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# EMPIRICAL ANALYSIS OF HOUSEHOLD SAVINGS DECISIONS IN CONTEXT OF UNCERTAINTY: A CROSS- SECTIONAL APPROACH

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# Empirical Analysis of Household Savings Decisions in Context of Uncertainty: A cross-sectional approach

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## Abstract

Most empirical studies of savings behaviour that take explicit account of uncertainty consider for identification data that describe the evolution of circumstances observed during an appreciable period of the life-course. Here we report results obtained using a dynamic programming model that has been adapted to permit identification of preference parameters based on data observed at a point in time for a given population cross-section. The behavioural margins used to identify key preference parameters are described, and the advantages of the approach are discussed. Our empirical results demonstrate the feasibility of the empirical approach in context of contemporary desktop computing technology.

**Key Words:** Dynamic Programming, Savings, Labor Supply

**JEL Classifications:** C51, C61, C63, H31

## 1 Introduction

Empirical analysis of intertemporal decision making is complicated by the effects of uncertainty on incentives. Where uncertainty is considered sufficiently important to warrant a central place in a structural model, then dynamic programming methods are now commonly employed. Studies of savings behaviour in this vein often limit the computational burden by focussing upon the evolving circumstances of individual birth cohorts. The computational advantage that is gained by limiting a dynamic programming model to focus on a single birth cohort is, however, off-set by attendant complications associated with the empirical specification. This paper explores the benefits for empirical analysis of a dynamic programming model of savings and labour supply that projects forward from a population cross-section. Matching a standard life-cycle model of the consumption/savings and labour/leisure margins to British survey data, we conclude that the advantages of projecting forward from a population cross-section can out-weigh the associated computational costs in context of contemporary personal computing technology.

A complex two-dimensional relationship exists between time, cohort, and age effects that characterise differences between heterogeneous population subgroups. Focussing upon the evolving circumstances of a single birth cohort is a useful way for empirical studies to cut through this complexity, as age, time and cohort effects are then described by a single dimension. Such a simplification is particularly appealing where the central subject of interest is complex, as is often the case for decision problems that have no

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closed form solution. Hence, the dynamic programming literature that explores savings behaviour in the context of uncertainty has focussed predominantly upon empirical analysis of cohort-specific structural models, following the seminal study by Gourinchas & Parker (2002).

Alternative data options exist for empirical analyses that focus on cohort-specific structural models of savings behaviour. An obvious choice is to parameterise a cohort-specific model with reference to data observed for a single birth cohort (e.g. Attanasio et al. (2005)).<sup>1</sup> There are, however, at least two important drawbacks of adopting this approach: it is usually difficult to obtain an adequate description of the evolving policy context; and it is uncertain how far results obtained for a single birth cohort can be generalised to the wider population.

These two drawbacks stem from fundamental features of the empirical problem in relation to savings behaviour. An empirical analysis of household savings decisions in context of uncertainty requires for identification data observed for an appreciable period of life. The longer is the period from which data for analysis are drawn, the greater is the scope for appreciable variation of the policy environment underlying observed behaviour. The greater is the variation of the policy environment over multiple dimensions, the stronger is the proposition that such variation is likely to be an important determinant of observed behaviour.

Aspects of the policy environment that typically exhibit substantial variation with time, and which are likely to influence savings decisions of the household sector, include taxes and benefits, (pre-transfer) rates of return, variation of employment opportunities, and the changing nature of family demographics. Obtaining comprehensive (pseudo) panel data regarding all of these factors usually represents a significant challenge, and integrating these data into a structural model in a coherent fashion is more challenging still. Furthermore, allowing for such variation can work to off-set any computational advantage that is derived from focussing on the circumstances of a single birth cohort. We are not aware, for example, of any dynamic programming model of household sector savings that includes an explicit account of reforms to tax and benefits policy implemented during the period of estimation.<sup>2</sup> Such complications hamper efforts to reflect adequately the savings and employment incentives that individuals face.

One popular way to identify results that generalise to the wider population is to conduct sensitivity analysis by exploring data reported for alternative birth cohorts (as in Attanasio et al. (2008)). Such an approach complicates the challenges involved in adequately describing the evolving policy context. Alternatively, empirical techniques can be used that are designed to estimate age-specific moments which

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<sup>1</sup>French (2005) applies a similar procedure, but uses regression techniques to improve estimated age profiles for his reference cohort by drawing upon data observed for near-by cohorts.

<sup>2</sup>An explicit allowance for evolving tax and benefits policy has, however, been implemented in dynamic microsimulation models based on analytical functional forms for behaviour; see, e.g., Nelissen (1998).

control for time and cohort effects (e.g. Sefton et al. (2008)). Collinearity between age, cohort and time effects requires an additional restriction to permit identification. One common restriction, suggested by Deaton (1997), is to assume that time effects average out over the long run. This assumption produces estimated age profiles that represent an average taken over all cohorts included in the panel data used for estimation. The averaging that such methods apply obscures the nature of the underlying policy environment, so that it is difficult – if not impossible – to ensure that the assumed structural specification provides an adequate representation of the incentives underlying observed behaviour.

A third approach that has been applied in the literature is designed to simplify identification of the incentives that underly observed behaviour, which is the principal drawback associated with the two alternatives referred to above. In this case, empirical analysis is based upon cross-sectional data that are adjusted to reflect assumptions about the relationship between the characteristics of the population cross-section and those of a single birth cohort (e.g. van de Ven (2010)). Focussing on cross-sectional data limits the incentives underlying observed behaviour to those that applied at a single point in time, which are relatively simple to document. The drawback of this approach, however, is that strong assumptions are required to derive a stylised relationship between the characteristics of the population cross-section and those of a single birth cohort; assumptions that are unlikely to hold in practice.<sup>3</sup>

The basic premise of this paper is that an overlapping-generations (OLG) model of savings behaviour in context of uncertainty can help to mitigate the weaknesses of the existing empirical literature that are discussed above. Such a model can project the circumstances of a population cross-section through time, and is therefore well adapted to consider implications for a broad segment of society, thereby mitigating concerns regarding the representativeness of results obtained. Furthermore, this modelling approach is capable of describing behaviour observed throughout the life-course at a single point in time, albeit for individuals drawn from different birth cohorts. As such, an OLG modelling approach permits preferences for savings to be identified on cross-sectional survey data, which considerably simplifies the task of describing the incentives underlying observed behaviour. In matching such a model to contemporary survey data observed for Britain, we have found that this estimation strategy is both computationally feasible in context of contemporary computing technology, and facilitates the process of parameterisation.

Although OLG models of savings in context of uncertainty are not new (e.g. Livshits et al. (2007), Hansen & Imrohorglu (2008), Feigenbaum (2008), Hairault & Langot (2008)), most of the associated studies focus on implications of theory, rather than the empirical task of matching models to survey data. We are not aware of any other study that exploits the empirical advantages of an OLG model of

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<sup>3</sup>Adjusting age profiles of income and consumption by trend growth, for example, rests upon the assumption that the economy is in a steady-state equilibrium, characterised by a stable growth path. This assumption is highly unlikely to hold for any modern economy.

savings behaviour that we discuss above.

In Section 2 we provide an overview of the model that is used to undertake the analysis. Details regarding the analytical mechanics that underly our empirical approach are described in Section 3. The data are described in Section 4, and results are reported in Section 5. Our discussion of results focusses on drawing out the ways that preference parameters influence alternative observable margins, which are crucially important for parameter identification. A concluding section provides a summary and directions for further research.

## 2 The Structural Model

The model considers the evolving circumstances of a sample of reference adults and their families, organised into annual snap-shots during the life-course. The decision unit of the model is the nuclear family, defined as a single adult or partner couple and their dependent children. Intra-family allocations are ignored. Decisions regarding consumption, labour supply, and pension scheme participation are endogenous, and are assumed to be made to maximise expected lifetime utility, given a family's prevailing circumstances, its preference relation, and beliefs regarding the future. Preferences are described by a nested Constant Elasticity of Substitution specification. Expectations are 'substantively-rational' in the sense that they are either perfectly consistent with, or specified to approximate, the intertemporal processes that govern individual characteristics. The model assumes a small open economy (appropriate for Britain), for which rates of return to labour and capital are exogenously given. Heterogeneous circumstances of reference adults are limited to the following nine characteristics:

- year of birth of reference adult
- age of reference adult
- relationship status
- education status
- wage potential
- non-pension wealth
- private pension wealth
- timing of pension access
- survival of reference adult

Three of the characteristics listed here are considered to be uncertain and uninsurable from one year to the next when evaluating expected lifetime utility (relationship status, wage potential, and time of death). This specification for the model was carefully selected to ensure adequate margins for empirical identification of unobserved preference parameters. Including year of birth in the list of heterogeneous family characteristics introduces the overlapping generations framework that is necessary to reflect the circumstances of a population cross-section. Age, wage potential, measures of wealth, and survival are all centrally important for any empirical analysis of savings and labour supply. Past experience with similar analytical frameworks has also emphasised the importance of relationship status when seeking to capture labour supply and consumption decisions. Finally, as discussed in Section 3, education status and pension scheme participation decisions feature in the empirical identification strategy employed in this paper. The remainder of this section describes key features of the model; technical details can be

found in the companion paper van de Ven (2013).

## 2.1 Preference relation

Expected lifetime utility of reference adult  $i$  at age  $a$  is described by the time separable function:

$$U_{i,a} = \frac{1}{1-\gamma} \left\{ u \left( \frac{c_{i,a}}{\theta_{i,a}}, l_{i,a} \right)^{1-\gamma} + E_a \left[ \sum_{j=a+1}^A \delta^{j-a} \left( \phi_{j-a,a}^b u \left( \frac{c_{i,j}}{\theta_{i,j}}, l_{i,j} \right)^{1-\gamma} + (1 - \phi_{j-a,a}^b) \zeta B_{i,j}^{1-\gamma} \right) \right] \right\} \quad (1a)$$

$$u \left( \frac{c_{i,a}}{\theta_{i,a}}, l_{i,a} \right) = \left( \left( \frac{c_{i,a}}{\theta_{i,a}} \right)^{(1-1/\varepsilon)} + \alpha^{1/\varepsilon} l_{i,a}^{(1-1/\varepsilon)} \right)^{\frac{1}{1-1/\varepsilon}} \quad (1b)$$

Observable characteristics of the preference relation are  $\phi_{j-a,a}^b$  the probability that a reference adult with birth year  $b$  will survive to age  $j$  given survival to age  $a$ ;  $c_{i,a} \in R^+$  discretionary composite (non-durable) consumption;  $l_{i,a} \in [0, 1]$  the proportion of family time spent in leisure;  $\theta_{i,a} \in R^+$  adult equivalent size based on the “revised” or “modified” OECD scale<sup>4</sup>; and  $B_{i,a} \in R^+$  the legacy that reference adult from family  $i$  would leave if they died at age  $a$ . Unobserved preference parameters are  $\gamma > 0$  the (constant) coefficient of relative risk aversion;  $\delta$  an exponential discount factor;  $\zeta$  the “warm-glow” model of bequests<sup>5</sup>;  $\varepsilon > 0$  the (intra-temporal) elasticity of substitution between equivalised consumption ( $c_{i,a}/\theta_{i,a}$ ) and leisure ( $l_{i,a}$ ); and  $\alpha > 0$  the utility price of leisure.  $E_a$  is the expectations operator and  $A$  is the maximum age that any individual may survive to.

