined by the effective tax rate. Where labour supply is held fixed, the approximations that are referred to above can approximately be taken to hold. This is an advantage in associated statistical studies, because changes in tax policy during a sample period will be of concern only insofar as they affect the effective tax rate. But where labour supply is variable, changes in labour status will typically have an important bearing on the linear segment of the tax and benefits schedule to which an individual is subject. In this case, it is important that the full tax structure be taken into account.

to age and cohort effects. Cohort effects are particularly relevant for the current study, because recent reforms to the UK pension system have substantially altered the state retirement benefits enjoyed by people distinguished by year of birth. To control for these cohort effects, the estimates focus upon data for individuals aged 25 to 45 in 2007.[16] This age band focuses upon the period in life when the illiquidity of pension wealth is likely to have the most pronounced influence on behaviour, is broad enough to describe a reasonable degree of variation, and is narrow enough to ensure that the data are not unduly contaminated by cohort effects. Finally, the data were adjusted to capture expectations that individuals may reasonably have over how financial circumstances are likely to evolve as they age (controlling for *age effects*). This involved deflating all financial statistics to 2007 prices (using the National Accounts consumption deflator[17]), and then adjusting to reflect real wage growth, taking age 20 in 2007 as the base for adjustment.[18] The specific data sources that were used for estimation are described in sections 5 and 6.

5 First Stage Parameter Estimates

The structural model is based upon a total of 395 parameters. 3 of these describe interest rates on liquid net worth; 13 parameters describe the evolution of household demographics (relationship status and dependant children); 101 parameters describe age specific probabilities of mortality; 50 parameters describe the earnings processes for singles and couples; 210 parameters describe the tax and benefits system; 13 parameters describe the nature of personal pensions; and 5 parameters describe household preferences. All but the five preference parameters were estimated exogenous of the structural model.

The 390 parameters estimated in the first stage are reported in Tables 2 to 5. The last two of these four tables report parameters that are likely to have an important bearing upon the estimates obtained in the second stage of the estimation, and which were consequently subject to sensitivity analysis. This sensitivity analysis is set out under two alternative regression scenarios that were considered in the second stage of the estimation.

5.1 Credit constraints, real interest and growth rates

Noting that the welfare effects of pension fund illiquidity increase, *ceteris paribus*, as access to credit is constrained, the sensitivity analysis includes variation of the assumed credit constraints. Estimation scenario 1 assumes that households cannot have negative net worth in excess of £2,000 at any age and scenario 2 relaxes the hard liquidity constraint, subject to the condition that all debts be repaid by age

[16] The financial crisis that began in August 2007 did not have a noticeable bearing on the profiles that are considered for estimation.

[17] See ONS code YBGA.

[18] For example, observed consumption data for a 35 year old in 2007 were adjusted for $(35-20) = 15$ years of compounded wage growth.

Table 2: Exogenously estimated model parameters – various characteristics

real interest & growth rates (% p.a.)

	credit cards	overdrafts	fixed rate deposits	return to capital	wages	benefits	tax threshold
average	15.28	13.92	**2.73**	**4.05**	**1.27**	**-0.08**	**0.33**
std deviation	3.15	1.31	1.21	0.79	0.97	1.73	0.84
minimum	12.08	**11.52**	1.25	2.59	-0.31	-3.79	-0.79
maximum	**19.81**	15.34	4.66	5.29	2.75	4.40	1.43
sample period	'96-'08	'96-'08	'96-'08	'88-'06	'90-'07	'78-'08	'97-'07

household demographics

proportion of households single at age 20*	**0.45**
all households single from age*	**100**

logit regression for singles / couples

variable	coefficient	std. error
constant	**-6.40607**	0.34372
age	**0.17634**	0.02226
age^2	**-3.76E-03**	4.47E-04
age^3	**2.66E-05**	2.79E-06
single	**6.89326**	0.03963
sample	97619	
R squared	0.7947	

non-linear regressions for number of children

	singles		couples	
variable	coefficient	std. error	coefficient	std. error
param0	**0.67268**	0.00041	**1.54100**	0.00053
param1	**-0.00776**	0.00001	**-0.00711**	0.00001
param2	**38.2792**	0.0056	**39.7949**	0.0037
sample	13527		10438	
R squared	0.203		0.5258	

*mortality probabilities from age 40**

age	probability	age	probability	age	probability	age	probability
40	**0.0001**	60	**0.0006**	80	**0.0105**	100	**0.2964**
41	**0.0000**	61	**0.0005**	81	**0.0116**	101	**0.3607**
42	**0.0000**	62	**0.0007**	82	**0.0129**	102	**0.4278**
43	**0.0001**	63	**0.0012**	83	**0.0167**	103	**0.4951**
44	**0.0000**	64	**0.0011**	84	**0.0176**	104	**0.5607**
45	**0.0001**	65	**0.0014**	85	**0.0225**	105	**0.6230**
46	**0.0001**	66	**0.0016**	86	**0.0243**	106	**0.6810**
47	**0.0000**	67	**0.0012**	87	**0.0262**	107	**0.7341**
48	**0.0001**	68	**0.0023**	88	**0.0310**	108	**0.7818**
49	**0.0002**	69	**0.0021**	89	**0.0408**	109	**0.8237**
50	**0.0002**	70	**0.0020**	90	**0.0503**	110	**0.8598**
51	**0.0001**	71	**0.0025**	91	**0.0548**	111	**0.8904**
52	**0.0002**	72	**0.0033**	92	**0.0610**	112	**0.9157**
53	**0.0003**	73	**0.0036**	93	**0.0632**	113	**0.9363**
54	**0.0002**	74	**0.0051**	94	**0.0834**	114	**0.9527**
55	**0.0003**	75	**0.0045**	95	**0.0935**	115	**0.9654**
56	**0.0004**	76	**0.0049**	96	**0.1139**	116	**0.9752**
57	**0.0003**	77	**0.0068**	97	**0.1449**	117	**0.9826**
58	**0.0005**	78	**0.0085**	98	**0.1865**	118	**0.9879**
59	**0.0008**	79	**0.0095**	99	**0.2375**	119	**0.9918**

Notes: model parameters in bold
* no standard errors obtained
benefits growth rate estimated on historical rates for unemployment benefits and the basic state pension
relationship status modelled as a logit regression, describing the risk of being single as a function of age, and whether single in preceding year
number of children by age described by the density function of the normal distribution: param0 exp(param1 (age - param2)) - see Section 5.5
mortality probabilities calculated on cohort life expectancies for couples where both members aged 35 in 2007.

Source: credit card interest, Bank of England IUMCCTL; overdraft interest, Bank of England IUMODTL
fixed deposit interest, Bank of England, IUMWTFA; wages growth, Office National Statistics, LNMQ
return to capital derived from Khoman and Weale (2008), based on National Accounts data income flows
historical data on value of unemployment benefits, basic state pension, and tax thresholds obtained from the Institute for Fiscal Studies
logit for relationship status estimated on weighted pooled data from waves 1 to 17 of the BHPS
equation for the number of children by age estimated on weighted data from the 2007/08 FRS
mortality rates based on historical survival rates to 2006 and ONS principal projections thereafter.

Table 3: Exogenously estimated model parameters – earnings process

*probability of low wage offer^*

	mean	std dev	sample
singles	**0.29382**	0.45551	3939
couples	**0.06523**	0.24694	3531

*weekly wages and working hours by relationship and employment status^*

relationship status	couple	couple	couple	couple	single	single
adults full-time emp	2	1	1	0	1	0
adults part-time emp	0	1	0	1	0	1
working hours						
mean	**85.10**	**67.09**	**44.73**	**19.03**	**42.40**	**20.07**
std. deviation	12.54	13.08	10.49	8.55	8.50	9.28
log wages						
mean	**6.822**	**6.612**	**6.175**	**4.841**	**5.924**	**4.707**
std. deviation	0.475	0.511	0.724	0.756	0.569	0.722
sample	2530	1814	1840	509	4352	1360

*distribution of wages at age 20^*

	singles		couples	
	coefficient	std. error	coefficient	std. error
mean of (log) full-time wage, age 20	**5.74605**	0.00043	**6.29821**	0.00161
standard deviation of full-time wage, age 20	**0.39571**	.	**0.10445**	.

*wage dynamics for households changing marrital status**

	newly weds		newly single	
	coefficient	std. error	coefficient	std. error
target equation				
constant	**0.06442**	0.06714	**0.02537**	0.08270
age	**-0.00797**	0.00198	**0.00016**	0.00180
employment (single) / employment (couple)				
part time / 1 part time	**-0.14154**	0.06627	**-0.02215**	0.12454
part time / 1 full time	**0.47775**	0.29080	**-1.55863**	0.21295
part time / 1 part time & 1 full time	**1.44259**	0.13195	**-1.50337**	0.06714
part time / 2 full time	**1.87653**	0.19665	**-1.65264**	0.21921
full time / 1 part time	**-1.61412**	0.42382	**0.65706**	0.04307
full time / 1 part time & 1 full time	**0.29650**	0.06387	**-0.34763**	0.04923
full time / 2 full time	**0.64900**	0.03275	**-0.63573**	0.03626
selection equation				
age	0.04772	0.02525	0.12171	0.02444
age squared	-0.00085	0.00032	-0.00156	0.00030
degree	-1.08084	0.12228	1.24433	0.11370
other further education	-1.07942	0.11253	1.15538	0.09038
higher school qualification (A level)	-1.07025	0.11781	1.10500	0.10204
lower school qualification (O level)	-1.12394	0.11623	1.01499	0.09083
other education	-1.61396	0.15082	0.82185	0.10304
poor health	-0.27916	0.11064	-0.30229	0.10154
accident	-0.17709	0.09139	0.45756	0.08773
childcare	-0.37326	0.09748	-0.27075	0.07306
care (other)	-0.10474	0.10116	0.00110	0.08468
woman	-0.80629	0.07546	1.51969	0.18730
constant	0.68686	0.46202	-5.81684	0.50812
summary statistics				
correlation	0.69441	0.07586	-0.09977	0.102915
standard error	**0.40089**	0.02385	**0.36413**	0.015331
Number of (weighted) observations	2742		2517	
Censored observations	2163		2012	
Uncensored observations	579		505	
Log pseudolikelihood	-1194.495		959.637	
Wald test of independent equations				
Chi squared statistic	34.17		0.93	
p value	0.00		0.34	

Notes: model parameters in bold
prob of low wage offer = proportion of households aged 25-45 with no adult employment
mean log income at age 20 estimated using sample selection model - reported in Appendix
std of log income at age 20 calculated from raw survey data, no std errors obtained
dependent variables in equations for wage dynamics = (ln(observed wage(t+1)) - ln(observed wage(t)))

Source: ^ author's calculations on data from 2007/08 wave of the FRS
* author's calculations on data from waves 1 to 17 of the BHPS

