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# OVERVIEW OF THE NIGEM-S MODEL: SCOTTISH VERSION OF THE NATIONAL INSTITUTE GLOBAL ECONOMETRIC MODEL

*The NiGEM-S model is based on the National Institute Global Econometric Model, NiGEM, a large-scale structural macro-econometric model of the world economy, which the National Institute has been developing since 1987. NiGEM is used for forecasting and policy analysis by NIESR and model subscribers, mainly in the policy community, including the ECB, the IMF, the OECD, the FSA, the Bank of England, and the central banks of France, Germany, Italy, Netherlands, Spain, Portugal and the Czech Republic.*

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# **Overview of the NiGEM-S Model: Scottish version of the National Institute Global Econometric Model**

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<sup>1</sup> For details check <http://nimodel.niesr.ac.uk/>

## 1. NiGEM OVERVIEW

NiGEM is a large-scale structural macro-econometric model of the world economy. It is essentially New-Keynesian in its approach, in that agents are presumed to be forward-looking, at least in some markets, but nominal rigidities slow the process of adjustment to external events. It has complete demand and supply sides, and there is an extensive monetary and financial sector. Rational or model-consistent expectations are used throughout the model, although users may also opt to use adaptive expectations for consumers, firms, wage setters and financial markets. As far as possible the same theoretical structure has been adopted for each of the major industrial countries, except where clear institutional or other factors prevent this. As a result, variations in the properties of each country model reflect genuine differences in data and estimated parameters, rather than different theoretical approaches.

Linkages between countries take place through trade, interacting financial markets and international stocks of assets. Hence a change in, say, US equity prices affects Europe through its effects on US imports, interest rates and exchange rates, and also on the wealth of the European personal sector. It is a ‘closed’ global model, in that all income, asset and trade inflows into one country are matched by outflows from another country, i.e. aggregation at the “world” level is internally consistent (world net assets and trade balance are approximately zero).

NiGEM is a global model. Most countries in the OECD are modelled separately, and there are also separate models of China, India, Russia, Brazil, Hong Kong, Taiwan, Singapore, South Africa, Estonia, Latvia, Lithuania, Slovenia, Romania and Bulgaria. The rest of the world grouped into 6 regional aggregates that cover the remaining countries in Sub-Saharan Africa, Europe, the Commonwealth of Independent States, the Middle East and North Africa, Asia and the Americas. All country and regional models contain the determinants of domestic demand, export and import volumes, prices, current accounts and net external assets. Economies are linked through trade, competitiveness and financial markets and are fully simultaneous.

For a macro-econometric model to be useful for policy analyses, particular attention must be paid to its long-term equilibrium properties. At the same time, we need to ensure that short-term dynamic properties and underlying estimated properties are

consistent with data and well-determined. Output is tied down in the long run by factor inputs and technical progress interacting through production functions, but is driven by demand in the short- to medium-term.

Policy rules for monetary and fiscal policies are essential for the operation of a coherent model of the economy. Interest rates can be set according to a number of targeting rules as discussed in Barrell et al. (2006). The default rule follows a ‘two-pillar’ strategy, targeting a combination of inflation and a nominal aggregate.

Government debt stocks exist for each of the major economies, and deficits flow onto them. It has been shown in Barrell and Sefton (1997) that the existence of an equilibrium in a forward-looking model requires that debt stocks do not explode. This requires a fiscal feedback rule ensuring that the deficit achieves an equilibrium trajectory. NiGEM fiscal rule ensures that the actual deficit slowly approaches the desired deficit using the direct tax rate as an instrument. A direct target on the debt to GDP ratio can also be imposed.

NiGEM is used to produce economic *forecasts* for the world economy. At the National Institute we prepare quarterly forecasts that are published in the *National Institute Economic Review*, along with a discussion of alternative scenarios around the central forecast and short notes based on recent model-based research. This work is also made available for model users on NIESR’s web-based product, NiGEMWEB (<http://nimodel.niesr.ac.uk>). Model users can produce their own forecasts over an extended forecast horizon. The central forecast baseline currently extends to 2039, but can be extended to 2059 in order to consider longer-term issues.