Although the preference relation defined by equation (1) is popular in the associated literature, it has also been the subject of considerable criticism. Four points can be singled out here. First, the assumption of time separability suppresses behavioural persistence, and has been the subject of an extensive debate (e.g. Deaton & Muellbauer (1980), pp. 124–5; Hicks (1939), p. 261). Any empirical study concerned with inter-relations between decisions through time would appropriately consider data observed thorough time, in contrast to the cross-sectional data that are the focus of interest here. Second, it is now increasingly common to allow preference parameters, including discount rates, to vary with individual specific characteristics (e.g. Gustman & Steinmeier (2005), who consider variation in relation to discount rates, and the relative attractiveness of alternative employment options). The associated literature suggests that suppressing this form of variation in an empirical analysis of preferences can be interpreted as a form of omitted variable bias. Third, the assumption that preferences are time consistent – as is implied by the preference relation defined by equation (1a) – has been

<sup>4</sup>The modified OECD scale assigns a value of 1.0 to the family reference person, 0.5 to their spouse (if one is present), and 0.3 to each dependent child. The OECD scale is currently the standard scale for adjusting before housing costs incomes in European Union countries, and is included here to reflect the impact that family size has been found to have on the timing of consumption (e.g. Attanasio & Weber (1995) and Blundell et al. (1994)).

<sup>5</sup>See, for example, Andreoni (1989) for details regarding the warm-glow model.

criticised for failing to reflect a growing body of empirical evidence (e.g. Thaler (1981), Ainslie & Haslam (1992), Green et al. (1994) and Kirby (1997), see Ainslie (1992) for a review). Adapting preferences to accommodate time-inconsistency has also been shown to affect behavioural margins that are used for empirical identification in this study (e.g. Laibson et al. (2007), van de Ven (2010)). Finally, the assumption of a CES specification for intertemporal preferences has been criticised for the restrictions that it imposes upon the relationship between relative risk aversion and the intertemporal elasticity of consumption.

The objective of this study is to demonstrate the advantages of an OLG framework for empirically identifying preferences concerning savings decisions in context of uncertainty. The preference relation defined by equation (1a) was assumed for this purpose, despite its various limitations, because it is both parsimonious (depending only on five preference parameters), and commonly applied in the associated literature. It is hoped that this study will encourage future empirical work to clarify the nature of preferences in relation to savings, including further exploration of the diverse considerations set out in the preceding paragraph.

## 2.2 The wealth constraint

Equation (1) is maximised, subject to an age specific credit constraint imposed on non-pension wealth,  $w_{i,a} \geq D_a$  for reference adult  $i$  at age  $a$ .<sup>6</sup> Non-pension wealth is a net figure measured over all financial assets and liabilities of a family, excluding assets held in private pensions and rights to state benefits. An important asset class included in this measure for the UK is owner occupied housing. The model abstracts from the peculiarities of housing assets that have been explored elsewhere (e.g. Flavin & Nakagawa (2008)).  $D_a$  is set equal to minus the discounted present value of the minimum potential future income stream up to the age threshold  $a_D$ .

Intertemporal variation of  $w_{i,a}$  is, in most periods, described by the simple accounting identity:

$$w_{i,a} = w_{i,a-1} - c_{i,a-1} + \tau_{i,a-1} \tag{2}$$

where  $\tau$  denotes disposable income net of non-discretionary expenditure. There are only two contexts that depart from equation (2). At the time a family first accesses its pension wealth, it is assumed to receive a tax-free lump-sum addition to its non-pension wealth; see Section 2.3. Alternatively, if a reference adult experiences a marriage transition prior to state pension age, then non-pension wealth is assumed to double in response to a new marriage, and to halve in response to a marital dissolution (it is unaffected by marital transitions from state pension age). The second of these effects is designed to account for the influence of divorce on  $w_{i,a}$ .

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<sup>6</sup>  $B_{i,a}$ , the legacy that reference adult  $i$  would leave if they were to die at age  $a$  is related to non-pension wealth in the model as:  $B_{i,a} = 0.2 + \max(0, w_{i,a})$ .

The tax function assumed for the model is represented by:

$$\tau_{i,a} = \tau(l_{i,a}, x_{i,a}, n_{i,a}, n_{i,a}^c, r_{i,a} w_{i,a}, pc_{i,a}, b, a) \quad (3)$$

which depends on labour supply  $(1 - l_{i,a})$ ; private non-capital income,  $x_{i,a}$ ; the number and age of adults,  $n_{i,a}$ ,  $a$ ; the number of dependent children,  $n_{i,a}^c$ ; the return to non-pension wealth,  $r_{i,a} w_{i,a}$  (which is negative when  $w_{i,a} < 0$ ); private pension contributions,  $pc_{i,a}$ ; and birth year,  $b$ .

Non-capital income  $x_{i,a}$  is equal to labour income  $g_{i,a}$  plus pension annuity income. Non-capital income is split between adult family members to reflect the taxation of individual incomes in the UK.

The interest rate,  $r_{i,a}$ , is treated differently depending on and whether  $w_{i,a}$  indicates net investment assets or net debts. Where  $w_{i,a}$  is (weakly) positive, then the interest rate is assumed to vary by age  $a$  and time  $t$ ;  $r_{a,t}^I$ . Age variation of  $r^I$  allows the model to accommodate important age-specific shifts in family portfolio allocations, and time variation allows it to reflect fluctuations in the macro-economy. When  $w_{i,a}$  is (strictly) negative, then the interest rate is designed to vary from  $r_t^{D-}$  at low measures of debt to  $r_t^{D+}$  when debt exceeds the value of working full time for one period ( $g_{i,a}^{ft}$ ):

$$r_{i,a}^s = \begin{cases} r_{a,t}^I & \text{if } w_{i,a} \geq 0 \\ r_t^{D-} + (r_t^{D+} - r_t^{D-}) \min \left\{ \frac{-w_{i,a}}{g_{i,a}^{ft}}, 1 \right\}, r_l^D < r_u^D & \text{if } w_{i,a} < 0 \end{cases} \quad (4)$$

Specifying  $r_l^D < r_u^D$  reflects a so-called ‘soft’ credit constraint in which interest charges increase with loan size.

Although all three of the interest rates referred to above are time variable, families are assumed to ignore this aspect of variation when evaluating their expectations (see Appendix A.1). This stylisation helps to ensure that the model is computationally feasible.

### 2.3 Private Pensions

Private pensions are modelled at the family level as defined contribution schemes. In each year, a family with earnings exceeding a minimum threshold,  $g_l^P$ , can choose whether to make fresh contributions to its pension scheme. If a family chooses to contribute to its pension, then a fixed share of its total pre-tax labour income,  $\pi^P$ , is added to its accumulated pension fund. Contributing families also receive an employer contribution to their pension fund, which is specified as a fixed share of pre-tax labour income,  $\pi_{ec}^P$ . Eligible employer contributions to a family’s pension fund in any given year are lost if the family chooses not to contribute to its scheme in the respective year. Wealth held in a private pension fund,  $w_{i,a}^P$ , is assumed to be illiquid, and attracts a fixed rate of return  $r^P$ . In most periods prior to pension receipt, pension wealth follows the accounting identity:

$$w_{i,a}^P = r^P w_{i,a-1}^P + (\pi^P + \pi_{ec}^P) g_{i,a-1} \lambda_{i,a-1}^P \quad (5)$$



where  $\lambda_{i,a-1}^P$  is an indicator variable, equal to one if the family of reference adult  $i$  at age  $(a-1)$  contributes to its pension, and zero otherwise. The only departures from equation (5) are following relationship transitions, where relationship formation doubles pension wealth and relationship dissolution halves it.

A family can choose to access its accumulated pension fund at any time between ages 55 and 75, at which time 25% of the fund is taken as a tax-free lump-sum transfer into non-pension wealth, and the remainder is used to purchase an inflation adjusted life annuity (which is assessable for income tax as discussed above). The annuity rates assumed for analysis are calculated with reference to the survival rates assumed for individual birth cohorts, an assumed return to capital, and an assumed transaction cost levied at the time of purchase.

This specification of private pension opportunities is designed to reflect in broad terms occupational pension schemes administered in the UK; see Appendix A.4 for further discussion.

## 2.4 Labour income dynamics

Wages are modelled at the family level, and are described by:

$$g_{i,a} = \lambda_{i,a}^{emp} \lambda_{i,a}^o \lambda_{i,a}^{ret} h_{i,a} \quad (6)$$

where  $h_{i,a}$  defines family  $i$ 's latent wage at age  $a$ ,  $\lambda^{emp}$  adjusts for (endogenous) labour supply decisions,  $\lambda^o$  is an adjustment factor to allow for uncertain wage offers, and  $\lambda^{ret}$  is the impact on earnings of accessing pension wealth.

Three labour supply options are considered for each adult family member, representing full-time, part-time and non-employment.  $l_{i,a}$  is a decreasing function of labour supply, and the wage factor  $\lambda_{i,a}^{emp}$  is an increasing function of labour supply;  $\lambda_{i,a}^{emp} = 1$  when all adult members are employed full-time.

$\lambda_{i,a}^o$  is included to allow for involuntary unemployment of the highest adult wage earner in each family. When the highest wage earner is identified as not receiving a wage offer, then  $\lambda_{i,a}^o$  adjusts to ensure that  $g_{i,a}$  is independent of their labour supply decision, implying non-employment where labour supply incurs a leisure penalty. Receipt of wage offers is stochastic and uncertain between years, with the probability depending only upon age and education status. We have found this feature to be important when matching the model to rates of employment during peak working years.

Access to pension wealth is assumed to incur a wage penalty for all subsequent periods of the life-course, represented by the wage factor  $\lambda_{i,a}^{ret}$ . The wage penalty defined by  $\lambda^{ret}$  is useful to match the model to rates of retirement described by survey data.