Table 4: Parameters of wage dynamics distinguished by estimation scenario

	estimation scenario 1				*estimation scenario 2*			
	singles		couples		singles		couples	
	coefficient	std. error	coefficient	std. error	coefficient	std. error	coefficient	std. error
target equation								
age*	**-0.0018**	0.0001	**-0.0012**	0.0001	**-0.0019**	0.0001	**-0.0013**	0.0001
experience effect								
1 full-time & 1 part-time emp	.	.	**-0.0101**	.	.	.	**-0.0081**	0.0020
1 ful-time employed	.	.	**-0.0120**	.	.	.	**-0.0094**	0.0025
1 part-time employed	**-0.0170**	.	**-0.0144**	.	**-0.0102**	0.0072	**-0.0064**	0.0080
not employed	**-0.0350**	.	**-0.0200**	.	**-0.0814**	0.0383	**-0.0543**	0.0516
constant	**0.1047**	0.0054	**0.0777**	0.0043	**0.1068**	0.0057	**0.0783**	0.0042
selection equation								
age*	0.0911	0.0072	0.1013	0.0061	0.0911	0.0072	0.1012	0.0061
age squared*	-0.0012	0.0001	-0.0012	0.0001	-0.0012	0.0001	-0.0012	0.0001
highest education qualification								
no education qual recorded	-0.1467	0.0889	-0.1303	0.0537	-0.1468	0.0890	-0.1312	0.0537
lower school (O-level D-E)	0.0494	0.1266	-0.0055	0.0664	0.0488	0.1267	-0.0074	0.0666
mid school (O-level A-C)	0.1763	0.0726	0.0228	0.0445	0.1755	0.0726	0.0217	0.0446
higher school (A-level)	0.1360	0.0809	0.0520	0.0561	0.1356	0.0809	0.0519	0.0562
post-school qualification	-0.0795	0.0646	-0.0748	0.0528	-0.0791	0.0646	-0.0749	0.0528
poor health	-0.6752	0.0701	-0.3693	0.0407	-0.6756	0.0701	-0.3683	0.0408
accident	-0.0173	0.0527	-0.0581	0.0295	-0.0173	0.0527	-0.0583	0.0295
childcare	-0.8101	0.0737	-0.2820	0.0369	-0.8094	0.0738	-0.2820	0.0369
care (other)	-0.0636	0.0675	-0.1411	0.0323	-0.0632	0.0675	-0.1411	0.0323
woman	-0.0709	0.0615	.	.	-0.0704	0.0614	.	.
Standard Occupational Classification								
manager, admin, prof	1.9272	0.0783	0.7528	0.0509	1.9275	0.0783	0.7535	0.0510
assoc prof, technical, clerical	1.4495	0.0727	0.6791	0.0481	1.4499	0.0727	0.6792	0.0481
craft, personal protective	1.6056	0.0720	0.6975	0.0464	1.6062	0.0720	0.6979	0.0464
sales, plant, machinery	1.6544	0.0793	0.7077	0.0497	1.6547	0.0793	0.7074	0.0498
constant	-3.9136	0.2534	-3.7755	0.2456	-3.9127	0.2535	-3.7718	0.2460
summary statistics								
correlation*	0.0706	0.0336	0.1078	0.0312	0.0617	0.0327	0.0901	0.0320
standard error*	**0.1153**	0.0023	**0.0928**	0.0013	**0.1152**	0.0023	**0.0927**	0.0013
Number of (weighted) obs	12671		20682		12671		20682	
Censored observations	6346		8385		6346		8385	
Uncensored observations	6325		12297		6325		12297	
Log pseudolikelihood	-5471.04		-8021.352		-5467.795		-8019.233	
Wald test of independent equations								
Chi squared statistic	4.38		11.75		3.55		7.86	
p value	0.0364		0.0006		0.0597		0.0051	
Wald test of linear constraints								
Chi squared statistic	2.42		2.87		.	.	.	.
p value	0.2979		0.5791		.	.	.	.

Source: Wage dynamics estimated on data from waves 1 to 17 of the BHPS

Notes: model parameters in bold

Estimates using a sample selection model with robust standard errors to control for clusting of individuals in pooled panel data

Endogenous variable of wage dynamic equation = (log labour income in period (t+2) - log labour income in period (t))

Experience effect of wage dynamic equation calculated on observed labour market status in periods t and (t+1)

Wage dynamics equation based on dummy variables, except those denoted by *

Table 5: Pension parameters and credit constraints distinguished by estimation scenario

	estimation scenario 1		*estimation scenario 2*	
	singles	couples	singles	couples
maximum credit	£2,000	£2,000	relaxed	relaxed
all debts repaid by age	65	65	65	65
state pension age*	68	68	68	68
value of flat-rate state pension (£2006 per week)	121.50	243.00	114.75	229.50
*means tested retirement benefits***				
maximum value (£2006 per week)	31.76	41.89	29.96	39.56
withdrawal rate of benefits on private income	40%	40%	40%	40%
terms of private pensions				
employee contribution rate (% of earnings)	8	8	8	8
employer contribution rate (% of earnings)	11	11	16	16
min earnings threshold for eligibility (% median)	75	75	50	50

Source: Terms of state retirement benefits based on Pensions White Paper, DWP (2006b)

Notes: * See DWP (2006b), paragraph 3.34

** paid on top of flat-rate state pension

no standard errors obtained

65. These credit limits are reported in Table 5.

Real interest and growth rates are reported in the top panel of Table 2. The lower limit cost of debt $\left(r_l^D\right)$ was set to 11.5 per cent per annum, and the upper limit $\left(r_u^D\right)$ to 19.8 per cent. This range of interest charges reflects the range of average real interest charges applied between January 1996 and January 2008 to credit card loans and overdrafts in the UK. Positive balances of liquid net worth were assumed to earn a return $\left(r^I\right)$ of 2.7 per cent per annum, equal to the average real return on fixed rate bond deposits held with banks and building societies during the period between January 1996 and January 2008. The return to pension wealth $(r_t^p = r^p)$ was set equal to 4.1 per cent per annum based on the average return to capital described by the UK National Accounts between 1988 and 2006, as reported by Khoman & Weale (2008). The real rate of wage growth, used to adjust cross-sectional sample statistics, was set to 1.3 percent per annum, equal to the real growth observed for the average earnings index between 1990 and 2007. Welfare benefits were assumed to fall very marginally with time (annual rate of 0.1%), to reflect historical data over the period 1978 to 2008 on the value of unemployment benefits and the basic state pension. Similarly, real tax thresholds were assumed to rise by 0.3 per cent per annum, based on growth of the income threshold for the highest rate of income tax over the period 1997 to 2007.

5.2 Household demographics

It was assumed that a household can be comprised of one or two adults to age 99, and of a single adult from age 100. The logit function that governs relationship transitions in the model was selected after considering various alternatives, and is described by equation (19). The intertemporal dynamics of relationship status that is described by equation (19) require data with a time dimension for estimation. The parameters of this logit model were consequently estimated on pooled data from waves 1 (1991) to 17 (2007) of the BHPS, reorganised by family unit (see Appendix B), and screened to omit any unit by year that had missing data.[19] The parameter estimates of this logit model are reported on the left hand side of the middle panel of Table 2.

The numbers of children by age and relationship status were described by equation (20) (the density function of the normal distribution). This function provides a close reflection of the average numbers of children of singles and couples by age described by survey data, as is reported in Appendix C. Estimation of equation (20) does not require data with a time-series dimension, and so associated estimates were calculated on cross-sectional data from the 2007/08 Family Resources Survey. As for the BHPS data referred to above, the FRS data were organised at the level of the family (benefit) unit, and screened to omit observations with inconsistent data, or where at least one adult member of the family was identified as self-employed or working in a public sector organisation with a non-contributory occupational pension. Estimates for equation (20) are reported on the right hand side of the middle panel of Table 2.

$$s_{i,t+1} = \alpha_0^A + \alpha_1^A t + \alpha_2^A t^2 + \alpha_3^A t^3 + \alpha_4^A s_{i,t} \quad (19)$$

$$n_{i,t}^c = \alpha_0^C \exp\left\{\alpha_1^C \left(t - \alpha_2^C\right)^2\right\} \quad (20)$$

5.3 Mortality probabilities by age

The survival probabilities assumed for estimating the model are based upon the cohort expectations of life published by the Office for National Statistics (ONS). These data were used to calculate the age specific probabilities of survival for a couple, where both members of the couple were aged 35 in 2007 (the middle of the target age band for estimation). The life expectancies are based on historical survival rates from 1981 to 2006, and calendar year survival rates from the 2006-based principal projections that embody official estimates for the trend improvement of future survival rates.

The official data permit survival rates to be calculated to age 94, whereas a maximum age of 120 is considered for the estimation. Age specific survival probabilities between 95 and 120 were exogenously adjusted to describe a smooth sigmoidal progression from the official estimate at age 94 to a 0 per cent survival probability at age 120. The mortality rates considered for analysis are reported at the bottom of Table 2.

[19] Households with adult members who were either self employed or employees in public sector organisations with access to non-contributory occupational pensions were also excluded. Public sector employees omitted from analysis were identified under Standard Industrial Classification codes 9100-9199 (1980) / 75 (1992).

5.4 The probability of a low wage offer

Previous experience in use of the structural model revealed that wages tend to be sufficient to motivate some labour supply by almost all households during the prime working years spanning ages 25 to 45. The probability of a low wage offer (see Section 3.5) was consequently set to the proportion of single adults and couples that were identified as not working within this age band, as described by data reported by the 2007/08 wave of the FRS (described in subsection 5.2). The associated sample statistics are reported in the top panel of Table 3.

5.5 Distinguishing the implications of alternative labour supply decisions

Single adults were considered to choose between full-time employment, part-time employment, and not employed. Couples were considered to choose between 2 full-time employed, 1 full-time and 1 part-time employed, 1 full-time employed and 1 not employed, 1 part-time employed and 1 not employed, and 2 not employed; the option to allow for 2 part-time employed adults in a household was omitted because very few households take up this option in practice. The influence of alternative labour supply decisions on leisure and income from employment were defined as non-stochastic and age invariant proportions of the respective statistics associated with the maximum employment decision (full-time employment of all adult household members). These proportions were estimated using data for households aged between 20 and 59 from the 2007/08 FRS, organised and screened as described in subsection 5.2. Weighted averages were calculated for the number of hours worked and log wages, distinguishing population sub-samples by the number of adults in a household and labour market status.[20] These statistics are reported toward the top of Table 3.

5.6 The distribution of wages at age 20

Each simulated household that was generated to calculate the simulated moments upon which the second stage estimation is based (discussed in Section 4) was allocated a latent wage at entry into the model by taking a random draw from a log normal distribution. The mean and variance of the distribution for singles and couples of log latent wages at age 20 were estimated on the same FRS data that were used to estimate the implications of alternative labour supply decisions (described above). A sample selection model that describes log wages as a cubic function of age was estimated separately for singles and couples (see Appendix D for associated regression statistics).[21] These estimates were used to calculate the means for singles and couples of log full-time wages at age 20 that were assumed in the

[20] The International Labour Organization (ILO) definition of labour market status was used for the estimations. Age invariant statistics were applied after observing little systematic variation by age.