NiGEM is also extensively used as a *simulation* tool. Typical simulations involve analysing the effects of changes in monetary or fiscal policy or changes in commodity prices such as an oil price shock.

NiGEM has also been used for more complex *policy analysis* exercises, such as modelling the sovereign debt crisis in Europe (Holland et al., 2011b), the macro-economic impact of migration following the EU enlargements of 2004 and 2007 (Holland et al., 2011a), and a cost-benefit analysis of banking regulation (Barrell et al., 2010).

The *stochastic* mode of NiGEM is used to construct error bounds around the central forecast baseline. The fan charts are based on the historical errors on all the key model equations.

There are two types of models within the NiGEM framework: *full* model and *reduced* model. There are full models for most OECD countries. The following countries have reduced model structure: Bulgaria, Brazil, China, Estonia, Hong Kong, India, Lithuania, Latvia, Mexico, Norway, New Zealand, Romania, Russia, South Africa, Korea, Slovenia, Slovakia, Switzerland, Taiwan, and regional aggregates.

Further technical details of NiGEM model are presented in Appendices A-C.

## **2. NiGEM-S OVERVIEW**

NiGEM-S is an extension of NiGEM that incorporates changes allowing scenarios associated with potential Scottish constitutional change to be executed. It has two additional countries/regions and one extra-regio sector – Scotland, the rest of the UK (RUK) and North Sea oil and gas sector (NS) – which are used in conjunction with the current UK model.

NiGEM-S is a simulation-only model which means that while it cannot be used to produce forecast for Scotland and RUK, it can be used to look at scenarios covering fiscal, monetary shocks and oil price change. Scotland and RUK models structurally very similar to other reduced models in NiGEM (see Appendix B) and are based on the same principles and theoretical approaches. The North Sea model is specific to NiGEM-S. The simulation period is from Q1 2014 until Q4 2028.

There are three constitutional arrangements for Scotland and RUK that are available in NiGEM-S:

- UK intact – status quo scenario when Scotland remains part of the UK. Scotland and RUK have identical fiscal and monetary policy and similar reaction to main shocks;
- Independence with informal currency union – Scotland becomes independent but forms a currency union with RUK. Scotland and RUK have autonomous fiscal

policy but Scotland does not have an independent monetary policy and follows RUK monetary instruments;

- Independence – Scotland becomes independent and introduces its own currency. Fiscal and monetary policies are independent in Scotland and RUK.

### *Scotland model*

The key model equations in NiGEM-S are estimated based on the publically available data for Scotland (mainly SNAP, GERS and Global connections survey). Some equations are based on the full UK model. The third group of equations are calibrated to produce theoretically predictable response in standard model simulations. The lack of some key data means simplifying assumptions have been necessary.

### *GDP*

As in other reduced models in the short and medium term, GDP is driven by the demand side and is an identity of domestic demand, exports and imports.

In the long run GDP is driven by the supply side.

### *Domestic demand*

When Scotland is part of the UK, its domestic demand behaves in line with that of the UK.

If Scotland is independent (with or without currency union), it has its own domestic demand equation, estimated on historical data

### *External sector*

Scottish exports and imports equations are based on the full UK model equations.

### *Price block*

When Scotland is part of the UK, it has the same prices as in the UK.

If Scotland is independent (with or without currency union), export and import deflators as well as export and import commodity prices are calibrated on the basis of the corresponding equations for the UK and Norway. Norway was chosen because it is a large European oil exporting economy. Scotland after independence also will be an oil exporting economy, while the UK as a whole is a net importer of oil. The price system reflects this by using a weighted average of UK and Norway weights in the corresponding price equations. Norway weights are scaled to reflect the relative importance of the oil sector in Scotland and in Norway. Export and import non-commodity prices are based on the full UK model equations.

### *Financial markets*

When Scotland is part of the UK or is independent but maintains a currency union with the UK, it has the same financial and monetary variables (interest and exchange rates) as in the UK. Note that this does not allow for endogenous risk premium, which might be important, as became evident in the Euro area.