## Latent wages, $h$

Latent wages are assumed to follow the stochastic process described by the equation:

$$\log\left(\frac{h_{i,a}}{m_{i,a}}\right) = \psi_{i,a-1} \log\left(\frac{h_{i,a-1}}{m_{i,a-1}}\right) + \omega_{i,a-1} \quad (7)$$

$$m_{i,a} = m(n_{i,a}, ed_{i,a}, a, b_i) \quad (8)$$

$$\psi_{i,a} = \psi(n_{i,a}) \quad (9)$$

$$\omega_{i,a} \sim N(0, \sigma_\omega^2(n_{i,a}, ed_{i,a})) \quad (10)$$

where the parameters  $m(\cdot)$  account for wage growth (and depend on relationship status  $n_{i,a}$ , education  $ed_{i,a}$ , age  $a$ , and birth year  $b$ ),  $\psi(\cdot)$  accounts for time persistence in earnings, and  $\omega_{i,a}$  is an identically and independently distributed family specific disturbance term. The variance  $\sigma_\omega^2$  is defined as a function of relationship status and education.

Equation (7) is a parsimonious specification that has been explored at length in the wider empirical literature. Nevertheless, the form of equation (7) differs from much of the related literature by its omission of transitory shocks. In the current context, transitory wage shocks are represented by the wage offers  $\lambda^\circ$  included in equation (6).

## 2.5 Allowing for family demographics

Family demographics in the model refers to three factors: survival of reference adults; the relationship status of reference adults; and the allowance made for dependent children.

### Modelling survival

The model focusses upon survival with respect to reference adults only; the mortality of the spouses of reference adults is aggregated with divorce to obtain the probabilities of a relationship dissolution (discussed below). Survival in the model is governed by age and year specific mortality rates, which are commonly reported components of official life-tables.

### Modelling relationship status

A ‘relationship’ is defined as a cohabitating partnership (including formal marriages and civil partnerships). The relationship status of each reference adult in each prospective year is considered to be uncertain. The transition probabilities that govern relationship transitions depend upon a reference adult’s existing relationship status, their education, age, and birth year. These probabilities are stored in a series of ‘transition matrices’, each cell of which refers to a discrete relationship/education/age/birth year combination.

## Modelling children

Children are modelled as deterministic functions of a reference adult’s age, relationship status and birth year. Non-parametric functions are assumed for dependent children, with a separate dummy variable representing each relationship/age/birth year combination. Hence, all reference adults with the same birth year, age, and relationship status are also assumed to have the same number of dependent children, which may take a non-integer value. This allowance for children ensures that the model is able to capture the hump-shape in consumption needs associated with peak child-rearing ages without increasing the computational burden of the dynamic programming problem.

## 3 Basic Mechanics of the Empirical Approach

In common with the existing dynamic programming literature, a two stage procedure was used to identify parameters that match our structural model to survey data.<sup>7</sup> The first stage identified a subset of parameters exogenously from the model structure. Most of these parameters are directly observable – e.g. marital transition rates, contribution rates to private pensions, the functional forms assumed for taxes and benefits – and are evaluated on survey data. The methods employed to identify this first set of parameters have changed little since the advent in the 1960s of ‘classical’ dynamic microsimulation models. Given the model parameters evaluated in the first stage, remaining model parameters were adjusted in a second stage so that selected ‘simulated moments’ implied by the structural model matched to ‘sample moments’ estimated from survey data. Conceptually, the second stage of the procedure involves adjusting unobserved model parameters to ensure that observable endogenous characteristics implied by the assumed theoretical framework best reflect a selected set of moments estimated from survey data.

The principal departure between the analysis reported in this paper and the related literature is that the model described in Section 2 is designed to consider the decisions of a population cross-section, rather than of a single birth cohort. Our analysis is motivated by the proposition that this cross-sectional approach facilitates evaluation of the second stage of the empirical procedure that is referred to above. In this section we describe how we have implemented this second stage, with emphasis on the relative advantages of taking a cross-sectional perspective.

### 3.1 Evaluating simulated moments

The approach taken to evaluate population moments implied by the assumed theoretical framework is now well established in the related literature. This section therefore provides a brief overview of the

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<sup>7</sup>This two-step procedure is well adapted to the extended computation times required to determine the implications of a given parameter combination and the large number of parameters upon which the model depends.

techniques employed; for further detail see, for example, Adda & Cooper (2003) or Christensen & Kiefer (2009).

Population moments implied by the model under a given set of model parameters were evaluated by: (i) solving the lifetime decision problem for any feasible combination of family specific characteristics; and (ii) using the solutions obtained in (i) to project endogenous characteristics for a reference population.

### Solving the decision problem

No analytical solution exists to the utility maximisation problem described in Section 2, and numerical solution routines were consequently employed. These solution routines are structured around a ‘grid’ that over-lays all feasible combinations of individual specific characteristics (the state space).<sup>8</sup> As noted in Section 2.1, the model assumes that there is a maximum potential age to which any individual may survive,  $A$ . At this age, the decision problem is deterministic, and trivial to solve. The solution routine that we employed starts by solving for utility maximising decisions at all intersections of the grid that correspond to this final period of life, and stores both the maximising decisions and optimised measures of utility (the value function). These solutions at grid intersections for age  $A$  are used to approximate solutions at age  $A$  more generally, via the linear interpolation routine that is described in Keys (1981).

Given results for age  $A$ , the solution routine that we used then considers decisions at intersections corresponding to the penultimate age,  $A - 1$ . Here, expected lifetime utility is comprised of the utility enjoyed at age  $A - 1$ , and the impact that decisions taken at age  $A - 1$  have on circumstances – and therefore utility – at age  $A$ . Given any decision set at age  $A - 1$ ,  $d_{A-1}$ , the solution routine projects forward the set of individual specific characteristics at age  $A$ ,  $z_A$ , that is implied by the processes assumed to govern intertemporal transitions (e.g. equation 2 for wealth, equation 7 for wage potential). If characteristics at age  $A$  are uncertain, then each potential characteristic vector  $z_A^p$  is projected forward with an assigned probability  $pr_A^p$ . Uncertainty in the model is either between a discrete set of alternatives (relationship status, wage offers, and death), or over a continuous normal distribution (wage potential). Expectations over normal distributions were approximated at 5 discrete points, using weights and abscissae implied by the Gauss-Hermite quadrature (implemented following Press et al. (1986)). These terms, combined with a von Neuman Morgenstern preference relation, allow the expected lifetime utility associated with any decision set  $d_{A-1}$  to be evaluated. A numerical routine (described below) was used to search over the set of feasible decisions to maximise expected lifetime

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<sup>8</sup>The grid assumed for analysis has the following dimensions: 26 points for non-pension wealth between ages 18 and 74, and 151 points between ages 75 and 130; 26 points for earnings potential between ages 18 and 74; 21 points for private pension rights from age 18 to 74, and 151 points between ages 75 and 130; 2 points for wage offers between ages 18 and 74; 2 points for pension receipt from age 55 to 75; 2 points for education status from age 18 to 74, 2 points for relationship status from age 18 to age 89. Hence, the grid considered for analysis comprised 10,409,209 individual cells. This problem was solved in 19.6 minutes on a desktop workstation purchased in 2011.

utility at each intersection of the grid corresponding to age  $A - 1$ . These solutions, and the associated measures of optimised utility are stored, and the solution routine then considers the next preceding age. Repeated application of this procedure obtained a numerical approximation of the solution to the lifetime decision problem at all intersections of the grid spanning the feasible state space.

The numerical search routine that was employed for this study is adapted to the decisions that are considered for analysis. As described in Section 2, families are assumed to decide over one continuous domain relating to the consumption/savings margin, and a series of discrete alternatives relating to labour supply, pension participation, and the take-up of pension benefits. The search routine considered each potential discrete alternative in turn, and searched for a local optimum in relation to consumption. Of all feasible alternative solutions, the one associated with the maximum numerical approximation of expected lifetime utility was taken as the solution to the lifetime decision problem.

As the value function of the utility maximisation problem considered here is not smooth, we used three alternative approaches to search over the eligible consumption domain for a local maximum to expected lifetime utility. The first uses Brent's method as described in Press et al. (1986); the second uses the simplex method of Lagarias et al. (1998); and the third applied Brent's method, the method by Lagarias *et al.*, and the multi-level coordinate search method described in Huyer & Neumaier (1999) (as implemented by the NAG library) in serial. All three approaches generated very similar results (which are available from the authors upon request).

### **Calculating simulated moments**

The simulated moments used to guide adjustment of the model's parameters were calculated using data generated by the model for a population of reference adults drawn from a nationally representative cross-sectional survey. The circumstances of each reference adult described by the survey were used to locate them within the grid structure that is referred to above. Given their respective grid coordinates, the linear interpolation methods that are also mentioned above were used to approximate each reference adult's utility maximising decision set, as implied by the numerical solutions identified at grid intersections. Given a family's characteristics (state variables) and behaviour, its characteristics were projected through time following the processes that are considered to govern their intertemporal variation. Where these processes depend upon stochastic terms, random draws were taken from their defined distributions in a process that is common in the microsimulation literature (sometimes referred to as Monte Carlo simulation).

### 3.2 Adjusting model parameters

The second stage of the model parameterisation involved identifying the parameters of the assumed preference relation and a selected set of parameters governing intertemporal evolution of latent wages. Preference parameters are unobservable, and are consequently prime candidates for the second stage of the parameterisation. Although wages are observable, a selected set of wage parameters were included in the second stage of the parameterisation to account of associated selection effects.

This stage of the empirical analysis is commonly conducted either by manual calibration or optimisation of a loss function using an econometric criterion.<sup>9</sup> Our focus in this paper is to explore how model parameters influence implied moments for a population cross-section. The results reported here were consequently obtained via a series of manual adjustments of model parameters, guided by graphical representations and sums of squared errors for a set of age specific population moments, following the approach described by Sefton et al. (2008). Although this approach sacrifices some objectivity in the specification of model parameters, it also facilitates a detailed understanding of the behavioural implications of alternative parameter combinations with which we are principally concerned, relative to a numerical “black-box”.

The assumed preference relation (see Section 2.1) includes five parameters: relative risk aversion,  $\gamma$ ; an exponential discount factor,  $\delta$ ; a parameter for the warm-glow model of bequests,  $\zeta$ ; the intra-temporal elasticity,  $\varepsilon$ ; and the utility price of leisure,  $\alpha$ . In contrast, the specification adopted for wages (see Section 2.4) includes a very large number of parameters. The persistence of latent wages,  $\psi$ , and the factor effects of alternative labour supply decisions,  $\lambda^{emp}$ , were identified in the first stage of the model parameterisation. This left the parameters governing wage growth  $m(\cdot)$ , earnings volatility  $\sigma_{\omega}^2(\cdot)$ , and the factor effects of pension take-up  $\lambda^{ret}$  to be identified in the second stage of the parameterisation. Following extensive experimentation, we settled upon the following step-wise procedure to identify these various parameters.