[21] The sample selection model controlled only for the incidence of non-employment. Households with adults who were less than full-time employed had their aggregate wage adjusted up on the basis of the respective statistics discussed in subsection 5.5.

second stage estimation. The standard deviations of the log-normal distributions were set equal to the FRS sample statistics observed for the respective population subgroups at age 20. These statistics are reported in the middle panel of Table 3.

5.7 Labour income dynamics

An experience effect was only taken into consideration where relationship status remained unchanged between adjacent periods. To estimate an experience effect over the extensive labour margin, recursive substitution was used to restate equation (10) as:

$$\begin{aligned}\ln\left(g_{i,t+2}\right)-\ln\left(g_{i,t}\right) &= \ln\left(\mu\left(emp_{i,t+2}\right)\right)-\ln\left(\mu\left(emp_{i,t}\right)\right)+.. \\ &\quad +f_h\left(n_{i,t}^a,t\right)+f_h\left(n_{i,t+1}^a,t+1\right)+.. \\ &\quad +\sum_{k=t}^{t+1}\sum_{j=1}^{n}\delta_j\left(emp_{i,k}^j\right)+\omega_{i,t+1}+\omega_{i,t} \qquad (21)\end{aligned}$$

where n is the number of potential labour states, $emp_{i,t}^j$ is a dummy variable that is equal to 1 if household i engages in employment state j at age t and zero otherwise, and all other variables are as defined previously.[22] Where relationship status was observed to change between adjacent periods, omission of an experience effect enabled equation (10) to be estimated directly.

The time dimension that is embedded in the specification of the equations that govern intertemporal wage dynamics made the FRS an unsuitable data source for estimation. Data from the BHPS for households aged between 20 and 64 were consequently used for estimation, organised and screened as described in subsection 5.2. The sample for estimation was extended beyond the 25 to 45 year old age band to limit the influence of boundary effects in relation to estimated polynomials by age, and to provide a plausible description of agent expectations regarding later ages.

The pooled BHPS data were divided into four population sub-groups distinguished by the marital transitions observed in adjacent years. Each sub-sample was then censored to omit extreme observations on the respective dependent variable ($\ln\left(g_{i,t+2}\right)-\ln\left(g_{i,t}\right)$ or $\ln\left(g_{i,t+1}\right)-\ln\left(g_{i,t}\right)$), resulting in sample sizes for estimation of 18,631 for continuously single adults, 27,831 for continuously married families, 3,850 newly married families, and 3,705 newly single families. Separate estimates were calculated on the data for each of these population subgroups, correcting for sample selection and heteroscedasticity of error terms.[23]

The results of unrestricted estimations are reported for newly married and newly single households in Table 3, and for continuously single / married households under estimation scenario 2 in Table 4.

[22] Estimates were also obtained for two recursive substitutions (a dependent variable of $\ln\left(g_{i,t+3}\right)-\ln\left(g_{i,t}\right)$), which were found to be qualitatively the same as those reported here.

[23] Full maximum likelihood estimation was undertaken using the "heckman" command in STATA 10, adjusting for enumeration weights, and allowing for clustering by enumerated individual in the error terms.

The second of these sets of statistics indicate that the effects of experience on prospective wages were estimated with relatively high standard errors. As experience effects play an important role in the second stage estimation, associated sensitivity analysis was conducted to the extent permitted by the data. The parameter restrictions that were assumed for estimation scenario 1 are also reported in Table 4. These parameters were altered, subject to the constraint that the resulting empirical specification was not rejected in preference for the unrestricted specification at any reasonable confidence interval.

5.8 Taxes and benefits

The wedge between gross private income and disposable income was calculated by dividing the life course into two periods: the working lifetime for ages below state pension age, $t < t_{SPA}$, and pension receipt, $t_{SPA} \leq t$. The treatment of each of these periods is described separately below, and graphical examples of the assumed tax and benefit schedules are reported in Appendix E.

- *The working lifetime*; $t < t_{SPA}$

Taxes and benefits during the working lifetime were structured to reflect schedules reported in the April 2007 edition of the *Tax Benefit Model Tables* (TBMT), issued by the Department for Work and Pensions.[24]

Take-home pay during the working lifetime was calculated from the taxable income of each adult household member via a five piece linear spline. This linear spline was specified to capture the effects of income taxes and National Insurance contributions.

Welfare benefits were calculated from aggregate household take-home pay via a four piece linear spline. Unlike the spline that was used to calculate take-home pay, the segments of the spline used to calculate benefits were defined to respond to the numbers of adults and children in a household. These parameters were selected to reflect the Child Benefit (a universal benefit paid in respect of dependant children), Working Tax Credit (a benefit paid to employees on modest earnings, excluding the childcare element), the Child Tax Credit, and Jobseekers Allowance (unemployment benefits).

Net childcare costs arising from non-discretionary childcare expenditure were calculated in a very similar way to welfare benefits, but with a three piece linear spline in aggregate take-home pay. The parameters of this part of the model – including the value of childcare costs and associated benefits (WTCs) – reflect the assumptions of the TBMTs.

Net housing costs were calculated to reflect the impact of Housing Benefit and Council Tax Benefit. Housing Benefit helps households meet rental costs on residential accommodation, and was withdrawn at a rate of 65% on income evaluated net of most other welfare benefits. Council Tax Benefit helps

[24] See http://www.dwp.gov.uk/asd/tbmt.asp.

households to meet the costs of Council Tax for which they are liable on residential accommodation, and was withdrawn at a rate of 20%, on a basis similar to that considered for Housing Benefit. Although the model does not explicitly reflect decisions regarding owner occupation, housing wealth or the incidence of council tax, these welfare schemes were included in the model as they can have an important bearing on the circumstances of low income households (and retirees in particular). Non-discretionary net housing costs were described by a three piece linear spline in aggregate net income before housing costs.

Asset tests on benefits were applied in the model by imputing an income stream from liquid net worth, which was then subject to the income tests on means tested benefits. The imputed income stream was arrived at by deducting an assets disregard from liquid net worth, and then applying a rate of return to any positive balance that remained. £50,000 for a single adult and £100,000 for a couple in 2007 prices were disregarded, which reflect the fact that residential housing was usually exempt from assets tests in the UK. The rate of return assumed for the assets test was calculated with regard to the real return assumed for liquid net worth (described above).

- *Pension receipt*; $t_{SPA} \leq t$

Take-home pay during the period of pension receipt was calculated from the taxable income of adult household members via a four piece linear spline, which was specified to capture the effects of income taxes in 2007.

Welfare benefits from state pension age were loosely defined around reforms set out in the 2006 Pensions White Paper (DWP, 2006b). This assumption reflects the fact that the White Paper was both freely available and widely publicised during the period covered by the estimation, and represents a sensible information source upon which an individual from the target cohort could have based their expectations. Despite a significant research effort, however, there remains a question mark over the fiscal sustainability of the arrangements that are set out in the White Paper, which has been exacerbated by the 2007 financial crisis.[25] Both of the estimation scenarios that are considered for analysis were consequently specified to imply a smaller fiscal burden than described in the White Paper. The specific terms of the respective estimation scenarios are reported in Table 5.

Both estimation scenarios assume a state pension age of 68, which is currently planned to apply in 2046. We have drawn the time horizon of the increase in the pension age forward to reflect the associated public policy debate (as referred to in the preceding paragraph). At this age, all individuals are assumed to be eligible to a full flat-rate state pension. This reflects the expanded coverage of state pensions implemented by the reforms described in the 2006 White Paper, and the coincident amendments to make state pensions a flat-rate benefit worth around £135 per week to a single pensioner

[25] This is made clear by a statement issued by the conservative opposition party in October 2009 that it would establish a commission to consider accelerating the planned increase in state pension age if elected in 2010 (see, Curry (2009)).

in 2006 earnings terms. Means-tested benefits subject to a 100% clawback rate are assumed to keep pace with the increased generosity of the flat-rate state pension, so that they can be ignored. The (real) value of means tested benefits subject to a 40% clawback rate are projected by the 2006 White Paper to grow with wages between 2008 and 2015, and to be frozen in real terms thereafter. Estimation scenario 1 applies a 10% discount to the value of these state retirement benefits, relative to the reforms set out in the White Paper, and scenario 2 applies a discount of 15%.[26]

5.9 Private pensions

There is a great deal of diversity in private pension arrangements in the UK, and in the details of occupational pensions in particular (see Section 2). These differences have an important bearing on incentives to participate in private pensions, and were consequently taken into account when specifying the two estimation scenarios. We start by describing a number of pension characteristics that were held constant between the alternative estimation scenarios, before discussing the terms that were subject to variation.

Pension contributions and associated investment returns were assumed to be tax exempt. The annuity rate, χ, was specified as actuarially fair, given mortality rates, the return on pension wealth, and subject to a capital charge of 4.7 per cent to reflect administration expenses and uncertainty over mortality rate projections.[27] The proportion of pension wealth used to purchase an annuity at state pension age was set to 75%, based on the maximum pension wealth that could be taken as a tax free lump-sum at retirement in 2006.

The pension contribution rate of employees who choose to participate in a private pension in any given year was set equal to the 'normal' contribution rate stated in the guidance to interviewers for the FRS. The rate of employer contributions to private pensions was varied between the estimation scenarios, to reflect the range of contribution rates described by survey data – see Table 1. Not all agents are considered to be eligible to participate in a private pension, with eligibility limited to those households with employment income in excess of an exogenously defined threshold. Each estimation scenario applies a different threshold, where the range of thresholds was selected with reference to the survey data reported in Figure 1. These terms are summarised in Table 5.

[26] The benefits adopted for analysis apply a discount relative to the following: a state pension of £135 per week per adult in current earnings terms, a means tested benefit subject to a claw back rate of 40% that is worth up to £35.29 per week for singles and £46.54 per week for couples. The upper bounds of means tested benefits were obtained by adjusting the maximum value of the savings credit payable in 2006 by a real growth rate of 1% per annum for 17 years (between 2008 and 2015).

[27] This resulted in an annuity rate of 6.06% for estimation. The 4.7% capital charge is based on "typical" pricing margins reported in the pension buy-outs market in the UK. See Lane et al. (2008), p. 22.

6 Estimated Preference Parameters

Up to five preference parameters were considered in the second stage of the model estimation; the two discount parameters β and δ, the intertemporal isoelastic parameter γ, the intra-temporal elasticity ε, and the utility price of leisure α. The parameters γ, ε and α were included in the final stage estimation despite being incidental to the central focus of concern due to the associated potential for omitted variable bias. Results reported by Bucciol (2009), for example, indicate that relaxing the preference structure that he considers to allow for temptation has a substantial impact on estimates for both the parameter of relative risk aversion and the exponential discount factor. Similarly, Laibson et al. (1998) find that decisions over DC pensions in the context of quasi-hyperbolic preferences are sensitive to the relative risk aversion.

This section begins by reporting the moments that were considered for the second stage estimation. Numerical approximations of the weighted loss function are then discussed, before presenting the regression results that were obtained.