If Scotland is independent, it has the same choice of short-term interest rate rules as any other country (see Appendix B). Our default setting is the two pillar strategy, targeting a combination of inflation and a nominal aggregate. Long interest rates and exchange rates are modelled based on the short-term interest rates in the same way as in every other country.

### *Public sector*

The Scottish public sector is modelled in more detail than in other reduced models and is modelled differently depending on the constitutional arrangement chosen.

When Scotland is part of the UK, we assume that it will continue to receive a block grant based on the Barnett formula (*Barn*). Future projections of this grant grow in line with UK government consumption (*UKGC*), corrected for the Scottish population share (*popshare*):

$$Barn_t = Barn_{t-1} + \Delta UKGC_t * popshare \quad (1)$$

Additional tax revenues are calculated as a share of nominal GDP and the budget deficit is equal to the Scottish population share of the UK budget deficit. Thus, total Scottish

public sector expenditures are equal to the sum of tax revenues, Barnett transfer and budget deficit.

If Scotland is independent (with or without currency union), it has an independent fiscal policy, collects all of its own taxes and adjusts tax rate according to the budget rule. Total on-shore tax revenues ( $TTAX$ ) are calculated by applying aggregate tax rate ( $TAXR$ ) to the nominal GDP. Public spending ( $TEXP$ ) grows in line with the long-term GDP growth. Budget deficit ( $BUD$ ) is equal to the difference between total revenues (Scottish share of NS tax revenues ( $CTO$ ) and on-shore tax revenues) and public sector spending:

$$BUD_t = CTO_t + TTAX_t - TEXP_t \quad (2)$$

The budget rule ensures that the government stays solvent in the long run, i.e. that the deficit and debt stock return to sustainable levels in all scenarios. The budget rule adjusts the aggregate tax rate when the public debt ratio ( $GBR$ ) deviates from its target ( $GBRT$ ). By default the target is the baseline value of the budget deficit, i.e. the level forecast before a shock. However, it is possible to set alternative budget deficit targets.

$$TAXR_t = f(GBR_t - GBRT_t) \quad (3)$$

### ***RUK model***

Most of the equations in the RUK model are adopted from the full UK model. The only differences are due to the reduced size of the model.

### ***NS model***

North Sea extra-regio sector is specific to NiGEM-S. Since in Scotland off-shore activity constitutes a significantly larger part of the economic output compared with the UK as a whole, the decision was taken to model it separately, unlike in the UK where no such differentiation exists. This also allows for the tracking of oil price shocks in more detail.



The NS sector contains a supply side which is determined by exogenous assumptions on the price of oil and NS oil production (level of production does not depend on prices of oil). It is assumed that all NS output is exported. NS output and tax revenues (calculated based on the tax rate calibrated from the historical data) are split between Scotland and RUK according to an exogenous parameter.

### *Available simulations*

In all three scenarios NiGEM-S allows to run standard shocks:

- monetary expansion/contraction
- fiscal expansion/contraction
- change in the price of oil
- borrowing cost premium (only for independent Scotland)

**APPENDIX A. NiGEM FURTHER TECHNICAL DETAILS. FORWARD-LOOKING BEHAVIOUR, TERMINAL CONDITIONS, BLACK HOLES AND GLOBAL BALANCE.**

World models are often constructed with an eye to the analysis of world trade, and although this was true for NiGEM, it is now much more commonly used to analyze issues such as the evolution of national income, the effect of government deficits and the role of monetary policy. The Lucas critique – based on Lucas (1976) – cast doubt on the performance on early macroeconomic models, as they rested on the assumption that the estimated parameters based on historical behavioural patterns were invariant to the policy environment. Rational expectations or forward-looking models evolved as a reaction to this criticism, and NiGEM incorporates many of the key features of these types of models.