We divided the calibrated model parameters into two sets; set  $A$  comprising the parameters governing wage growth and earnings volatility, and set  $B$  comprising all other calibrated parameters. We began by setting all wage growth parameters  $m(\cdot) = 1$ , and made initial guesses for earnings volatility,  $\sigma_{\omega}^2(\cdot)$ . Given these assumptions for set  $A$  parameters, and the model parameters identified exogenously from the model structure in the first stage of the analysis, we adjusted the parameters in set  $B$  to reflect behaviour observed at a single point in time for a reference population cross-section. Having obtained first approximations for set  $B$  parameters, we then adjusted the parameters  $m(\cdot)$  and  $\sigma_{\omega}^2(\cdot)$  to reflect historical earnings data. This procedure was then repeated until convergence in the two sets

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<sup>9</sup>Econometric methods include Simulated Minimum Distance (Lee and Ingram, 1991), Method of Simulated Moments (Stern, 1997), Indirect Estimation (Gourieroux et al., 1993) and Efficient Method of Moments (Gallant and Tauchen, 1996).

of parameters  $A$  and  $B$  was obtained. We found that it was not necessary to iterate between these two sets of model parameters more than two times to obtain convergence, a property that is attributable to the invariance of the cross-sectional population characteristics considered for adjusting parameters in set  $A$ , which we discuss further below.

### **Parameters identified on data for a reference population cross-section**

All five preference parameters of the model and the factor effects of pension take-up  $\lambda^{ret}$ , were identified by matching the model to moments evaluated on survey data reported for a single (reference) population cross-section. This is notable, given that preference parameters are often a central focus of interest in the related literature. It is also extremely useful because it simplifies specification of the policy context underlying the behaviour considered for identification, and omits the feed-back effects that can otherwise complicate parameter adjustments.

The feed-back effects that are mentioned here complicate any empirical analysis that refers to dynamic behaviour described for an appreciable period of time. Suppose, for example, that we were interested in matching a structural model of savings and retirement to data observed during the life-course of a single birth cohort. If a given set of model parameters implied savings early in the life course that over-stated observed data, then this might suggest that preferences should reflect greater impatience. Adjusting preferences in this way might then imply lower wealth later in life, and thereby influence model implications for the timing of retirement. Such feed-back effects can be ignored in an empirical analysis of household sector savings that focuses exclusively on behaviour described for a single point in time (as population characteristics such as wealth holdings are exogenously defined), which considerably simplifies the identification problem.

Our calibration of parameters identified on cross-section survey data started with the assumption of a high value for  $\gamma$  ( $=5$ ), a high value for  $\delta$  ( $=1$ ), a low value for  $\zeta$  ( $=0$ ), and a moderate value for  $\varepsilon$  ( $=0.5$ ). Parameterisation then proceeded in four concentric ‘loops’.

(1) The inner-most loop, which was repeated most frequently, focussed on adjusting  $\alpha$  and  $\lambda^{ret}$ . Increasing the utility price of leisure  $\alpha$  tends to decrease labour supply throughout the working lifetime. Exaggerating the wage discount for families that have previously accessed their private pensions tends to decrease labour supply late in the working lifetime. These two model parameters provide a high degree of control over the employment profile throughout the life-course, and were jointly adjusted to match the model to age and relationship-specific means for employment participation.

(2) The second loop of our calibration jointly adjusted  $\delta$  and  $\zeta$  to reflect age and relationship specific geometric means for consumption. Increasing the discount factor  $\delta$  makes families more patient, and consequently tends to decrease consumption throughout the working lifetime. In contrast, exaggerating

the bequest motive by increasing  $\zeta$  tends to lower consumption late in the life course when the probability of imminent mortality is appreciable. Taken together  $\delta$  and  $\zeta$  provide a high degree of control over the age profile of consumption implied by the structural model.

(3) The third loop of the calibration strategy adjusted the parameter of relative risk aversion  $\gamma$ .  $\gamma$  has an important influence on savings incentives throughout the life course, in common with the discount factor  $\delta$ . Raising  $\delta$  *ceteris paribus* tends to imply lower consumption and higher pension scheme participation as families are made more patient. Raising  $\gamma$ , in contrast, exaggerates precautionary savings motives, implying lower consumption *and* lower pension scheme participation (due to the illiquidity of pension wealth). Hence, if the rates of pension scheme participation implied by the model following the second loop of the calibration were too low (high), we reduced (increased)  $\gamma$  and returned to the inner-most loop. Otherwise we proceeded to the outer-most loop.

(4)  $\varepsilon$  was adjusted to match the model to distributional variation described by data for the ratio between equivalised consumption and leisure. If the utility maximisation problem was separable, and labour supply was a decision on a continuous domain, then the preference relation defined by equation (1) would imply the following relationship between the decision variables  $c$  and  $l$  in the region of the optimum:

$$\alpha \frac{\hat{c}_{i,a}}{l_{i,a}} = \hat{h}_{i,a}^\varepsilon \quad (11)$$

where  $\hat{c}$  denotes equivalised consumption and  $\hat{h}$  is the equivalised post-tax and benefit wage rate. This relationship will approximately hold late in the simulated working lifetime, when families exhibit substantial variation over labour supply decisions and continue to possess multiple periods over which they can choose between (discrete) labour supply alternatives. The relationship defined by equation (11) can be used to compare the decisions taken by any two families, 0 and 1, as described by the ratio:

$$\left(\frac{\hat{c}}{\hat{l}}\right)_1 / \left(\frac{\hat{c}}{\hat{l}}\right)_0 = \left(\frac{\hat{h}_1}{\hat{h}_0}\right)^\varepsilon \quad (12)$$

Equation (12) indicates that increasing  $\varepsilon$  will tend to shift period specific expenditure in favour of (equivalised) consumption, relative to leisure, for families with relatively high (equivalised) wage rates.

Model implications were consequently evaluated for the ratio between equivalised consumption and leisure for every family with a reference adult aged 55 to 60 in the reference population cross-section. Two separate averages were calculated over these ratios, distinguishing families with and without reference adults educated to graduate level. If the value of the ratio of the graduate average divided by the non-graduate average was too low (high), then  $\varepsilon$  was increased (decreased). The calibration then proceeded back to the inner-most loop, and the entire process repeated until a convergence was obtained.



## Parameters identified on historical earnings data

The drift parameters,  $m(\cdot)$ , and the dispersion parameters,  $\sigma_{\omega}^2(\cdot)$ , were calibrated against historical data by projecting the reference population cross-section backward through time. The drift parameters were adjusted to reflect geometric means of employment income, distinguished by age, year, relationship status and education status. The model includes a separate drift parameter for each age, year, education and relationship combination, so that a close match could be obtained to the associated sample moments. Given the large number of model parameters involved, this stage of the parameterisation was undertaken using an automated procedure. First, age, year, education and relationship specific means of log employment income implied by the model under any given parameter combination were calculated from simulated panel data projected back in time for the reference population cross-section. These simulated moments were subtracted from associated sample moments estimated from survey data. The differences so obtained were then multiplied by a ‘dampening factor’, equal to 0.4.<sup>10</sup> The exponent of the result was taken, and multiplied by the prevailing drift parameter to obtain an updated value for the parameter. This procedure was repeated until the average absolute variation of parameters over ages for any year, education, and relationship combination fell below 5 percentage points.

Similarly, the variance parameters were adjusted to reflect age, year, and relationship specific variances of log employment income calculated from survey data. Unlike the drift parameters, however, only four parameters – distinguish singles from couples, and graduates from non-graduates – were adjusted to reflect the dispersion of employment income. These model parameters were adjusted manually.

## 4 Survey Data

This section defines the cross-sectional data selected for analysis, before describing the sample moments used to conduct the second stage of the model calibration.

### 4.1 The reference population cross-section

Data for the reference population cross-section were drawn from wave 1 of the Wealth and Assets Survey (WAS). This survey is designed to provide representative data for households and individuals in Great Britain, and includes information concerning demographics, income, assets and debts. As such, the survey is ideally suited for the analysis that is undertaken here. Wave 1 of the survey was drawn from the Postcode Address File, specified to reflect the population accommodated in private households in Great Britain, excluding Scotland north of the Caledonian Canal, the Scottish Islands and the Isles of Scilly. The survey was designed to over-sample from high wealth households, and information was

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<sup>10</sup>A dampening parameter often improves convergence properties of iterative search routines like the one considered here.

solicited from all individuals aged 16 or over in each responding household (excluding full-time students between 16 and 18 years of age). Data were collected continually between July 2006 to June 2008, and the survey achieved a response rate of 55 per cent, reporting information for 71,268 individuals in 30,595 households.<sup>11</sup>

The survey data reported by the WAS were subject to three key adjustments: the sample was constrained; the units of analysis were altered; and missing earnings data were imputed. All of these adjustments were made using the Stata statistical package, and the associated code is available from the authors upon request.

### **Data limits imposed**

The WAS sample covers the period from July 2006 to June 2008. Although unemployment remained reasonably stable throughout this period, and the stock market did not crash until October 2008, measures of consumer confidence did fall substantially in the second year of the sample.<sup>12</sup> To avoid contaminating the calibration with behaviour observed in context of unusually pronounced negative sentiment, we therefore limit our calibration to the population cross-section observed during the year between July 2006 and June 2007.

We further restrict the WAS cross-sectional sample to omit any household with a member reported to be self-employed, due to well-recognised difficulties in evaluating their financial circumstances. Households with a member who was recorded as having a non-contributory pension scheme were also excluded to limit the heterogeneity in savings incentives described by our sample (primarily constituting public-sector employees).

### **Family units**

The WAS organises data by survey household, which is a broader unit of analysis than the nuclear families that we focus on here. To account for this mis-match, we began by identifying family units, defined as married couples and their dependent children (under age 18), from the micro-data reported by the WAS. Most family-level statistics required for the model could be obtained by summing over the individual specific data reported by the WAS within each family unit. The notable exception was home ownership, in which case the value of the main home was allocated to the family unit of the household reference person (identified by the survey).

Gender neutrality is a guiding principal adopted for our empirical analysis, and we consequently represented each person aged 18 or over reported by the WAS as a reference adult of a separate family

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<sup>11</sup>Although data from wave 2 of the Wealth and Assets Survey were not publicly available when this analysis was conducted, these were released in the summer of 2012.

<sup>12</sup>In the two years to July 2007, the GfK Consumer Confidence Index was approximately stable at -4 (negative being an indication of pessimism on average). The index fell between August 2007 and July 2008, from -4 to -39.

unit in the population cross-section from which model projections were made. Model characteristics specific to the reference adult, including year of birth, age, and education status, were equated to the characteristics of the respective adult reported by the WAS. All other model characteristics were set equal to their values reported for each adult’s family.<sup>13</sup> This approach meant that family characteristics of couples reported in the WAS were represented twice in our data – once for each partner. To avoid over-sampling couples in our empirical analysis, we divided the weighting factor attached to each couple by two. The total sample size considered for analysis, following the sample selection described above, was 22,689 family units.

### **Imputing missing earnings**

We are fortunate that the WAS provides almost all of the detail that we require to describe the characteristics of the reference population cross-section. The most notable exception applies to adults who are not reported as working full-time in the WAS, in which case the survey does not report their full earnings potential. We imputed the missing data wherever necessary, using reduced-form regressions estimated on the wider WAS data. Associated regression results are reported in Appendix B.