6.1 Moments for the second stage estimation

Four sets of moment conditions were included in the second stage estimations. The first three of these require little explanation. A set of age and relationship specific rates of *pension scheme membership* were included on the hypothesis that these might be important in identifying the short-run discount factor. Rates of pension membership were considered in place of the scale of pension saving because data on the magnitude of pension contributions – and the contributions of employers in particular – are scarce for the UK. Age and relationship specific means of log household *consumption* are important in determining discount factors and the isoelastic parameter γ, given first-stage estimates for rates of investment return. And moments of *employment status by age* and relationship status relate closely to the utility price of leisure, and may also bear upon the short-run discount factor.

The fourth moment condition concern rates of *employment participation by wealth quintile* observed late in the working lifetime. These moments were included following the observation that the preference relation described in Section 3.1 implies that preferences can be approximated by the following relationship as households near retirement:

$$\frac{l}{c} = \left(\frac{\overline{h}}{h}\right)^{\varepsilon} \left(\frac{\overline{l}}{\overline{c}}\right) \tag{22}$$

where population average terms are indicated by $\overline{*}$ (bars). Thus for high income households $(\overline{h}/h) < 1$, increasing the intratemporal elasticity decreases the demand for leisure relative to consumption (equivalent to later retirement), but for low income households $(\overline{h}/h) > 1$ it increases the demand (equivalent to earlier retirement). As h is not observed when a household chooses not to work, wealth was used as a proxy.

The moments that were considered for the second stage estimation are reported in Table 6.

6.2 Properties of the weighted loss function

The approach that was taken to fit the model to sample moments is based upon Brent's method for identifying a local optimum in n dimensions, as described in Press et al. (1986). Brent's method is a robust search routine, so long as the surface over which an optimum is sought is fairly well behaved. As there is no *a priori* description of the properties of the weighted loss function, care was taken to explore its numerically derived properties. This sensitivity analysis was conducted by undertaking a grid search over a sample of parameter combinations $(\gamma, \varepsilon, \beta)$, with the remaining parameters (δ, α) estimated by Brent's method. Table 7 reports the results of the grid search under estimation scenario 1, which are qualitatively the same as those obtained under scenario 2.[28]

Begin by considering the results of the grid search that focuses upon exponential discounting that are reported in Panel A of Table 7. The results that are reported in Panel A reveal a reasonably well identified optimum in the of the intra-temporal elasticity ε of around 0.55, for any given value of the isoelastic parameter γ. In contrast, given $\varepsilon = 0.55$, the loss function is reasonably flat between the values $\gamma = \{1.2, ..., 2.2\}$, so that the isoelastic parameter was relatively poorly identified. These observations reflect the mixed results reported in the broad literature that has attempted to estimate time separable isoelastic utility functions, an issue that is returned to below.

The local description of the loss function that was identified under the assumption of exponential discounting (reported in Panel A of Table 7), permitted a more focussed grid search to be conducted over the full set of preference parameters $(\gamma, \varepsilon, \beta)$. Results of the grid search that was extended over the short-run discount factor β are reported in Panel B of the table, with the β dimension suppressed to focus on two dimensions – a full tabulation of the results obtained during the grid search are available from the author upon request. The statistics that are reported in Panel B support the general proposition that relaxing the restriction over β does not have a very pronounced impact on the values of γ and ε that minimise the weighted loss function. Ignoring for the moment differences in the scale of the loss function reported for exponential and quasi-hyperbolic discounting in Table 7, the two Panels of the table indicate that relaxing the assumption of exponential discounting is likely to have little bearing upon estimated values for γ and ε under scenario 1.

Differences in the scale of the estimates of the loss function that are reported in Panels A and B of Table 7 provide our first indication that allowing for quasi-hyperbolic discounting provides an improved fit between the model and the associated survey data. The assumption of exponential discounting

[28] Sensitivity of the parameter values identified by Brent's method to starting values is reported in Appendix F. Results of the grid search obtained under estimation scenario 2 are available from the author upon request.

Table 6: Moments considered for final stage estimation

	estimate	variance	sample
males aged 50 to 59 not economically active: lowest wealth quintile / highest wealth quintile	2.2429	0.0650	379

	proportion participating in employer sponsored pensions						*mean ln(consumption)*					
	singles			couples			singles			couples		
age	estimate	variance	sample	estimate	variance	sample	estimate	variance	sample	estimate	variance	sample
25	0.1483	0.1263	262	0.4071	0.2414	78	5.2273	0.7022	61	6.1993	0.4252	16
26	0.1980	0.1588	287	0.4012	0.2402	95	5.2845	0.8906	58	5.9442	0.4234	21
27	0.1988	0.1593	224	0.4294	0.2450	135	5.2998	0.9692	61	6.1538	0.5407	35
28	0.2464	0.1857	192	0.4934	0.2500	147	5.5013	0.6704	62	6.1765	0.5091	43
29	0.3242	0.2191	195	0.5494	0.2476	105	5.3634	0.9119	58	6.3905	0.4750	45
30	0.2247	0.1742	178	0.5770	0.2441	146	5.6775	0.8520	44	6.2908	0.4693	46
31	0.3536	0.2286	163	0.5428	0.2482	127	5.6052	0.7938	42	6.3497	0.5038	49
32	0.2827	0.2028	156	0.5325	0.2489	156	5.5502	0.7894	38	6.5598	0.3619	49
33	0.3203	0.2177	161	0.5174	0.2497	162	5.5827	0.7678	44	6.4610	0.4157	43
34	0.3336	0.2223	171	0.6308	0.2329	174	5.8206	0.6098	25	6.3963	0.5789	54
35	0.2910	0.2063	180	0.5582	0.2466	191	5.7254	0.9171	51	6.3657	0.5303	58
36	0.2907	0.2062	196	0.6112	0.2376	201	5.5911	0.8021	50	6.5152	0.5086	67
37	0.2581	0.1915	171	0.5291	0.2492	230	5.4818	0.8427	34	6.5286	0.4897	57
38	0.2924	0.2069	193	0.5885	0.2422	206	5.7905	0.6925	48	6.5678	0.4835	61
39	0.2521	0.1886	163	0.5664	0.2456	234	5.6120	0.8574	51	6.6305	0.4655	50
40	0.3029	0.2112	170	0.5840	0.2429	205	5.7306	0.7470	44	6.6838	0.5741	58
41	0.2951	0.2080	178	0.6234	0.2348	214	5.7790	0.6744	48	6.5583	0.4752	77
42	0.3581	0.2299	215	0.5788	0.2438	252	5.9342	0.7383	52	6.5614	0.6287	59
43	0.3268	0.2200	210	0.6386	0.2308	220	5.8971	0.8861	48	6.4836	0.4362	51
44	0.3986	0.2397	171	0.6795	0.2178	171	5.7790	0.8138	54	6.6471	0.5647	61
45	0.3434	0.2255	185	0.6209	0.2354	207	5.5147	0.7423	48	6.6077	0.5090	69

	proportion employed full-time						*proportion employed part-time*					
	singles			couples			singles			couples		
age	estimate	variance	sample	estimate	variance	sample	estimate	variance	sample	estimate	variance	sample
25	0.6649	0.2228	262	0.7202	0.2015	78	0.1059	0.0947	262	0.1088	0.0969	78
26	0.6063	0.2387	287	0.7057	0.2077	95	0.1199	0.1055	287	0.1051	0.0941	95
27	0.6131	0.2372	224	0.7097	0.2060	135	0.1059	0.0947	224	0.1170	0.1033	135
28	0.6737	0.2198	192	0.7731	0.1754	147	0.0949	0.0859	192	0.0757	0.0700	147
29	0.6018	0.2396	195	0.7002	0.2099	105	0.1056	0.0944	195	0.1105	0.0983	105
30	0.6259	0.2341	178	0.7345	0.1950	146	0.0758	0.0700	178	0.1044	0.0935	146
31	0.6936	0.2125	163	0.7148	0.2039	127	0.0618	0.0580	163	0.1305	0.1134	127
32	0.6559	0.2257	156	0.7366	0.1940	156	0.0858	0.0784	156	0.0930	0.0844	156
33	0.6240	0.2346	161	0.6490	0.2278	162	0.0834	0.0765	161	0.1324	0.1149	162
34	0.6573	0.2253	171	0.7117	0.2052	174	0.0820	0.0753	171	0.1347	0.1165	174
35	0.6089	0.2381	180	0.6710	0.2208	191	0.0926	0.0840	180	0.1062	0.0949	191
36	0.5826	0.2432	196	0.6611	0.2240	201	0.1022	0.0918	196	0.1456	0.1244	201
37	0.5726	0.2447	171	0.6512	0.2271	230	0.1144	0.1013	171	0.1553	0.1312	230
38	0.5400	0.2484	193	0.6304	0.2330	206	0.1644	0.1374	193	0.1525	0.1292	206
39	0.4748	0.2494	163	0.6334	0.2322	234	0.1688	0.1403	163	0.1776	0.1461	234
40	0.5264	0.2493	170	0.6080	0.2383	205	0.1480	0.1261	170	0.1802	0.1477	205
41	0.5029	0.2500	178	0.6114	0.2376	214	0.1569	0.1323	178	0.1753	0.1445	214
42	0.5444	0.2480	215	0.6503	0.2274	252	0.1484	0.1264	215	0.1808	0.1481	252
43	0.5759	0.2442	210	0.6494	0.2277	220	0.1720	0.1424	210	0.1947	0.1568	220
44	0.5404	0.2484	171	0.6232	0.2348	171	0.1477	0.1259	171	0.1811	0.1483	171
45	0.5009	0.2500	185	0.6398	0.2304	207	0.1448	0.1239	185	0.1881	0.1527	207

Source: employment and pension statistics estimated on FRS data, 2007/08
all consumption moments estimated on 2007 EFS data, for households aged 25 to 45
economic activity by wealth quintile derived from Marmot, *et al.* (2003, p. 156).

Table 7: The weighted loss function optimised over the exponential discount factor and the utility price of leisure, for given values of the isoelastic parameter γ , the utility price of leisure α, and the short-run discount factor β; estimation scenario 1

Panel A: exponential discounting, restricting $\beta = 1$

relative risk aversion	*utility price of leisure* 0.25	0.35	0.45	0.50	0.55	0.60	0.65	0.75	0.85
1.1	8.22	7.95	7.39	6.86	7.05	8.17	11.01	17.14	13.23
1.2				5.92	5.68	5.77			
1.4			6.53	5.93	5.60	5.63	5.85		
1.6				5.86	5.69	5.81			
1.8	6.52	7.04	6.79	5.98	5.64	5.70	5.93	6.55	7.63
2.0				6.02	5.66	5.74			
2.2			6.85	5.97	5.63	5.70	5.92		
2.4				6.02	5.74	5.70			
2.6			6.97	6.05	5.78	5.79	6.12		
3.0		7.26			5.82			7.62	
3.5									
4.0									
4.5	8.67				6.74				9.99

Source: Author's calculations

Panel B: quasi-hyperbolic discounting, grid search over β

isoelastic parameter	*utility price of leisure* 0.45	0.5	0.55	0.6	0.65
1.1	6.97		6.80		9.60
1.2		5.52'	5.28'	5.35'	
1.4		5.52'	5.10'	5.35	
1.6		5.48"	5.10"	5.30'	
1.8	6.10"'	5.28"'	5.19'	5.35'	5.54'
2.0					
2.2					
2.4					
2.6	5.91"'		5.17"'		5.62"

Source: Author's calculations

Notes: grid search expanded above and below 1.0 over the domain of the short-run discount factor, in increments of 0.05

' denotes loss function where short-run discount factor = 0.85

" denotes loss function where short-run discount factor = 0.80

"' denotes loss function where short-run discount factor = 0.75

values for the loss function reported in Panel B without a superscript were optimised at a short-run discount factor > 0.85

implies an unambiguously higher estimate for the weighted loss function across the range of parameter combinations considered by the grid search, with the data supporting a point estimate for β in the region of 0.75 to 0.85. The issue of statistical significance is addressed in Section 6.3.