- The model allows for forward-looking wage bargainers, financial markets (exchange rates, long term interest rates, equity prices and house prices), consumers and targets in policy rules. Each forward-looking variable needs an extrapolation or terminal condition at the terminal date in order that a rational solution exists. Forward-looking solutions will not normally solve if the model has not achieved some sort of equilibrium toward the end of a simulation run, but this condition is not imposed. The only terminal condition involves the rate of growth of the relevant variables.
- The model is theoretically coherent, and this requires that there are no financial ‘black holes’ to absorb imbalances. The world current account must be balanced in order that assets and liabilities match, and the model should be approaching an asset equilibrium by the terminal date. This in turn requires that the stock of government debt does not explode.
- The world current account is brought into approximate balance by the model. Global trade is driven by import demand, and the volume of world exports approximately equals the volume of world imports, as we impose the unit elasticity on demand in the export equations. Final exports are adjusted to distribute any discrepancy in the change in the export-to-import ratio proportionately across countries based on their share of world trade. World flows of property income balance because all assets are matched by liabilities, revaluations of liabilities match those of assets and income flows match payments.

## APPENDIX B. REDUCED MODEL DESCRIPTION

### *GDP*

In the short to medium term, GDP is driven by the demand side:

$$Y = DD + XVOL - MVOL \quad (a1)$$

In the longer term, GDP is driven by the supply side:

$$\Delta \ln(YCAP) = \alpha - \lambda [\ln(YCAP)_{-1} - techl + \ln(POPT)] - \beta \ln(USER) \quad (a2)$$

where *USER* (long real interest rate plus a risk premium) acts as a proxy for capital stock, *techl* is trend productivity growth, *POPT* is population and acts as a proxy for labour input.

### *Domestic demand*

Domestic demand depends on the user cost of capital (*USER*), export income (*XREV*) (especially important for commodity exporters), wealth (proxied by net foreign asset ratio) (*NAR*), population developments (*POPT*) and government spending (*G*).

$$\Delta \ln(DD)_t = \alpha_2 - \lambda \left[ \begin{array}{l} \ln(DD)_{-1} + \beta_1 \ln(USER)_{-1} - \delta_1 XREV \\ - \delta_2 NAR \end{array} \right] + \delta_3 \Delta \ln(POPT) + \delta_4 \Delta \ln(G) \quad (a3)$$

### *External trade*

Trade volumes and prices are linked by Armington matrices, based on 2007 trade patterns. External market size (*S*) is a weighted average of imports in other countries. Relative export prices (*RPX*) are domestic export prices relative to weighted average of competitor prices. Relative import prices (*RPM*) are domestic consumer prices relative to weighted average of export prices in import source countries. Total final expenditures (*TFE*) are sum of domestic and export demand.

$$\Delta \ln(XVOL) = \alpha_1 - \lambda[\ln(XVOL)_{-1} - \beta_1 S + \beta_2 RPX] + \delta_1 \Delta \ln(S) - \delta_2 \Delta \ln(RPX) \quad (a4)$$

$$\Delta \ln(MVOL) = \alpha_2 - \lambda_2[\ln(MVOL)_{-1} - \beta_3 \ln(TFE)_{-1}] + \delta_3 \Delta \ln(TFE) + \delta_4 \ln(RPM) \quad (a5)$$

### Capacity utilisation

Capital utilisation is the ratio of output to potential output. It feeds into the price system to bring demand in line with supply.

$$CU = \frac{Y}{YCAP} \quad (a6)$$

### Prices

Consumer prices ( $CED$ ) determined by import prices ( $PM$ ), capacity utilisation ( $CU$ ) and inflation expectations ( $CED^e$ ).

$$\begin{aligned} \Delta \ln CED_t = & \alpha_1 - \lambda[\ln CED_{t-1} - \ln PM_{t-1} - \delta_1 CU_{-1}] \\ & + \beta_1 \Delta \ln PM_t + \beta_2 \Delta \ln CED_{t-1} + (1 - \beta_1 - \beta_2) \Delta \ln CED^e_{t+1} \\ & + \beta_3 \Delta CU \end{aligned} \quad (a7)$$