## **4.2 Sample moments**

Our calibration strategy is described in Section 3.2. This strategy was implemented with reference to the following sample moments:

1. The proportion of adult family members employed, by age and relationship status; estimated on data for the population cross-section observed in 2006.
2. The geometric mean of family employment income, by age, education and relationship status; estimated on data for population cross-sections observed from 1978 to 2010.
3. The variance of family log employment income, by age, education and relationship status; estimated on data for the population cross-sections observed from 1978 to 2010.
4. The geometric mean of family consumption, by age and relationship status; estimated on data for the population cross-section observed in 2006.
5. The proportion of families reporting to contribute to private pensions, by age and relationship status; estimated on data for the population cross-section observed in 2006.

These sample moments were estimated on survey data from the Family Expenditure Survey (FES) and the Family Resources Survey (FRS). In common with the WAS, the FES and FRS are conducted

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<sup>13</sup>The characteristics obtained for each family unit from the WAS were: age, relationship status (single/couple), total net non-pension wealth, full-time earnings (imputed if not reported), total private pension wealth, whether income received from private pension, and a population weighting factor.

by the Office for National Statistics, use similar sampling frames and methods, and typically achieve similar response rates to the WAS. The most significant departures between the sampling approaches implemented by the three surveys are the over-sampling of high wealth households by the WAS, and the time period covered by the respective sampling frames: while we focus on the WAS data reported for the year extending between July 2006 and June 2007, the FES reports data for calendar years, and the and FRS reports data for the UK financial year (starting in April). We ignore the mismatch between the time frames covered by these alternative data sources.

The FES is the principal source of micro-data for domestic expenditure in the UK. In addition to expenditure, it provides detailed information regarding family demographics, employment, and earnings, and covers a relatively long time-series, reporting at annual intervals from 1978. Most of the sample moments used for calibrating the model parameters were consequently estimated on FES data. The exception concerns participation rates in private pensions, which are more adequately described by the FRS than the FES.

## 5 Calibrated Preference Parameters

This section reports the calibrated model parameters that were adjusted endogenously to the structural model, and which were identified using data observed for a reference population cross-section. As discussed in Section 3.2, this includes all of the parameters of the assumed preference relation and the factor effects of pension take-up. All other model parameters are reported in Appendices A to C.

### 5.1 Calibrated parameters

Calibration of the model parameters to behaviour observed for the reference population cross-section required testing over 263 alternative parameter combinations. Our preferred parameter set is reported in Table 1.

The calibrated value for the parameter of relative risk aversion  $\gamma = 1.675$  is within the broad range

Table 1: Calibrated model parameters adjusted to match behaviour reported for the British population cross-section in 2006

relative risk aversion (gamma)	1.6750
intra-temporal elasticity (epsilon)	0.3000
utility price of leisure (alpha)	1.3000
discount factor (delta)	0.9590
bequest motive (zeta)	5090
factor effects of pension take-up	
at age 55	0.6
from age 65	0.0

identified by the associated literature. Simulations undertaken by Auerbach & Kotlikoff (1987), for example, are based upon a coefficient of risk aversion of 4, while Cooley & Prescott (1995) consider a value of 1. Grossman & Shiller (1981) and Blundell et al. (1994) report estimates just over 1.0, while Hansen & Singleton (1983), Mankiw et al. (1985), and Ziliak & Kniesner (2005) report estimates of approximately 1. Values of the coefficient of risk aversion required to explain the equity premium puzzle (Mehra & Prescott (1985)) are high by comparison, supported by econometric estimates reported by Mankiw (1985) and Hall (1988). Nevertheless, evidence from attitudinal surveys suggest that the value is unlikely to be greater than 5 (Barsky et al. (1997)).

The value obtained for the intra-temporal elasticity of substitution implies that consumption and leisure are direct complements.<sup>14</sup> The utility price of leisure is in the region of 1.0 by construction<sup>15</sup>, and the discount factor implies a higher rate of time discounting than the long-run returns to safe assets (1.5% p.a.) or pension assets (3.5% p.a.). The bequest motive is quite important in capturing the age profile of consumption displayed by survey data, as indicated by the calibrated parameter value. The factor effects of pension take-up apply a 40% discount to earnings at age 55, and increase linearly to 100% from age 65.

Numerical simulations indicate that the calibrated model parameters imply an inter-temporal elasticity of consumption of 0.274 measured at the population means.<sup>16</sup> This is in contrast to the controversial finding by Hall (1988) that the inter-temporal elasticity may not be very different from zero (see also Dynan (1993), Grossman & Shiller (1981), and Mankiw (1985)). Other studies have, however, found evidence to support substantially higher inter-temporal elasticities than reported here. Attanasio & Weber (1993), for example, find that focussing upon cohort data for individuals who are less likely to be liquidity constrained than the wider population obtains an estimate for the inter-temporal elasticity of consumption of 0.8 on UK data, and Attanasio & Weber (1995) report estimates between 0.6 and 0.7 for the US. Other empirical studies that support higher rates for the inter-temporal elasticity include Blundell et al. (1993) (0.5), Blundell et al. (1994) (0.75), Engelhardt & Kumar (2007) (0.75), Hansen & Singleton (1983) and Mankiw et al. (1985) (just over 1). The meta-analysis by Havranek et al. (2013) includes 34 studies that report 242 estimates for the intertemporal elasticity of substitution calculated on UK data, with a mean of 0.487 and a standard deviation of 1.09.

<sup>14</sup>The preference relation described by equations (1a) and (1b) implies that  $U_{cl} = (1/\varepsilon - \gamma)U_c U_l / u^{1-\gamma}$ , which is negative when  $1/\varepsilon < \gamma$ .

<sup>15</sup>The equivalence scale is multiplied by 470, so that equivalised consumption is rescaled to an order of magnitude of 1.0.

<sup>16</sup>This statistic was estimated by numerically calculating the derivative  $d(\Delta \ln c_{i,t})/d \ln r_{i,t}$ , where  $\Delta \ln c_{i,t} = \ln c_{i,t} - \ln c_{i,t-1}$ , for the reference population cross-section. The derivative was taken by perturbing interest rates up by 0.5 percentage points (giving an elasticity estimate of 0.263), and down by 0.5 percentage points (giving an estimate of 0.285). The average between these two estimates is reported here.

## 5.2 Identification

Table 2 reports a set of summary statistics that is specified to indicate the influence of model parameters on the moments considered for identification. The table is designed to be read from top to bottom, and from left to right. The top-left corner of the table reports our preferred parameter combination, re-stating Table 1. Below the model parameters are measures of fit for the proportion of the population not employed, which were used to identify the utility price of leisure  $\alpha$  and the factor effects of pension take-up  $\lambda^{ret}$  in the first ‘calibration loop’ described in Section 3.2. Sensitivity to alternative assumptions concerning  $\alpha$  and  $\lambda^{ret}$  is reported in the next two sets of columns to the right of the ‘preferred’ parameter combination.

The ‘high alpha’ series describes sensitivity of the model fit to increasing  $\alpha$  from 1.3 (the preferred value) to 1.6. Comparing the fitted moments for employment reveals that the higher value for  $\alpha$  is associated with higher values for the difference between the simulated and sample moments of non-employment for both singles and couples throughout the working lifetime. Hence, as preferences for leisure are strengthened, the proportion of the population choosing non-employment is projected by the model to rise, with the most substantial effects reported for peak working years. This variation is projected to increase the root mean squared error between the simulated and sample moments for non-employment by just over 1 percentage point.

The ‘no retirement effects’ series describes sensitivity of the model fit to setting  $\lambda^{ret} = 1$  throughout the life-course, so that pension take-up is assumed to have no impact on the wages that families can earn. Comparing the fitted moments for employment reveals that the suppression of wage responses to pension take-up has a substantive impact on the match obtained late in the working lifetime. Without tying pension take-up with wage potential, retirement rates implied by the model are much lower than those described by survey data, resulting in a rise in the root mean square error of non-employment of 8 percentage points.

Below the measures of fit for non-employment are those for age and relationship specific geometric means of consumption. These moments were used to adjust the discount factor  $\delta$  and the warm-glow bequest parameter  $\zeta$  in the second calibration loop described in Section 3.2. Sensitivity of the model fit to alternative assumptions concerning  $\delta$  and  $\zeta$  is reported in the two sets of columns to the right of the ‘no retirement effects’ parameter combination.

The ‘high delta’ series describes sensitivity of the model fit to increasing  $\delta$  from 0.959 to 0.975. The measures of fit for consumption that are reported for this series indicate that the higher value of  $\delta$  is associated with lower measures of consumption throughout the life course for both singles and couples. Hence, as preferences exhibit more patience, consumption in the reference cross-section is projected

to decline, with the largest effects in absolute terms projected for couples late in the life course. The root mean square error evaluated for this parameter alternative is 46.8, up from 39.1 in the preferred parameter specification.

The effects on simulated consumption moments of raising  $\delta$  are qualitatively similar to those projected under the high alpha series that is discussed above. This is observed because a strong preference for leisure implied by a high value of  $\alpha$  reduces lifetime income, which depresses period specific consumption. Taking the high alpha and high delta results together suggests that the effects on consumption of a high value of  $\alpha$  can be off-set by assuming a low value for  $\delta$  (reflecting less patience). The employment effects projected under a the high delta series, however, suggest that this would increase simulated rates of non-employment, thereby exaggerating the higher rates of non-employment simulated in context of a high value for  $\alpha$ . In practical terms, the inter-dependence of the effects on fitted moments that this discussion reveals between alternative assumptions concerning  $\alpha$  and  $\delta$  indicates the need to iterate between the respective calibration ‘loops’ to identify a preferred parameter combination.

The ‘low zeta’ series describes sensitivity of the model’s fit to reducing the value of  $\zeta$  from 5090 to 2000. The consumption moments that are reported in Table 2 indicate that reducing  $\zeta$  resulted in higher consumption, with the strongest effects applying late in the life-course. Whereas consumption increased by an average of £56 per week for couples (£29 for singles) between ages 55 and 74, it increased by just £17 per week for couples (£9 for singles) between ages 20 and 29. These shifts produced an increase in the root mean square error for the geometric mean of consumption from 39.1 to 42.6.