6.3 Regression results

Each of the two estimation scenarios that are described in Section 5 were estimated twice; once assuming exponential discounting by imposing the restriction $\beta = 1$, and a second time to relax the restriction to accommodate quasi-hyperbolic discounting. Headline results of the analysis are reported in Table 8.

Starting with the results reported for the restricted specification of estimation scenario 1, the point estimate of the discount factor implies a discount rate of 3.17 percent per annum. This is 0.3 percent higher than the rate of return to positive balances of liquid net worth, which is within a single standard error of the parameter estimate. The point estimate of the isoelastic parameter γ is 1.438 and the associated standard error indicates that it is not significantly different from 1.0, which implies a log transformation of intratemporal utility. Furthermore, the point estimate for γ is less than the inverse of the point estimate of the intratemporal elasticity ε, 1.823. This implies that leisure and consumption are direct complements in utility, but note that 1.823 is also within a single standard error of the point estimate obtained for γ.

The estimated parameters imply an intertemporal elasticity of consumption of 0.13 measured at the population means.[29] The extensive empirical literature concerning this statistic reports a wide range of estimates. The controversy can be traced to an influential paper by Hall (1988), which suggests that the intertemporal elasticity may not be very different from zero. This result is controversial in part because it has particularly important implications for policy – for example, if savings do not respond to interest rates, then the dead-weight loss of capital taxes can potentially be ignored. A number of subsequent studies have supported the hypothesis that the intertemporal elasticity of consumption is small (e.g. Dynan (1993), Grossman & Shiller (1981), and Mankiw (1985)).[30]

However, a number of studies that focus upon sensitivity of Hall's results have found evidence to support substantially higher intertemporal elasticities. Attanasio & Weber (1993), for example, find that focussing upon cohort data for individuals who are less likely to be liquidity constrained than the wider population obtains an estimate for the intertemporal elasticity of consumption of 0.8 on UK data, and Attanasio & Weber (1995) report estimates between 0.6 and 0.7 for the US.[31] A similar test was

[29] This statistic was estimated by numerically calculating the derivative $d(\Delta \ln c_{i,t}) / d \ln r_{i,t}$, where $\Delta \ln c_{i,t} = \ln c_{i,t} - \ln c_{i,t-1}$, and weighting age specific averages by the associated survival probabilities.

[30] Values of the coefficient of risk aversion required to explain the equity premium puzzle (Mehra & Prescott (1985)) tend to be large (implying a small intertemporal elasticity), although evidence from attitudinal surveys suggest that the value is unlikely to greater than 5 (Barsky et al. (1997)).

[31] Other empirical studies that support higher rates for the intertemporal elasticity include Blundell et al. (1993) (0.5), Blundell et al. (1994) (0.75), Engelhardt & Kumar (2007) (0.75), Hansen & Singleton (1983) and Mankiw et al. (1985) (just over 1).

performed here, by focussing on consumption responses to an interest rate perturbation by households aged 80 years and under, and in the top three quintiles by liquid wealth. This population restriction doubled the estimated intertemporal elasticity to 0.27.

Relaxing the specification to allow for quasi-hyperbolic discounting obtains an estimate for the excess short-run discount factor of 0.8458, which is significantly less than one. Hence the regression results provide empirical support to the proposition that the discount rate associated with the first prospective year – at 21 percent – exceeds the long-run discount rate – at 2.5 percent per annum (slightly lower than the rate of return to positive balances of liquid net worth). The the disparity between estimates for the short-run and long-run discount factors that is reported here is less pronounced than those reported by Laibson et al. (2007) (0.674 / 0.687 c.f. 0.958 / 0.960) , or Fang & Silverman (2007) (0.296 / 0.308 c.f. 0.875 / 0.868), which is probably explained by the fact that the estimates reported here are based upon a broader subgroup of the population than is considered by either of the alternative studies.

Comparing the target moments that are reported in the bottom half of the panel reveals that allowing for quasi-hyperbolic discounting improves the match obtained between the model and sample moments over pension participation and labour supply; the match to moments for consumption, by contrast, deteriorate very slightly. These results are consistent with the set of hypotheses upon which the empirical study is based; that an allowance for sophisticated myopia might help to better explain observed behaviour over margins that have the potential to serve as commitment mechanisms, consumption obviously not being one of these.

The regression statistics that are reported for scenario 2 indicate that the estimates obtained for both the short-run and long-run discount factors are robust to the sensitivity analysis that is conducted over the policy environment.[32] The differences between the policy environments considered by the respective estimation scenarios have the most pronounced impact on regression estimates for the intratemporal elasticity ε, which are higher under estimation scenario 2 than scenario 1. The rise in ε tends to reduce the extent to which consumption and leisure are direct complements in utility, but does not go so far as to make them direct substitutes.

7 Conclusions

If current employment tends to increase the expected value of future wages through an experience effect, then labour supply can be interpreted as an investment in an illiquid asset. Furthermore, pensions represent one of the most important illiquid assets in which most people choose to invest during the course of their lives. In the context of uncertainty and time-consistent behaviour, the illiquidity of

[32] The estimates reported for the unrestricted model under estimation scenario 2, may be subject to revision as associated robustness checks are on-going at the time of writing.

Table 8: Structural estimation of preference parameters

	estimation scenario 1				*estimation scenario 2*			
	restricted		unrestricted		restricted		unrestricted	
parameter	estimate	std error	estimate	std error	estimate	std error	estimate	std error
short-run excess discount factor	1.0000	.	0.8458	0.0401	1.0000	.	0.8502	0.0387
long-run (exponential) discount factor	0.9693	0.0053	0.9760	0.0041	0.9692	0.0074	0.9751	0.0048
intertemporal isoelastic parameter	1.4380	0.5212	1.3760	0.2964	1.3808	0.7550	1.4288	0.2309
intra-temporal elasticity	0.5485	0.0909	0.5500	0.0453	0.6294	0.0953	0.6012	0.0566
utility price of leisure	1.4003	0.0940	1.3900	0.0336	1.2823	0.1131	1.3210	0.0428
target moments								
consumption	1.270E-02		1.305E-02		1.375E-02		1.456E-02	
pension participation	8.308E-03		7.762E-03		1.078E-02		9.904E-03	
part-time employment	3.675E-03		3.471E-03		3.534E-03		3.030E-03	
full-time employment	7.313E-03		6.678E-03		5.311E-03		5.737E-03	
non-emp of 1st to 5th wealth quintiles	4.407E-02		1.583E-02		2.450E-01		1.957E-01	
Loss function	5.5339		5.0291		5.1426		4.9848	
J statistic	866.37		775.86		778.98		766.24	
*Test of over-identifying restrictions**	0.0000		0.0000		0.0000		0.0000	

Notes: * p-values

such assets represents an unambiguous welfare cost, to the extent that they exaggerate the influence of liquidity constraints. In the context of sophisticatedly myopic preferences, however, asset illiquidity can be welfare improving if it provides a commitment mechanism that favours current over future selves. This study considers how far allowing for sophisticated myopia can improve the estimation of a structural model of savings and labour supply, where the estimation is specified to take account of data on decisions regarding saving, labour supply, and pension scheme participation.

Particular care was taken not to bias the case in favour of time-inconsistency of preferences. An important aspect of this is in the modelling of the private pensions, which were designed to capture tax advantages, matching employer contributions, and excess returns assumed to be earned by long-term investment strategies, in addition to pension fund illiquidity. It was consequently not clear *a priori* how far the structural model would struggle to reflect rates of pension scheme membership described by survey data, and so the gap that an allowance for myopic preferences might help to fill. This analytical approach was predicated on the view that, if behavioural myopia is as important as the wealth of experimental evidence suggests, then we should not have to look too hard to find evidence of it in field data.

It is consequently of note that the regression statistics reported here strongly support the hypothesis that rates of temporal discounting are better described by quasi-hyperbolic discount factors than the model of exponential discounting. The excess short-run discount factor was estimated at 0.85, subject to a standard error of 0.04. The results imply a point estimate for the discount rate associated with the first prospective year of 21 percent, compared with a long-run discount rate of 2.5 percent per annum. There is currently little to compare these results against, as there are so few comparable studies that estimate quasi-hyperbolic discount rates on field data.

On the empirical side, this study highlights the need to identify moments that help to identify the intertemporal isoelastic parameter γ. One aspect of behaviour that we have used for this task in past work (Sefton et al. (2008)) is the profile of consumption about retirement. One might, for example, consider adopting a measure of anticipated consumption response to the change in labour supply around retirement to improve the estimate for γ, and to consider how far so-called excess sensitivity of the consumption response at retirement can be explained as the product of myopic preferences. Furthermore, the estimated discount factors that are reported in this study are robust to the sensitivity analysis that was conducted over the policy environment. It would be interesting to extend the robustness checks to help better understand the behavioural considerations driving the results that reported here.

More broadly, it is of interest to explore the simulated micro-data that underly the analysis. How far does myopia create a demand for pension saving? What impact does an experience effect have on labour supply at different times during the life course? What are the implications of considering naïve in place of sophisticated consumers? These are just some of the issues that remain for further research.