Export ( $PX^*$ ) and import ( $PM^*$ ) prices are modelled as a weighted average of commodity (??COM) and non-commodity prices (??NCOM):

$$PX = \alpha_1 PXCOM + (1 - \alpha_1) PXNCOM \quad (a8)$$

$$PM = \alpha_2 PMCOM + (1 - \alpha_2) PMNCOM \quad (a9)$$

Commodity import and export prices are a weighted average of 5 commodities (oil ( $WDPO$ ), food ( $WDPFDV$ ), beverages ( $WDPFLD$ ), agricultural non-food ( $WDPANF$ ), metal and minerals ( $WDPMM$ )). Import prices are multiplied by exchange rate ( $rx$ ).

$$PMCOM = \left\{ \begin{aligned} & \delta_1 WDPO + \delta_2 WDPFDV + \delta_3 WDPFLD + \delta_4 WDPANF + \\ & (1 - \delta_1 - \delta_2 - \delta_3 - \delta_4) WDPMM \end{aligned} \right\} rx \quad (a10)$$

$$PXC\text{OM} = \beta_1 W\text{DPO} + \beta_2 W\text{DPFDV} + \beta_3 W\text{DPFLD} + \beta_4 W\text{DPANF} + (1 - \beta_1 - \beta_2 - \beta_3 - \beta_4) W\text{DPMM} \quad (\text{a11})$$

Non-commodity import prices modelled as weighted average of non-commodity export prices from all other countries/regions on the model.

### *Monetary framework and financial markets*

Even reduced models have a choice of interest rate ( $r$ ) rules (with exception of regional aggregates). Short-term interest rates set by a central bank. Feedback rules depend on inflation ( $INFL$ ), output gap ( $Y/YCAP$ ), price level ( $PL$ ), nominal aggregate ( $NOM$ ).  $T$  at the end of the variable name stands for “target”.

- Two Pillar Strategy

$$- \quad r = c*(INFL-INFLT) + d*(NOM-NOMT) \quad (\text{a12})$$

- Taylor Rule

$$- \quad r = a*(INFL-INFLT) + b*Y/YCAP \quad (\text{a13})$$

- Price Level Targets

$$- \quad r = e*(INFL-INFLT) + f*(PL-PLT) \quad (\text{a14})$$

Forward looking long rates are modelled as consistent with the expected path for short term rates (assumes zero term premium).  $T=40$  (10 years):

$$(1 + LR_t) = \left[ \prod_{j=1}^T (1 + r_{t+j}) \right]^{\frac{1}{T}} \quad (\text{a15})$$

Simulations are generally run with forward looking exchange rates for consistency. Backward-looking or fixed exchange rates are available as options. Forecast baseline produced with fixed interest rates and backward looking exchange rates. Exchange rates evolve according to uncovered interest rate parity (UIP) condition adjusted by risk premium. If interest rates are above those in the US ( $rus$ ), exchange rate is expected to depreciate (unless there is a positive risk premium ( $rp$ )).

$$E\left(\frac{rx_{t+1}}{rx_t}\right) = \left(\frac{1 + r_t}{1 + rus_t}\right)(1 + rp_t) \quad (\text{a16})$$

## APPENDIX C. FULL MODEL DESCRIPTION

### *Production and price setting*

GDP ( $Y$ ) is driven by demand in the short-run, and is modelled as an identity relationship, summing the components of demand:  $C$  (consumption),  $PSI$  (private sector investment),  $GC$  (government consumption),  $GI$  (government investment),  $DS$  (stock building),  $XVOL$  (exports),  $MVOL$  (imports),  $RES$  (chainbasing residual)

$$Y = C + PSI + GC + GI + DS + XVOL - MVOL + RES$$

In the long-run, output is tied down by an underlying production function that describes the supply-side of the economy. The major country models rely on an underlying constant-returns-to-scale CES production function with labour-augmenting technical progress. This is embedded within a Cobb-Douglas relationship to allow the factors of production to interact with oil usage:

$$YCAP = \gamma \left[ s(K)^{-\rho} + (1-s)(Le^{\lambda t})^{-\rho} \right]^{-1/\rho} M^{1-\alpha} \quad (\text{a17})$$