Below the measures of fit for the geometric means of consumption in Table 2 are those for participation rates in private pensions. These moments were used to adjust the (constant) parameter of relative risk aversion  $\gamma$  in the third calibration loop described in Section 3.2. Sensitivity of the model fit to the assumed value for  $\gamma$  is reported in the ‘high gamma’ series, which is displayed immediately to the right of the ‘low zeta’ series that is discussed above. The high gamma series assumes a value of  $\gamma$  equal to 2.0, up from 1.675 in our preferred specification. As discussed in Section 3.2, the three parameters  $\gamma$ ,  $\delta$ , and  $\zeta$  all have an important bearing on simulated moments for both consumption and pension participation. Furthermore, the above discussion reveals that these parameters also influence preferences concerning labour supply. As the parameters adjusted in each of the calibration loops discussed above were identified taking the value of  $\gamma$  as given, they are re-specified in the high gamma series to clarify the effects underlying the assumed identification strategy. This involved increasing impatience by reducing  $\delta$ , which off-sets the heightened precautionary savings motive associated with greater risk aversion, increasing  $\zeta$  to force down consumption late in life (where modelled uncertainty is less pronounced), and reducing  $\alpha$  to off-set associated employment effects. This combination of adjustments

ensures that simulated age profiles for geometric mean consumption and rates of non-employment are approximately the same under the high gamma series as those obtained for the preferred parameter combination.

The measures of fit for pension participation that are reported for the high gamma series indicate that high relative risk aversion discourages pension participation during prime working years. Increasing  $\gamma$  from 1.675 to 2.0 reduces simulated participation in private pensions by a margin of approximately 10 percentage points for singles aged 20 to 54, and by almost 20 percentage points for couples. In contrast, simulated participation rates in private pensions late in the working lifetime are relatively little affected, which can be attributed to the coincident reduction in the time at which pension wealth can be accessed and the muted uncertainty that older families face. As a result of these effects, the root mean squared error of pension participation identified for the high gamma series is 8 percentage points higher than in our preferred parameter specification.

Below the measures of fit reported for participation in private pensions in Table 2 are those that were referenced to adjust the intratemporal elasticity  $\varepsilon$ . As discussed in Section 3.2,  $\varepsilon$  was adjusted in the final loop of the calibration to match simulated to sample statistics for the mean equivalised consumption to leisure ratio of graduates aged 55 to 60, divided by same statistic for non-graduates. Sensitivity of the model fit to assuming a value for  $\varepsilon$  equal to 0.5 rather than 0.3 (as in our preferred parameter combination) is reported in the ‘high epsilon’ series displayed at the far-left hand side of Table 2. These statistics indicate that the employment, consumption, and pension participation statistics referenced to adjust the other parameters discussed here are all broadly insensitive to the assumed value of  $\varepsilon$ . In contrast, assuming a value of  $\varepsilon$  of 0.5 rather than 0.3 increases the ratio of equivalised consumption to leisure statistic reported at the bottom of the table from 0.018 to 0.169. The sensitivity of the equivalised consumption to leisure ratio to  $\varepsilon$  ensures that the parameter is tied down tightly by the calibration procedure, and the lack of sensitivity of the alternative moments considered for the calibration to  $\varepsilon$  limits the need to iterate repeatedly through the alternative calibration loops that are mentioned above.



Table 2: Measures of fit between simulated and sample moments, by parameter combination, relationship status and age band

	preferred		high alpha		no retirement effects		high delta		low zeta		high gamma		high epsilon	
	singles	couples	singles	couples	singles	couples	singles	couples	singles	couples	singles	couples	singles	couples
gamma	1.675		1.675		1.675		1.675		1.675		2.000		1.675	
epsilon	0.300		0.300		0.300		0.300		0.300		0.300		0.500	
alpha	1.300		1.600		1.300		1.300		1.300		1.100		1.300	
delta	0.959		0.959		0.959		0.975		0.959		0.940		0.959	
zeta	5090		5090		5090		5090		2000		154970		5090	
factor effects of pension take-up														
at age 55	0.600		0.600		0.600		0.600		0.600		0.600		0.600	
from age 65	0.000		0.000		0.000		0.000		0.000		0.000		0.000	
	assumed model parameters													
	proportions of population not employed													
age 20-29	0.051	-0.031	0.051	0.003	-0.031	0.050	-0.050	0.051	-0.021	0.051	-0.039	0.050	-0.031	
age 30-54	-0.062	-0.006	-0.045	0.077	-0.062	0.005	-0.070	-0.055	-0.053	0.036	-0.066	-0.022	-0.065	-0.008
age 55-74	0.055	0.064	0.072	0.093	-0.231	-0.292	0.044	0.048	0.067	0.085	0.053	0.057	0.058	0.061
RMSE	0.0888		0.1004		0.1735		0.0947		0.0941		0.0892		0.0902	
	geometric mean consumption (£2006 per week)													
age 20-29	-0.754	-4.626	-4.373	-26.349	-0.605	-4.185	-4.863	-18.836	8.321	12.593	-13.382	-13.322	0.599	5.696
age 30-54	34.050	-28.544	25.648	-69.484	36.270	-22.233	23.662	-53.205	48.088	-3.015	31.380	0.603	39.860	-2.407
age 55-74	-11.603	-14.366	-16.552	-30.228	6.150	30.723	-20.907	-38.961	17.930	41.396	-19.810	-6.185	-6.082	6.293
RMSE	39.0985		50.8405		39.2045		46.8225		42.5921		38.2799		37.8690	
	proportions of population participating in private pensions													
age 20-29	-0.092	-0.084	-0.077	-0.091	-0.099	-0.101	-0.016	0.114	-0.073	-0.023	-0.153	-0.246	-0.089	-0.091
age 30-54	-0.037	0.064	-0.070	0.018	-0.043	0.050	0.023	0.152	-0.029	0.084	-0.131	-0.125	-0.026	0.075
age 55-64	-0.025	-0.065	-0.037	-0.087	-0.046	-0.074	0.001	-0.051	-0.011	-0.069	-0.041	-0.060	-0.023	-0.063
RMSE	0.0951		0.0961		0.1044		0.1096		0.0925		0.1756		0.0975	
	equalised consumption to leisure ratio of graduates to non-graduates*													
Difference	0.0177		-0.0218		-0.0678		0.0669		-0.0177		0.0645		0.1693	

Notes: all statistics other than RMSEs report simulated moments less sample moments

RMSE reports root mean square error of age and relationship specific moments, aggregated by population weights

gamma = parameter of (constant) relative risk aversion; epsilon = intratemporal elasticity; alpha = utility price of leisure; delta = exponential discount factor; zeta = warm-glow bequest parameter

\* simulated less sample statistics for mean equalised consumption to leisure ratio of graduates aged 55 to 60, divided by same statistic for non-graduates

## 6 Conclusions

This paper reports the results of a calibration conducted for a structural model of savings and labour supply to cross-sectional survey data for Britain. The structural model is specifically designed to consider behaviour for a population cross-section, with the objective of testing the conjectures that such a framework is computationally feasible on contemporary technology, and helps to address empirical complications that are discussed for cohort-specific models of behaviour. Importantly, our analysis indicates that preference parameters for a structural model of savings and employment – including the parameter of relative risk aversion – can be identified on behavioural margins observed for a population cross-section at a single point in time. We argue here that the additional complications involved in extending a dynamic programming model of savings to allow for heterogeneous birth cohorts are more than off-set by the conceptual advantages derived when bringing such a model to survey data.

The model that we consider here is based upon a preference relation that is standard in the literature, and we set out a concise calibration strategy that focuses upon specific and important behavioural margins. Our preferred parameter specification is shown to match to observed behaviour over employment, consumption, and pension scheme participation. It is notable that we match to all of these behavioural margins through the adjustment of just seven model parameters, five of which describe model preferences. Furthermore, the preference parameters obtained are broadly in line with those calculated in the associated empirical literature.

Parameterising a structural dynamic programming model of savings on data observed for a population cross-section at a point in time opens up a range of exciting empirical possibilities. One such possibility is to consider whether the intertemporal elasticity of substitution exhibits systematic variation with the economic cycle. This might help to explain the wide diversity of estimates that have previously been reported for this important preference parameter, with important behavioural and policy implications. Improvements in computing technology, and advancements in empirical methods will hopefully offer a wealth of opportunities during the next few decades to further our understanding of the decisions that people make.

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## **A Exogenously Identified Model Parameters**

### **A.1 Interest rates on non-pension wealth**

Real rates of return to positive net non-pension wealth are evaluated in a way that takes into consideration the importance of housing in UK private sector balance sheets. Age specific weights of net wealth held in owner occupied housing were calculated from the WAS cross-sectional data from which model projections are made.<sup>17</sup> These age specific averages are assumed to remain constant through time, and are used to dis-aggregate positive balances of net non-pension wealth into housing and non-housing wealth. Each of these measures of wealth is assumed to attract a separate rate of return.

Nominal returns to housing wealth were calculated from the “mix-adjusted housing price index” reported for the United Kingdom between 1970 and 2010 by the ONS (House Price Index Reference Table 8). Nominal returns to non-housing (non-pension) wealth are set equal to the yield on long-dated Gilts reported between 1970 and 2010 by the ONS (code AJLX). The lower bound interest charges on debt were set equal to the interest rates paid on personal loans and the upper bound charges were set equal to the interest rates paid on sterling credit card lending reported for the period 1995 to 2010 by the Bank of England (codes IUMCCTL and IUMHPTL). All of these interest rates were discounted for inflation using the National Accounts final consumption expenditure deflator (code YBGA). Any rates applicable to periods outside of the years reported above were set equal to the respective averages of the observed rates. As discussed in Section 2.2, although time-variation of each interest rate is taken into account when simulating families through time, each family is assumed to expect interest rates that are time-invariant. Family expectations concerning each interest rate are set equal to the respective averages taken over the observed periods referred to above.

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<sup>17</sup>See folder C:\MyFiles\NIESR\projects\HMT\analysis\WAS

## A.2 Wage parameters

The specification of latent wages is defined as a random walk with drift, so that  $\psi = 1.0$ . Full-time employment of all adult family members is assumed to reduce family leisure time by 40%. This assumption is based upon the view that there are 16 hours available for allocation each day, and that full-time employment consumes nine hours per day, five days per week. Part-time employment is assumed to be equivalent to 40% of a full-time job, reducing both leisure time and earnings in the same proportion,  $\lambda^{emp} = 0.4$ .<sup>18</sup>

## A.3 Tax and benefits policy

Taxes and benefits are formally modelled on the transfer system that applied in the UK in 2006/7. Simulated transfer policy distinguishes between two periods of the life-course, subject to an age threshold set equal to state pension age  $t_{SPA}(b)$ . The program code adopted to simulate taxes and benefits in the model can be obtained from the authors upon request.

### Taxes and benefits during the working lifetime

Prior to  $t_{SPA}$ , taxes and benefits are based upon the *Tax Benefit Model Tables* (TBMT) produced by the Department for Work and Pensions, which are designed to capture the key elements of the transfer system that applied to the healthy working-aged population. These include income taxes, national insurance contributions, the working tax credit, the child tax credit, the child benefit, housing benefit, council tax benefit, Jobseeker's allowance, healthy start allowances, and free school meals.