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A The Covariance Matrix of Second Stage Estimators and the Optimal Weighting Matrix

The current derivation follows Appendix B of Gourinchas & Parker (2002). From equation (16), the loss function is given by:

$$F = g(\theta, \hat{\chi})' W g(\theta, \hat{\chi}) \tag{23}$$

$\hat{\theta}$ is then the value of the parameter vector that minimises F. Assuming that F is differentiable in the region of the optimum, we have (assuming that W is symmetric):

$$g_\theta\left(\hat{\theta}, \hat{\chi}\right)' W g\left(\hat{\theta}, \hat{\chi}\right) = 0 \tag{24}$$

where $g_\theta(.)$ is the $k \times l_2$ matrix of partial derivatives with respect to the l_2 parameters of the vector θ. Expanding $g\left(\hat{\theta}, \hat{\chi}\right)$ around the (theoretically) true parameter vector θ_0:

$$g\left(\hat{\theta}, \hat{\chi}\right) = g(\theta_0, \hat{\chi}) + g_\theta(\bar{\theta}, \hat{\chi})\left(\hat{\theta} - \theta_0\right) \tag{25}$$

where $\theta_0 \lesseqgtr \bar{\theta} \lesseqgtr \hat{\theta}$. Substituting (25) into (24), and rearranging:

$$\left(\hat{\theta} - \theta_0\right) = -\left[g_\theta\left(\hat{\theta}, \hat{\chi}\right)' W g_\theta(\bar{\theta}, \hat{\chi})\right]^{-1} g_\theta\left(\hat{\theta}, \hat{\chi}\right)' W g(\theta_0, \hat{\chi}) \tag{26}$$

Expanding $g(\theta_0, \hat{\chi})$ around χ_0:

$$g(\theta_0, \hat{\chi}) = g(\theta_0, \chi_0) + g_\chi(\theta_0, \bar{\chi})(\hat{\chi} - \chi_0) \tag{27}$$

where $\chi_0 \lesseqgtr \bar{\chi} \lesseqgtr \hat{\chi}$, and $g_\chi(.)$ is the $k \times l_1$ matrix of partial derivatives with respect to the l_1 parameters of the vector χ. Substituting (27) into (26):

$$\begin{aligned}\left(\hat{\theta} - \theta_0\right) &= -\left[g_\theta\left(\hat{\theta}, \hat{\chi}\right)' W g_\theta(\bar{\theta}, \hat{\chi})\right]^{-1} g_\theta\left(\hat{\theta}, \hat{\chi}\right)' W \times \\ &\quad [g(\theta_0, \chi_0) + g_\chi(\theta_0, \bar{\chi})(\hat{\chi} - \chi_0)]\end{aligned} \tag{28}$$

By the central limit theorem, $\sqrt{\hat{I}}\left(\hat{\theta}-\theta_0\right)\underset{d}{\rightarrow}N\left(0,V_\theta\right)$. Therefore:

$$\begin{aligned} V_\theta &= E\left\{\hat{I}\left[g_\theta\left(\hat{\theta},\hat{\chi}\right)'Wg_\theta\left(\bar{\theta},\hat{\chi}\right)\right]^{-1}g_\theta\left(\hat{\theta},\hat{\chi}\right)'W\times\right. \\ &\quad \left[g\left(\theta_0,\chi_0\right)+g_\chi\left(\theta_0,\bar{\chi}\right)\left(\hat{\chi}-\chi_0\right)\right]\times \\ &\quad \left[g\left(\theta_0,\chi_0\right)+g_\chi\left(\theta_0,\bar{\chi}\right)\left(\hat{\chi}-\chi_0\right)\right]'\times \\ &\quad \left.Wg_\theta\left(\hat{\theta},\hat{\chi}\right)\left[g_\theta\left(\bar{\theta},\hat{\chi}\right)'Wg_\theta\left(\hat{\theta},\hat{\chi}\right)\right]^{-1}\right\} \end{aligned} \tag{29}$$

Assume that the first and second stage estimators are uncorrelated. Then, in the limit as $\hat{\theta}\rightarrow\theta_0$ and $\hat{\chi}\rightarrow\chi_0$:[33]

$$\begin{aligned} V_\theta &= \left[G_\theta'WG_\theta\right]^{-1}G_\theta'W\times \\ &\quad \left[E\left\{\hat{I}g\left(\theta_0,\chi_0\right)g\left(\theta_0,\chi_0\right)'\right\}+\hat{I}G_\chi\hat{J}^{-1}V_\chi G_\chi'\right]\times \\ &\quad WG_\theta\left[G_\theta'WG_\theta\right]^{-1} \end{aligned} \tag{30}$$

where $G_\theta=g_\theta\left(\theta_0,\chi_0\right)$ and all other terms are as defined previously. From (30), the optimal weighting matrix is given by:

$$W_{opt}=\left[E\left\{\hat{I}g\left(\theta_0,\chi_0\right)g\left(\theta_0,\chi_0\right)'\right\}+\hat{I}G_\chi\hat{J}^{-1}V_\chi G_\chi'\right]^{-1}$$

B Data Sources

B.1 Expenditure and Food Survey

The Expenditure and Food Survey (EFS) was introduced in 2001, as an amalgamation of the Family Expenditure Survey (FES – introduced in 1957) and the National Food Survey. The structure of the EFS is based upon the FES, and the survey reports detailed information regarding demographics, income, and expenditure for a cross-sectional sample of approximately 6,500 households in the United Kingdom. The basic unit in the survey is the household, with households being selected at random from the Post Office's list of addresses (for Great Britain, excluding the Scottish Isles and the Isles of Scilly; Northern Ireland is sampled through the Valuations and Lands Agency list), and participation being voluntary. The EFS defines a household as: "a group of people living at the same address with common housekeeping that is sharing household expenses such as food and bills, or sharing a living room." All individuals aged 16 and over in participating households are asked to keep a diary of expenditure covering a two week period, with children aged 7 to 15 also being asked to keep a simplified diary since 1998. Regular expenditure, demographic, and income data are recorded at a household interview, and retrospective information is collected on expenditure of selected large and infrequent purchases. The

[33] Where the central limit theorem is applied again, with respect to $\left(\hat{\chi}-\chi\right)\underset{d}{\rightarrow}N\left(0,\hat{J}^{-1}V_\chi\right)$

survey is collected on a continuous basis, and was reported at annual intervals from April to March covering the UK financial year between 2001 and 2006, and on a calender year basis from 2006.

The representative nature of the EFS for the UK population is affected by a number of factors. Firstly, people in institutions – such as retirement homes, the military, or prison – are omitted from the survey. Also, people with no fixed address (the homeless) are not surveyed. Furthermore, the voluntary nature of the survey typically obtains a response rate of those initially approached in the region of 50-60 per cent, and has been found in the past to be not uniformly distributed across the population. Foster (1996) compared the characteristics of households responding to the 1991 FES with information derived from the 1991 Census, and found that response was lower than average in Greater London, higher in rural areas, and that the response rate tended to increase with the age of the household reference person. Low response rates were also found for ethnic minorities, the lower educated, self employed, and the manual social class.[34] Nevertheless, data from the EFS appear to provide a reasonably comprehensive picture of household income and expenditure in the UK, accounting for approximately 90 per cent of income and 85 per cent of expenditure estimated in the National Accounts.[35] The EFS does not, however, adequately report income for the self-employed or from investments.

Units for analysis

The EFS reports data at the household level. As implied by the definition of a household that is quoted above, this can include a group of disparate adults sharing the same residence. As the model considered here is framed around the family unit, it was necessary to disaggregate some of the data that are reported at the household level by the EFS. We defined a family unit as a household comprised of a single adult, or partner couple, plus their dependant children. Dependant children were defined as any child under the age of 16, or any individual under the age of 18 and in full-time education. The EFS defines the *household reference person* as:

> the householder, i.e. the person who:
>
> a. owns the household accommodation, or
>
> b. is legally responsible for the rent of the accommodation, or
>
> c. has the household accommodation as an emolument or perquisite, or
>
> d. has the household accommodation by virtue of some relationship to the owner who is not a member of the household.
>
> If there are joint householders the household reference person will be the one with the higher income. If the income is the same, then the eldest householder is taken.

[34] This issue is partially corrected for by weights that are supplied with the EFS.
[35] See Banks et al. (1998) and Foster (1996) for detailed discussion.

We analysed the demographics of each household reported by the EFS, and excluded from analysis any household comprised of:

- family units not including the reference person, but including dependant children
- partner couples that did not include the reference person
- more than five family units

This selection resulted in a sample size of 7,218 family units, drawn from the 6,136 households for which data are reported by the 2007 EFS. Excluding any household with any member identified as self employed reduced the sample to 6,471 family units; it was not possible to censor out public sector employees with access to non-contributory occupational pensions due to miss-coding of the Standardised Industrial Classification Index (SIC) in the 2006 and 2007 waves of the EFS.

Consumption

One of the key reasons for consulting EFS data was to obtain sample statistics for consumption. The composite consumption variable with which the model is concerned is best interpreted as expenditure on all goods and services, including rent and mortgage interest on the family's residence, but excluding any mortgage capital repayment. The EFS reports two measures of aggregate consumption: the ONS definition, $p550tp$, and the National Accounts definition, $p560tp$. These two variables differ mainly in that the ONS definition includes expenditure on various taxes, charges, and fines (e.g. Council Tax, Stamp Duty, motoring fines), and unrequited domestic transfers that are not included in the National Accounts definition. As many of the taxes and charges that are included in the ONS definition are not explicitly allowed for by the model considered here (i.e. are not taken into account by the tax function considered for analysis), we have adopted the ONS definition for analysis.

B.2 Family Resources Survey

The Family Resources Survey (FRS) was introduced in 1992, and reports data at annual intervals covering the UK financial year (beginning in April) regarding the demographic, employment, income and financial circumstances of households in the United Kingdom (Great Britain prior to 2002). Like the EFS, data for the FRS are collected on a continuous basis from a cross-sectional sample of voluntary participating households that is designed to be representative of the wider United Kingdom population. The FRS sample is, however, just over 4 times as large as that of the EFS. Information is collected at both the household and individual level. Household level questions are asked of one adult (usually the household reference person / head), and individual level data are collected from all non-dependant people aged in excess of 16 years. The FRS also includes a 'benefit unit' identifier, which is very similar to the family unit identifier constructed for analysis of EFS data, and was used for calculating the associated sample statistics reported in the study.[36]

Limiting the sample to households (benefit units) with between one and two adults, and with no missing data on any of the characteristics of concern resulted in a sample of 29,562 observations for estimation.

B.3 British Household Panel Survey

The British Household Panel Survey (BHPS) reports detailed panel data for a sample of households at annual intervals since 1991, and is administered by the ESRC UK Longitudinal Studies Centre. Fieldwork for each wave is conducted between 1 September and the end of April of the following year, with most data collected by the end of December. The original sample was comprised of 10,264 interviewed individuals drawn from 5,538 households, and was selected to be representative of the UK cross-sectional population in 1991.[37] Subsequent waves of the survey have followed the evolving experience of members of the original sample and their households. Each member aged 16 or over in a household containing an original sample member is solicited to complete the survey, with children aged 11 to 15 subject to a special interview from wave 4 (1994). Additional sub-samples were added to the panel in 1997 (from the European Community Household Panel, including Northern Ireland and low income sub-samples), 1999 (extensions for Scotland and Wales), and 2001 (from the Northern Ireland Household Panel Survey). Major topics covered by the survey include household organisation, labour market participation, income, wealth, housing, health, and socio-economic status.

Data on wealth have been collected for the BHPS every five years, with the most recently available data reported for 2005 (wave 15). This wave of the survey provides information for 15,627 adults drawn from 8,709 households. Re-organising the data along the same lines as described above for the EFS, and excluding households with self-employed members, or members in public sector jobs produced 7,851 observations for estimation.