where  $YCAP$  is capacity output,  $K$  is the total capital stock,  $L$  is total hours worked,  $t$  is an index of labour-augmenting technical progress and  $M$  is oil input. This constitutes the theoretical background for the specifications of the factor demand equations, forms the basis for unit total costs and provides a measure of capacity utilization, which then feeds into the price system. Barrell and Pain (1997) show that the elasticity of substitution is estimated from the labour demand equation, and in general it is around 0.5. Demand for labour and capital are determined by profit maximisation of firms, implying that the long-run labour-output ratio depends on real wage costs and technical progress, while the long-run capital output ratio depends on the real user cost of capital:

$$\ln(L) = c_1 + \ln(Q) - (1-\sigma)\lambda t - \sigma \ln(w/p) \quad (\text{a18})$$

$$\ln(K) = c_2 + \ln(Q) - \sigma \ln(c/p) \quad (\text{a19})$$

where  $c_1$  and  $c_2$  are constant terms related to the other parameters in the model,  $w/p$  is the real wage and  $c/p$  is the real user cost of capital. The user cost of capital is influenced by corporate taxes, depreciation and risk premia and is a weighted average of the cost of equity finance and the margin adjusted long real rate, with weights that vary

with the size of equity markets as compared to the private sector capital stock. Business investment is determined by the error correction based relationship between actual and equilibrium capital stocks. Government investment depends upon trend output and the real interest rate in the long run. Prices are determined as a constant mark-up over marginal costs in the long term.

### *Labour market*

NiGEM assumes that employers have a right to manage, and hence the bargain in the labour market is over the real wage. Real wages, therefore, depend on the level of trend labour productivity as well as the rate of unemployment. Labour markets embody rational expectations and wage bargainers use model consistent expectations. The dynamics of the wage market depend upon the error correction term in the equation and on the split between lagged inflation and forward inflation as well as on the impact of unemployment on the wage bargain (Anderton and Barrell, 1995). There is no explicit equation for sustainable employment in the model, but as the wage and price system is complete, the model delivers equilibrium levels of employment and unemployment. An estimate of the NAIRU can be obtained by substituting the mark-up adjusted unit total cost equation into the wage equation and solving for the unemployment rate. Labour supply is determined by demographics, migration and the participation rate.

Derived from the underlying production function, the profit maximizing condition from the labour side sets the real wage equal to the marginal product of labour. This forms the core long-run solution to 3 equations: labour demand (*EE*), nominal wages (*WAGE*), and unit or marginal costs (*UTC*).

$$\begin{aligned} \Delta \ln(ee * hours) = & \varepsilon_1 + \varepsilon_2 \left\{ \ln(ee_{-1} * hours_{-1} / ycap_{-1}) + 0.5 \ln \left( \frac{wage_{-1}}{py_{-1}} \right) + 0.5 * techl_{-1} \right\} \\ & + \sum \varepsilon_{3i} (\Delta \ln(y) - \varepsilon_{6i} \Delta techl)_{-i} + \sum \varepsilon_{4i} \Delta \ln(ee_{-i} * hours_{-i}) + \sum \varepsilon_{5i} \Delta \ln \left( \frac{wage}{py} \right)_{-i} \end{aligned} \quad (a20)$$

$$\begin{aligned} \Delta \ln(wage) = & \omega_1 + \omega_2 \left\{ \ln \left( \frac{wage_{-1}}{py_{-1}} \right) + techl_{-1} - 2 \ln(ycap_{-1} / (e_{-1} * hours_{-1})) \right\} \\ & + \omega_{3a} \Delta \ln(CED)_t + (1 - \omega_{3a}) \Delta \ln(CED^e)_t + \omega_4 U_{-1} \end{aligned} \quad (a21)$$

$$\begin{aligned} \Delta \ln(utc) = & \pi_1 + \pi_2 \{ \ln(utc)_{-1} - \ln(wage)_{-1} - 2 \ln(e_{-1} * hours_{-1} / ycap_{-1}) - techl_{-1} \} \\ & + \sum \pi_{3i} \Delta \ln(wage)_{t-i} + \pi_4 \ln(cu_{-1}) \end{aligned} \quad (a22)$$

Where *PY* is the GDP deflator and *CED* is the consumer expenditure deflator, *U* is the unemployment rate and *CU* is capacity utilisation (output gap).