The allowance made for child tax credit requires assumptions to be made about the child-care costs to which a family is subject. Similarly, the allowance made for housing benefit and council tax benefit require assumptions to be made about housing and council tax costs. These costs are all assumed to be non-discretionary, and are based on the assumptions reported in the April 2006 edition of the TBMT. Beyond the assumptions made by the TBMT, it was necessary to assume that child-care costs are incurred by any family with at least one dependent child, and where all adult household members work full-time.

As a brief overview, the disposable income of a family is calculated by:

1. evaluating aggregate *take-home pay* from the taxable incomes of each adult family member – this reflects the taxation of individual incomes in the UK
2. calculating *benefits receipt* (excluding adjustments for child care and housing costs) from aggregate

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<sup>18</sup>The *2007 Annual Survey of Hours and Earnings* reports that median weekly earnings of all part-time employees were 31.5% of full-time employees (143.9/456.7), and that mean hours of all part-time employees were 46.7% of full-time employees (18.4/39.4).

household take-home pay – this reflects the fact that benefits tend to be provided at the level of the family unit

3. calculating non-discretionary *net child care costs* (after adjusting for child care related benefits) from aggregate take-home pay
4. calculating non-discretionary *net housing costs* (after adjusting for relevant benefits receipt) from aggregate take-home pay plus benefits less child care costs – this reflects the fact that housing benefit and council tax benefit in the UK are means tested with respect to income net of most other elements of the tax and benefits system
5. household *disposable income* is then equal to aggregate take-home pay, plus benefits, less net child care costs, less net housing costs.

As families approach state pension age, two alternative income support schemes to Jobseeker’s allowance are considered for analysis. These schemes are included to capture early-retirement incentives, and both are consequently applied only to families that do not supply any labour. Any family with a reference adult within 10 years of state pension age is considered eligible to the incapacity benefit in place of Jobseeker’s allowance. The incapacity benefit pays £78.50 per week in 2006 to a single adult, and £125.45 per week to a couple. These figures are appreciably higher than Jobseeker’s allowance, which pays £57.45 per week to a single adult, and £90.10 per week to a couple, which helps to support early retirement. Similarly, the model takes into consideration eligibility to the Guarantee Credit component of the Pension Credit, which could be obtained from age 60 in 2006.

### **A.3.1 Taxes and benefits from state pension age**

A similar approach was taken to model taxes and benefits from state pension age as described above for the working lifetime. Unlike for the working lifetime, however, the specification of transfer payments from state pension age could not be based on the TBMT, as these do not cover retirement benefits. Rather, we referred to official rates and thresholds of the transfer schemes that applied in practice to specify this aspect of the transfer system.

Five transfer schemes are explicitly taken into account by the transfer system considered for analysis from state pension age. Income taxes take a step-wise rate structure similar to those applied in the working lifetime (but subject to a different tax-free minimum income threshold). The Pension Credit (comprising both the Guarantee Credit and Savings Credit) is a means-tested benefit scheme, which is withdrawn at the rate of £1 for every £1 of private income up to a minimum threshold, and then at the rate of £0.40 for every £1 of private income thereafter, until the benefit is exhausted. Housing benefits



and council tax benefits are modelled in the same way as described for the working lifetime, including the associated assumption regarding the incidence of non-discretionary housing and council tax costs.

Finally, allowance is made for state contributory pension schemes. In practice, two schemes were applied in the UK in 2006; the basic state pension was subject to a maximum benefit value payable in respect of a minimum contributions history; and the state second pension was an income related benefit, rights to which were accrued in respect of national insurance contributions paid during the working lifetime. To avoid adding two state variables to the decision problem we represented both of these schemes by a single flat-rate state pension paid from state pension age, and set equal in value to the full basic state pension. This stylisation reflects the intention of policy reforms set out in the May 2006 Pensions White Paper published by the DWP, which appreciably relaxed the conditions required to obtain the full basic state pension, and severed the earnings link of the state second pension.

### **Transfer policy through time**

Although much of the empirical analysis with which this study is concerned focusses on behaviour observed at a single point in time, the structural model upon which the analysis is based requires transfer policy to be described over an extensive time period. The transfer policy described above for 2006 is assumed to vary through time in two ways. First, the evolution of benefit values and income thresholds are assumed to describe constant growth rates. After experimenting with various alternatives, we settled upon the assumption that most of the associated growth rates equal 1.2% per annum, reflecting trend real earnings growth. The key motivation for this assumption is that it ensures that the transfer system maintains pace with wages, omitting marginalisation of welfare provisions or extensive tax bracket creep. Sensitivity analysis indicated that our results are not qualitatively sensitive to the reasonable alternative of setting growth rates for benefits values and tax thresholds to reflect historical trends.

The only departure to the growth rates referred to above is the inter-temporal treatment of the rates and thresholds assumed for the Pension Credit, which is applied from state pension age (as discussed above). As mentioned above, the Pension Credit is comprised of two elements; the Guarantee Credit, which is withdrawn at a rate of 100% in respect of private income, and the Savings Credit, which is withdrawn at a rate of 40%. We assume that the Guarantee Credit grows at 1.7% per annum and that the Savings Credit is held fixed in real terms. These assumptions are designed to reflect proposals for reform put forward by the Pensions Commission in 2005, and associated policy reforms set out in the May 2006 Pensions White Paper.

The second aspect of the policy environment that is subject to change through time concerns state pension age. The model reflects prevailing plans to increase the minimum age of eligibility to the

Guarantee Credit from 60 in 2010 to 65 in 2020 (in line with state pension age of women), and to increase the state pension age from 65 in 2020 to 68 in 2046 (e.g. see reforms reported in the May 2006 Pensions White Paper).

#### A.4 Private pensions

Private pensions in the model depend upon six parameters: the rate of return to pension wealth  $r^P$ , the minimum earnings threshold for pension contributions  $g_l^P$ , the rate of private contributions to pensions out of employment income  $\pi^P$ , the rate of employer pension contributions  $\pi_{ee}^P$ , the return assumed for calculating the price of pension annuities, and the fixed capital charge associated with purchasing a pension annuity.

There is a great deal of diversity in private pension arrangements in the UK, and in the details of occupational pensions in particular. Panel A of Figure 1 reveals that – although not universal – a sizeable majority of employees were offered some form of contribution in respect of participation in an employer sponsored pension. Eligibility to an employer sponsored pension is reported to increase from a low of between 30 and 40 per cent among individuals on less than half of median earnings (increasing by age group), to between 75 and 85 per cent among individuals on more than one and a half times median earnings. The figure also indicates that eligibility to an employer pension contribution exhibited a stronger relationship with employee earnings than it did with age. Following these observations, we set  $g_l^P$  equal to 75% of median earnings.

Panel B of Figure 1 indicates that, for employees who received an employer pension contribution, the distribution of employer pension contributions was dominated by a single mode between 12.5 and 15 per cent of employee wages. Bearing in mind that the decision by an employee not to participate in their employer’s sponsored pension plan would usually result in the forfeiture of any matching employer pension contributions on offer, the scale of the employer contributions reported in Panel B provides an indication of how important these contributions were in supporting the UK system of private sector pension provisions. Panel B of Figure 1 also reveals that there was very little difference between the distributions of employer pension contributions offered in low-pay industries and the wider labour market, with the principal disparity being that employer contributions in excess of the mode were less frequent among employees in low pay industries. We consequently set the rate of employer contributions to 14%; the rate of private contributions to pension wealth was set to the ‘normal’ contribution rate stated in the guidance to interviewers for the FRS, equal to 8%.

We set the return assumed to pension wealth during the accrual phase,  $r^P$ , to 2.5% per annum, which is between the long-run real return to government debt (1.5%) and the return to equities (4.7%) observed between 1970 and 2010. The capital return assumed for calculating the price of pension

annuities was set equal to 1.5%, reflecting the average rate of return to long-term government debt observed between 1970 and 2010, and the associated capital charge was set to 4.7% based on “typical” pricing margins reported in the pension buy-outs market (see Lane et al. (2008), p. 22).

## A.5 Demographics

Three demographic characteristics were parameterised exogenously from the model structure: life expectancy; relationship status; and numbers of dependent children.

### Life expectancy

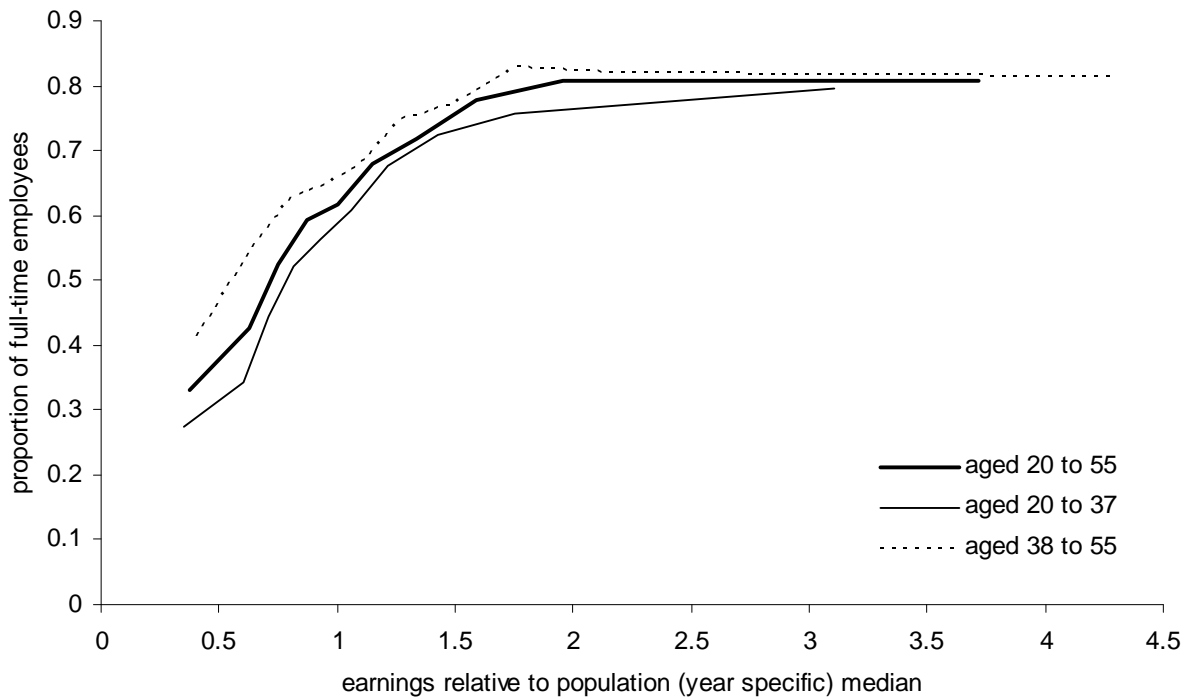
The model requires age and birth year specific survival rates to simulate the risk of mortality of reference adults. At the time of writing, the Office for National Statistics (ONS) reports period mortality rates for the UK that are distinguished by sex and age, at annual intervals between 1951 and 2060 inclusive, and between ages 0 and 100. The rates to 2010 are based on observed survival rates, and are projections thereafter. Three series of projections are reported by the ONS; a principal projection, a high life expectancy variant, and a low life expectancy variant. We focus on the principal projections here.