Wave 15 of the BHPS solicits information regarding the wealth held in housing, referring to the value of a family's place of residence (for owner occupiers) and their second home (if they have one). It also asks about the outstanding value of any associated mortgage commitments. The illiquid nature of wealth held in residential property complicates accurate evaluation for survey purposes. Various alternative methods can be devised to impute an 'expected' or 'reasonable' value of housing wealth from alternative data, such as the size of a household's residence, its region, and the value at purchase.

[36] The definition of a "benefit unit" reported by the FRS differs to our assumed definition of the family unit, by identifying 18 year old offspring in full-time education as dependant children.

[37] The first sample was drawn from the small users Postal Address file for Great Britain south of the Caledonian canal (excluding Northern Ireland). Due to the repeated survey methods employed, the most recent waves of the BHPS no longer provide a representative sample of the UK population. See Taylor (2005) for further details regarding the BHPS.

The BHPS places the onus of such imputations upon the survey respondent by asking them: "About how much would you expect to get for your home if you sold it today?" (Household Questionnaire, Wave 15, p. 2). It is clear that answers to this sort of question may be inaccurate, which the BHPS takes into consideration by asking respondents whether their expected value falls into one of a specific range of values.

Wave 15 of the BHPS also asks about any wealth held in tradeable securities, with specific reference to the following:

- Savings or deposit accounts
- National Savings Bank (Post Office)
- TESSA or ISA
- National Savings Certificates
- Premium Bonds
- Unit Trusts / Investment Trusts
- Personal Equity Plan,
- Shares (UK or foreign)
- National Savings Bonds
- Government or company securities

Individuals are asked to reveal the value of their aggregate holdings, and whether they are owned jointly or in their sole name. Importantly, no explicit reference is made by interviewers to statements or listed market values. Rather, respondents are asked questions such as; "Thinking now about the investments you have ... about how much is the total value of these investments?" (Household Questionnaire, 2006, Wave 16 , p. 102).

Similarly, a range of associated debts are reported, with specific reference to the following:

- Hire purchase agreements
- Personal loans
- Credit cards
- Catalogue or mail order purchase agreements
- DSS Social Fund loan

- Overdrafts
- Student loan
- Any other loans for a private individual (not including mortgages)

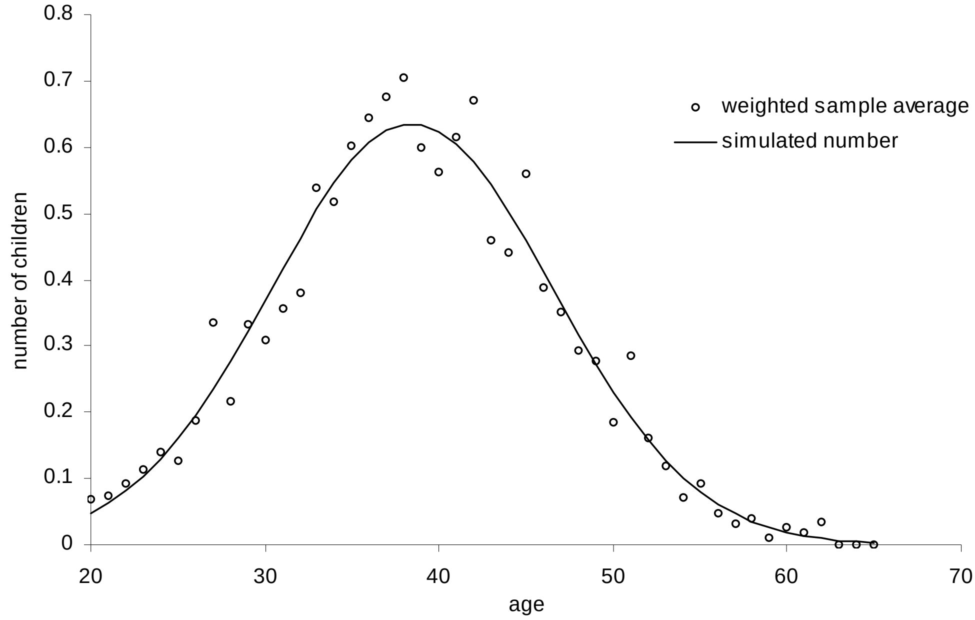
For the purposes of analysis, any asset or liability that is described by the BHPS as a joint holding was divided by two to obtain the survey respondent's share. Housing wealth is taken from responses of the household reference person, and securities wealth was aggregated over all adult members of a benefit unit.[38] Statistics by age were obtained with reference to the age of the household reference person.

B.4 Employers' Pension Provision Survey

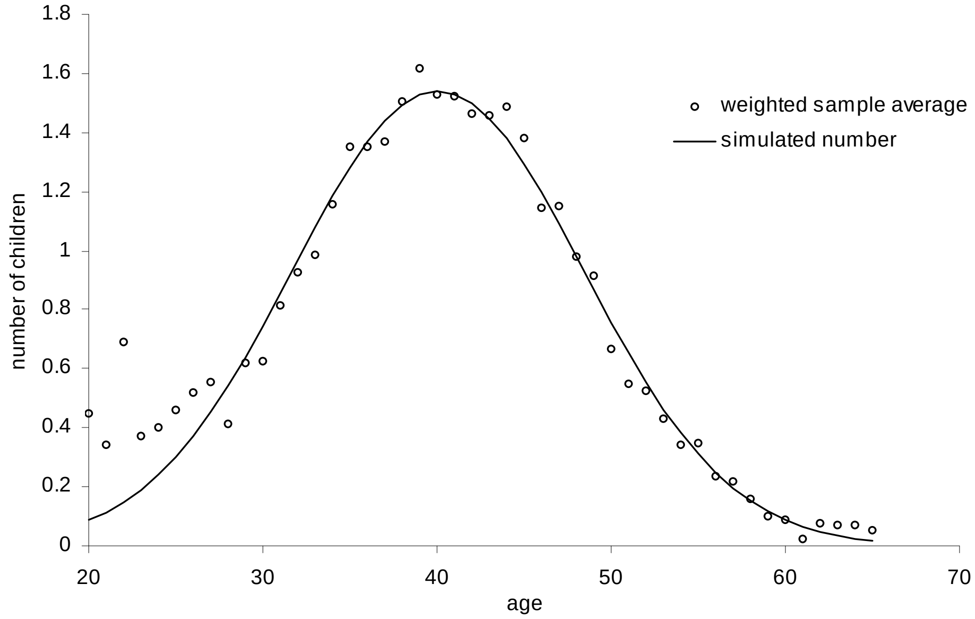
The Employers' Pension Provision Survey (EPPS) is designed to provide information regarding the pension provisions that employers provide to their employees. The EPPS was first introduced in 1994, and has been conducted approximately biennially since that date, with the most recent wave published for 2007 (the survey was undertaken in 2003 rather than 2002). The population for the 2007 wave of the survey was defined as all private sector employers in Great Britain, excluding small businesses without employees and all public sector organisations. The sample was drawn from the Inter-Departmental Business Register (IDBR), which is widely acknowledged to be the most complete register of businesses available. 4,213 institutions were originally approached, and 2,360 provided complete interviews (response rate of 56%). Organisations were selected at random from the IDBR, with over-sampling of large employers as these are relatively scarce in the macro-economy, but have an important impact on the labour market. Interviews were conducted were conducted by telephone between May and September of 2007, and were undertaken after having issued a letter that contained some of the most complex questions to allow time for the requested information to be gathered. Further detail regarding the 2007 wave of the EPPS is described in Forth & Stokes (2008)

[38] The household reference person is defined as the person legally or financially responsible for the accommodation, or the elder of two people equally responsible.

C Modelling the Number of Children



Panel A: single adults



Panel B: adult couples

Source: Author calculations based on data from the 2007/08 FRS

Figure 3: Simulated and sample average number of children per household, by age and relationship status

D Supplementary Estimates for Labour Income Processes

Table 9: Estimates to control for sample selection in evaluating geometric mean labour income of singles at age 20

target equation	Coef.	Std. Error
age	0.14903	0.01875
age squared	-0.00292	0.00049
age cubed	1.93E-05	3.98E-06
constant	3.78099	0.22543
selection equation		
age	0.12307	0.00576
age squared	-0.00172	0.00007
any education certificates	0.400209	3.16E-02
professional / vocational training	0.35408	0.02802
tertiary education	0.46786	0.03854
any dependant children	-0.65470	0.03435
constant	-2.04555	0.11179
correlation*	-0.83297	0.01476
standard error*	0.76708	0.014289
Number of (weighted) observations	13527	
Censored observations	7815	
Uncensored observations	5712	
Log pseudolikelihood	-1.32E+07	
Wald test of independent equations		
Chi squared statistic	617.15	
p value	0.00	

Source: author's calculations on data from 2007/08 FRS

Notes: Sample section model with robust standard errors

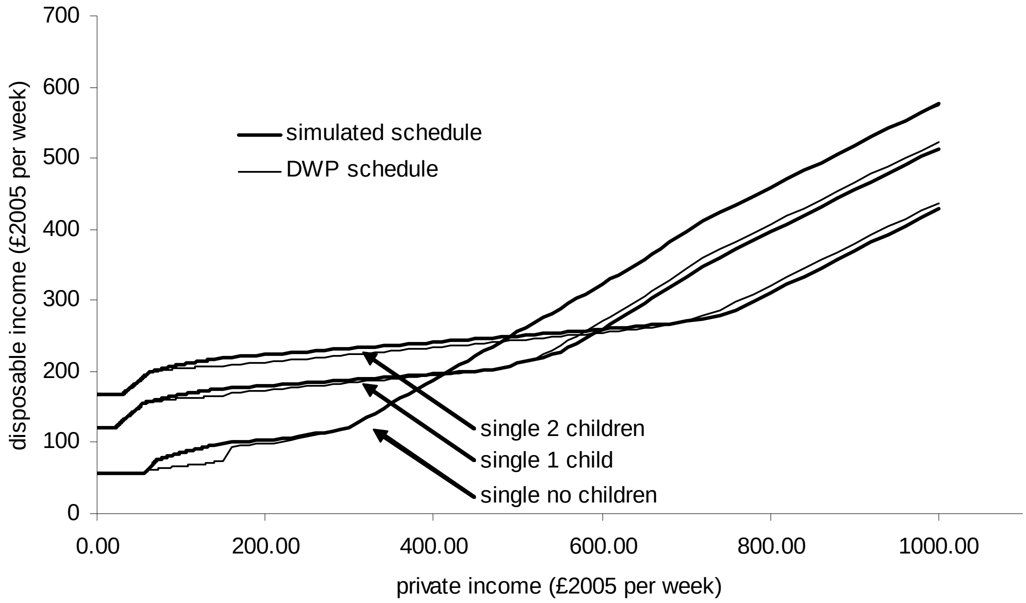
Table 10: Estimates to control for sample selection in evaluating geometric mean labour income of couples at age 20

target equation	Coef.	Std. Error
age	0.17496	0.02065
age squared	-0.00339	0.00047
age cubed	0.000021	3.51E-06
constant	3.98628	0.28754
selection equation		
age	0.29663	0.04932
age squared	-0.00485	0.00106
age cubed	1.68E-05	7.11E-06
any education certificates	0.34560	0.04636
professional / vocational training	0.33094	0.04402
tertiary education	0.39121	0.05247
any dependant children	-0.27394	0.04962
constant	-4.07843	0.69988
correlation*	-0.57599	0.04739
standard error*	0.59981	0.01466
Number of (weighted) observations	10438	
Censored observations	3745	
Uncensored observations	6693	
Log pseudolikelihood	-9.99E+06	
Wald test of independent equations		
Chi squared statistic	85.67	
p value	0.00	