A number of cross equation restrictions are imposed on the parameters in the three equations, to ensure that the model delivers a unique NAIRU. Allowing  $\varepsilon_{6i}$  to differ from 1 allows the growth rate of productivity to affect the NAIRU. This is consistent with finding by Ball and Mankiw (2002) and others.

### Prices

Consumer prices (*CED*) are a weighted average of import prices (*PM*) and unit costs (*UTC*), adjusted for changes in the VAT rate (*ITR*):

$$\begin{aligned} \Delta \ln(CED) = & \alpha - \lambda [ \ln(CED)_{-1} - \ln(1 + ITR)_{-1} - \beta \ln(UTC)_{-1} - (1 - \beta) \ln(PM)_{-1} ] \\ & + \Delta \ln(1 + ITR) + \sum \gamma_{1i} \Delta \ln(UTC)_{-i} + \sum \gamma_{2i} \Delta \ln(PM)_{-i} + \sum \gamma_{3i} \Delta \ln(CED)_{-i} \end{aligned} \quad (a23)$$

where all gammas sum to 1.

*GDP deflator (PY)* is a weighted average of consumer, export and import prices.

Aggregate import prices (*PM*) are an identity relationship that weights commodity and non-commodity prices:

$$PM = \alpha PMNCOM + (1 - \alpha) PMCOM \quad (a24)$$

Commodity import prices (*PMCOM*) are modelled as a weighted average of 5 commodity prices (oil, metals and minerals, food, beverages, agricultural non-foods)



Non-commodity import prices ( $PMNCOM$ ) are a weighted average of non-commodity export prices from the other countries and regions.

Aggregate export prices ( $PX$ ) are an identity relationship that weights commodity and non-commodity prices:

$$PX = \alpha PXNCOM + (1 - \alpha) PXCOCM \quad (a25)$$

Commodity export prices ( $PXCOCM$ ) are modelled as a weighted average of 5 commodity prices (oil, metals and minerals, food, beverages, agricultural non-foods)

Non-commodity export prices ( $PXNCOM$ ) error correct on a weighted average of domestic consumer prices – excluding the indirect tax rate ( $CED/(1+ITR)$ ) and competitor prices ( $CPX$ ). The impact on the current account from various shocks is sensitive to this equation.

$$\begin{aligned} \Delta \ln(PXNCOM) = & \alpha - \lambda \left[ \ln(PXNCOM)_{-1} - \beta \ln\left(\frac{CED}{1+ITR}\right)_{-1} - (1 - \beta) \ln(CPX)_{-1} \right] \\ & + \sum \gamma_{1i} \Delta \ln\left(\frac{CED}{1+ITR}\right)_{-i} + \sum \gamma_{2i} \Delta \ln(CPX)_{-i} + \sum \gamma_{3i} \Delta \ln(PXNCOM)_{-i} \end{aligned} \quad (a26)$$

where all gammas sum to 1.

### *Consumption, personal income and wealth*

Consumption decisions are presumed to depend on real disposable income and real wealth in the long run. Total wealth is composed of both financial wealth and tangible (housing) wealth where the latter data is available.

$$\ln(C) = \alpha + \beta \ln(RPDI) + (1 - \beta) \ln(RFN + RHW) \quad (a27)$$

where  $C$  is real consumption,  $RPDI$  is real personal disposable income,  $RFN$  is real net financial wealth and  $RTW$  is real tangible wealth. The dynamics of adjustment to the long run are largely data based, and differ between countries to take account of differences in the relative importance of types of wealth and of liquidity constraints.