We assume a maximum potential age of life of 130 years for the analysis.<sup>19</sup> The age specific mortality rates reported by the ONS were extended beyond age 100, using a smooth sigmoidal function to equal 1.0 (certain death) at age 130. Furthermore, the time series dimension of the age specific mortality rates reported by the ONS was extended to all age and year combinations feasible for any modelled birth cohort by assuming a constant exponential growth factor of 0.975 from the most approximate year described by the ONS data to exogenously assumed age and sex specific asymptotes for the distant past and future.

The model specification does not distinguish reference adults by their gender. The gender specific mortality rates that are reported by the ONS were consequently combined into a single series based on implied gender weights. Consider, for example, the cohort born in 1960. Assuming zero migration and equal numbers of males and females at age 16, the gender specific mortality rates reported for this birth cohort by the ONS can be used to project the ratio of men to women through time. This ratio was used to obtain a weighted average of the gender specific mortality rates reported by the ONS for each modelled birth cohort. To avoid imposing unwarranted structure on the parameters, the mortality rates were stored in the form of a transition matrix, comprised of 111 rows (representing ages 20 to 130), and 112 columns (representing years 1951 to 2060, with two additional rows to represent the distant past and future). The transition probabilities used can be obtained from the authors upon request.

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<sup>19</sup>The oldest age to which a human is documented to have survived is 122 years and 164 days, reported for Jeanne Calment of France, who died in 1997.



Panel A: eligibility to any employer pension contribution



Panel B: employer contribution where some contribution was made

Source: Panel A: author's calculations on individual level data from the 2006/07 and 2007/08 waves of the FRS

Panel B: author's calculations using data from waves 2005 to 2009 of the Annual Survey of Hours and Earnings

Notes: Earnings deciles defined within survey waves, and averaged across waves

Excludes employees in the public sector and the self-employed

low pay industries as defined by the Low Pay Commission (2010), Appendix 4

Figure 1: Eligibility rates of full-time employees to employer sponsored pensions by age and earnings

## **Relationship status**

The model requires rates of marriage formation and dissolution by age, year, and education status. At the time of writing, the ONS reports historical data for the number of marriages in England and Wales by age, sex and calendar year at annual intervals between 1851 and 2009. The ONS also makes available for modelling purposes the component factors that underlie its population projections, which describe official estimates for the number of marriages by age and sex at annual intervals between 2008 and 2033. Furthermore, ONS population estimates by age, sex and marital status are available for England and Wales at annual intervals between 1971 and 2033. These statistics permit age and gender specific marital rates to be calculated for England and Wales at annual intervals between 1971 and 2033 inclusive.

Marriage dissolution in the model accounts for both divorce and death of a spouse. The ONS reports age and sex specific divorce rates for England and Wales at annual intervals between 1950 and 2010, which can be extended to 2032 by the component factors of the ONS population projections that are referred to above. Combined with the mortality statistics that are referred to in the preceding subsection, these two series of data provide sufficient information to compile age, sex and year specific marital dissolution rates between 1951 and 2032.

The rates of marriage formation and dissolution that are described above are imperfect for modelling purposes in (at least) three important respects. First, the marriage rates calculated on historical data do not account for marriages that are performed abroad. Secondly, it is well recognised that mortality rates are correlated with marital status, and the required detail to take this into account is not provided by the information that is referred to above. And thirdly, the majority of the statistics that are reported by the ONS focus on legal marital status for the entire population, and do not extend to include civil partnerships or cohabitation, nor do they permit a distinction by education status.

The first and second problems identified above were addressed by adjusting marriage rates to age 44, and marital dissolution rates from age 45, to align age, sex, and year specific proportions of the population identified as married in the model to population estimates reported by the ONS (which are based primarily upon Census data). The focus of ONS statistics on legal marriage is problematic for modelling purposes due to the rise of civil partnerships and cohabitation, and the fact that couples who share the same address often engage in some pooling of consumption and income. This pooling of financial resources is recognised by the system of social security in the UK, which treats cohabitating couples in the same way as registered married couples when determining eligibility for most benefits (excluding state pensions and bereavement allowances). We consequently applied a final set of adjustments to account for this issue.

The Family Expenditure Survey (FES) provides detailed micro-data that can be used to determine age, sex, and education specific proportions of the population married between 1978 and 1989, and married or cohabitating between 1990 and 2009. Starting from the rates of marriage and marital dissolution calculated for registered marriages (described above), it is possible to compute implied age, sex, education, and year specific proportions of the population married on the assumption of zero migration. These proportions of the population married by age, year, education, and sex were compared against the associated proportions calculated for FES data (allowing for cohabitation). Marriage rates were then adjusted to age 44, and marital dissolution rates were adjusted from age 45, to match the implied proportions of the population in a (cohabitating) relationship to the proportions calculated on FES data. As the associated adjustments were exactly identified (involving the same number of model parameters as fitted moments), a precise fit to the sample moments was obtained.

The gender specific marital and marriage dissolution rates derived via the above procedure were aggregated into a gender neutral series in the same way as described above for mortality rates. Similarly, like mortality rates, the probabilities upon which change in relationship status depend were stored in four transition matrices, one for each of graduates and non-graduates, and one for each of marriage and marital dissolution. The transition matrices for marriage are comprised of 69 rows (representing ages 16 to 84) and 35 columns (representing years 1977 to 2009 with two additional columns to represent the distant future and distant past). The transition matrices for marital dissolution are comprised of 86 rows (representing ages 16 to 101; all adults are assumed to be single from age 101) and 35 columns.

### **Number of dependent children**

The numbers of dependent children are modelled as a deterministic function of age, year, and relationship status. This function is stored in the form of a matrix over these three dimensions, with dimensions 59 (representing ages 20 to 78) by 41 (representing years 1971 to 2009 with two additional elements to represent the distant past and future) by 2 (representing singles and couples). The elements of this matrix were set equal to averages reported in the FES.

## **B Regression Models Used to Input Missing Earnings Data**

Four regression equations were estimated on the cross-sectional WAS data, which were used to impute earnings for individuals who were not reported to be working full-time in the sample; separate equations for men and women, and separate equations for those aged under 50 from those aged 50 years or over. The specifications adopted for this analysis were constrained only by the information reported by the WAS, which includes a high degree of financial detail. After experimenting with various alternatives, regression results for the assumed earnings equations are reported in Table 3.

Table 3: Regression Estimates for log Earnings, Controlling For Sample Selection

	women aged 18-49		women aged 50+		men aged 18-49		men aged 50+	
	est	std error	est	std error	est	std error	est	std error
part-time work	-0.9515	0.0330	-0.9631	0.0472	-1.1867	0.0693	-1.4185	0.1095
no qualifications recorded	-0.1523	0.0406	-0.1014	0.0575	-0.1340	0.0294	-0.0702	0.0557
graduate qualifications	0.3199	0.0288	0.2950	0.0666	0.1804	0.0252	0.1944	0.0631
student	-0.0973	0.0609	0.1083	0.1096	-0.1887	0.1112		
student under age 23	-0.1699	0.1252			-0.0292	0.1525		
self-reported health vgood	0.0490	0.0315	-0.1217	0.0776				
self-reported health good	0.0574	0.0311	-0.1477	0.0712				
self-reported health fair			-0.2029	0.0858				
self-reported health bad			-0.6148	0.3467				
self-reported health vbad			-0.2515	0.2358				
SEC 1 of 3	0.2316	0.0264	0.2922	0.0537	0.1987	0.0247	0.1890	0.0456
SEC 2 of 3	0.1174	0.0336	0.1810	0.0609	0.0603	0.0268	0.0134	0.0675
self-reported saver	0.0504	0.0250	0.1275	0.0481	0.0823	0.0197		
partner not working	0.0083	0.0212			-0.0511	0.0171		
wealth under £10000							-0.1990	0.0739
own accommodation (£)	0.1168	0.0271			0.0910	0.0211		
total net wealth (£)	4.76E-07	8.51E-08	1.14E-07	7.11E-08	8.25E-07	8.72E-08	2.92E-07	8.59E-08
total private pension wealth (£)							1.08E-07	4.47E-08
outstanding mortgage (£)			1.47E-06	3.46E-07			1.72E-06	3.14E-07
constant	9.4671	0.0553	8.7632	0.1775	9.8266	0.0397	9.7207	0.2211
rho	0.0582	0.0303	-0.0477	0.0334	0.0110	0.0383	-0.0960	0.0407
sigma	0.6145	0.0288	0.8033	0.0434	0.5193	0.0216	0.7282	0.0435
lambda	0.0358	0.0189	-0.0383	0.0274	0.0057	0.0200	-0.0699	0.0298
Sample	5916		6084		5389		5300	
Censored observations	1976		4468		1324		3561	
mean dependent variable	9.4115		9.2189		9.8577		9.7363	
std dev of dependent variable	0.8939		1.0902		0.7785		1.0756	

Regression estimates for target equations controlling for sample selection

Regression estimates calculated on Wealth and Assets Survey data, using the "heckman" command in Stata

Table omits age specific dummy variables; SEC = Socio Economic Class

Robust standard errors reported

All statistics are dummy variables, except for the financials indicated by the (£) symbol

The parameter values reported in Table 3 indicate that earnings are positively correlated with education, lower for students, and tend to vary positively with health and socio-economic status. Self-reported savers tend to earn more than non-savers, and earnings are positively related to aggregate wealth, home ownership, and mortgage value. Part-time employment tends to imply an earnings penalty of 60% for women, and 70% for men; these compare with the model assumption that part-time work returns 40% of an individual's full-time wage (described in Appendix A.1).

The estimates obtained for rho – the correlation between the residuals of the target and selection equations – are interesting in their own right. These coefficients suggest that censoring tends to be more likely for low income individuals early in life, and more likely for high income individuals later in life, where the effects are not insignificant at the 90% confidence interval for women under age 50 or for men over age 49. Comparing the estimates obtained for sigma with the standard deviations of the associated dependent variables indicates that the regression models selected for analysis help to explain around 30% of the observed variation between individuals.

## C Endogenously Identified Wage Parameters

As noted in Section 3.2, two sets of wage parameters were calibrated by matching moments of employment income implied by the structural model against associated moments estimated on time-series data: the drift parameters  $m(\cdot)$ , and earnings volatility,  $\sigma_{\omega}^2(\cdot)$ . We calibrated the drift parameters to match the model to age, year, and relationship specific geometric means of employment income estimated on data reported by the Family Expenditure Survey between 1978 and 2010. We calibrated the associated earnings parameters to moments calculated for singles to age 60 and for couples to age 64, to omit issues associated with small sample effects at higher ages. The large number of parameters involved make reporting here impractical, and associated statistics can therefore be obtained from the authors upon request.