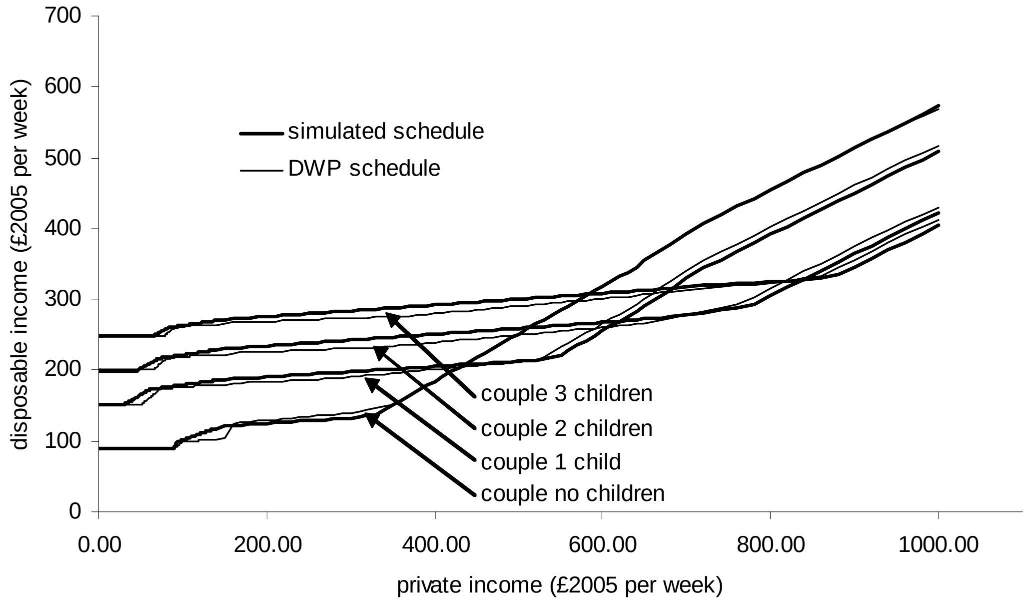
Source: author's calculations on data from 2007/08 FRS

Notes: Sample section model with robust standard errors

E Modelling Taxes and Benefits



Panel A: single adults

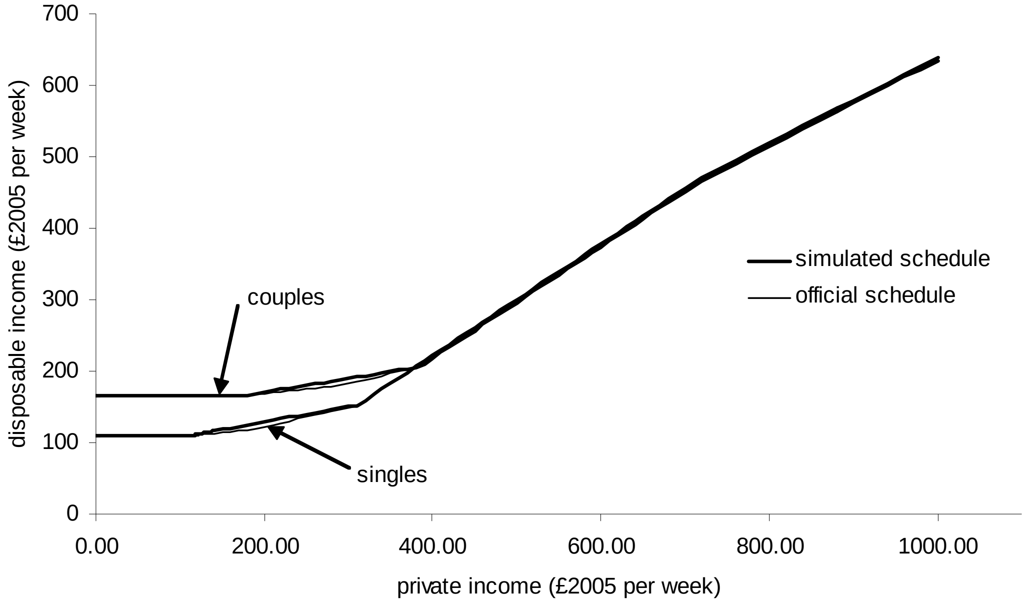


Panel B: adult couples

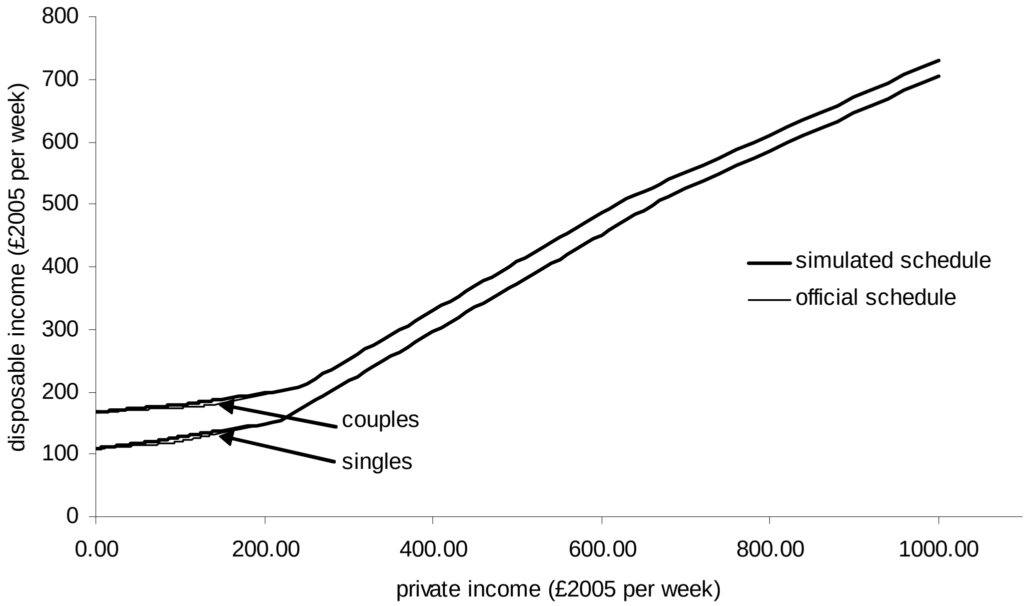
Source: Stylised characterisation of profiles described by Tax Benefit Model Tables

The author may be contacted for further details

Figure 4: Simulated tax functions – working lifetime



Panel A: early retirement and the Pension Guarantee



Panel B: later retirement and the Pension Credit

Source: Stylised characterisation of the Pension Guarantee and Pension Credit
The authors may be contacted for further details

Figure 5: Tax functions for early and later retirement

F Sensitivity of Brent's Search Routing to Parameter Starting Values

Table 11 reports statistics that indicate the scale of the sensitivity to starting values of Brent's method over values of (δ, α), for a sample of parameter combinations $(\gamma, \varepsilon, \beta)$ under estimation scenario 1.

Table 11: Sensitivity to starting values of the automated search routine over the exponential discount factor and the utility price of leisure, estimation scenario 1

					starting values		*optimised values*		*differences in optimised values*		
estimation scenario	relative risk aversion	intratemporal elasticity	short run discount factor	loss function	long run discount factor	utility price of leisure	long run discount factor	utility price of leisure	loss function	long run discount factor	utility price of leisure
1	1.2	0.55	0.85	5.277	0.9500	1.4000	0.9728	1.3749	0.0118	0.0008	0.0165
1	1.2	0.55	0.85	5.289	1.0200	0.9300	0.9720	1.3914	.	.	.
1	1.4	0.55	0.85	5.114	0.9728	1.3872	0.9748	1.3805	0.0155	0.0012	0.0113
1	1.4	0.55	0.85	5.099	1.0300	1.5000	0.9761	1.3918	.	.	.
1	1.8	0.45	0.75	6.101	0.9100	1.5500	0.9834	1.4661	0.1816	0.0016	0.0374
1	1.8	0.45	0.75	6.282	1.0200	0.9200	0.9849	1.4287	.	.	.
1	1.8	0.50	0.75	5.550	0.9851	1.3768	0.9837	1.3765	0.0313	0.0011	0.0105
1	1.8	0.50	0.75	5.519	1.0100	0.9500	0.9849	1.3869	.	.	.
1	1.8	0.55	1.00	5.644	0.9704	1.3922	0.9728	1.3972	0.0488	0.0004	0.0065
1	1.8	0.55	1.00	5.693	1.0500	0.9500	0.9724	1.4038	.	.	.
1	1.8	0.60	0.85	5.431	0.9811	1.3976	0.9817	1.3921	0.0771	0.0003	0.0069
1	1.8	0.60	0.85	5.353	1.0250	0.9200	0.9819	1.3852	.	.	.
1	1.8	0.65	0.85	5.540	0.9800	1.4000	0.9812	1.3622	0.0532	0.0004	0.0250
1	1.8	0.65	0.85	5.593	1.0200	1.5000	0.9808	1.3872	.	.	.
1	2.6	0.45	0.75	5.908	0.9926	1.5367	0.9928	1.4843	0.0730	0.0003	0.0076
1	2.6	0.45	0.75	5.981	1.0200	1.0000	0.9925	1.4919	.	.	.
1	2.6	0.50	1.00	6.048	0.9200	1.5500	0.9801	1.4234	0.0591	0.0006	0.0091
1	2.6	0.50	1.00	6.107	0.9813	1.4155	0.9795	1.4143	.	.	.
1	2.6	0.55	0.75	5.226	0.9800	0.9700	0.9932	1.4058	0.0517	0.0002	0.0066
1	2.6	0.55	0.75	5.174	0.9937	1.3757	0.9930	1.3991	.	.	.
1	2.6	0.65	0.80	5.625	0.9200	0.9500	0.9922	1.3712	0.0436	0.0001	0.0047
1	2.6	0.65	0.80	5.668	0.9941	1.3893	0.9920	1.3759	.	.	.
1	3.5	0.40	1.00	7.652	0.9200	0.9500	0.9864	1.3916	0.0982	0.0014	0.0027
1	3.5	0.40	1.00	7.750	1.0500	1.4000	0.9877	1.3889	.	.	.

The statistics reported in Table 11 indicate that the exponential discount factor identified by Brent's method varies by up to 0.2% depending upon the assumed starting values, which is less than half a standard deviation of the parameter estimates that are reported in Section 6. Similarly, the utility price of leisure varies by up to 0.04, which is just under a single standard deviation of the associated parameter estimates that are reported below. Furthermore, the average absolute variation between the loss functions reported for alternative starting values is 0.06.