### *Financial markets*

We generally assume that exchange rates are forward looking, and ‘jump’ when there is news. The size of the jump depends on the expected future path of interest rates and exchange rate risk premia, solving an uncovered interest parity condition, so that the expected change in the exchange rate is given by the difference in the interest earned on assets held in local and foreign currencies.

$$rx = rx_{+1} \left( \frac{1 + r^*}{1 + r} \right) (1 + rp) \quad (\text{a28})$$

where  $rx$  is the bilateral exchange rate (defined as domestic currency per unit of foreign currency),  $r$  is the short-term nominal interest rate at home set in line with a policy rule,  $r^*$  is the interest rate abroad and  $rp$  is the exchange rate risk premium.

Interest rates are determined by policy rules adopted by monetary authorities as discussed in Barrell et al. (2006). Nominal short-term interest rates are set in relation to a standard forward looking feedback rule. Our default rule follows a ‘two-pillar’ strategy, targeting a combination of inflation and a nominal aggregate. Forward looking long-term interest rates (LR) are a forward convolution of expected short-term interest rates:

$$(1 + LR) = \left[ \prod_{j=1}^T (1 + r_{+j}) \right]^{\frac{1}{T}} \quad (\text{a29})$$

We assume that equity markets are also forward looking, with equity prices determined by the discounted present value of expected profits, adjusted by an equity risk premium.

### *Public sector*

We model corporate (CTAX) and personal (TAX) direct taxes and indirect taxes (ITAX) on spending, along with government spending on investment and on current consumption, and separately identify transfers and government interest payments. Each

source of taxes has an equation applying a tax rate ( $TAXR$ ) to a tax base (profits, personal incomes or consumption). As a default we have government spending on investment ( $GI$ ) and consumption ( $GC$ ) rising in line with trend output in the long run, with delayed adjustment to changes in the trend. They are re-valued in line with the consumers' expenditure deflator ( $CED$ ). Government interest payments ( $GIP$ ) are driven by a perpetual inventory of accumulated debts. Transfers ( $TRAN$ ) to individual are composed of three elements, with those for the inactive of working age and the retired, depending upon observed replacement rates. Spending minus receipts give us the budget deficit ( $BUD$ ):

$$BUD = CED*(GC+GI)+TRAN+GIP-TAX-CTAX-MTAX \quad (a30)$$

We have to consider how the government deficit ( $BUD$ ) is financed. We allow either money ( $M$ ) or bond finance ( $DEBT$ ), so that the debt stock is related to historical deficits:

$$BUD = \Delta M + \Delta DEBT \quad (a31)$$

rearranging gives:

$$DEBT = DEBT_{t-1} - BUD - \Delta M \quad (a32)$$

In all policy analyses we use a budget rule to ensure that Governments remain solvent in the long run (Barrell and Sefton, 1997). This ensures that the deficit and debt stock return to sustainable levels after any shock. A debt stock target can also be implemented. The tax rate equation is of the form:

$$TAXR = f(\text{target deficit ratio} - \text{actual deficit ratio}) \quad (a33)$$

If the Government budget deficit is greater than the target, (e.g. -3 % of GDP and target is -1% of GDP) then the income tax rate is increased.

### *External trade*

International linkages come from patterns of trade, the influence of trade prices on domestic price, the impacts of exchange rates and patterns of asset holding and

associated income flows. The structure of the trade block ensures overall global consistency of trade volumes by imposing that the growth of import volumes is equal to the growth of export volumes at the global level. Trade volumes and prices are linked by Armington matrices, based on 2007 trade patterns. The volumes of exports and imports of goods and services are determined by foreign or domestic demand, respectively, and by competitiveness as measured by relative prices or relative costs. The export demand variable is constructed as a weighted sum of other countries' imports, which ensures approximate balance, and any discrepancy is allocated to exports in proportion to the country's share of world trade. Import prices depend on a weighted average of global export prices, and this ensures that the ratio of the value of exports to the value of imports remains at around its historical level. It is assumed that exporters compete against others who export to the same market as well as domestic producers via relative prices. Imports depend upon import prices relative to domestic prices and on domestic total final expenditure. The overall current balance depends upon the trade balance and net property income from abroad, which comprises flows of income onto gross foreign assets and outgoings on gross foreign liabilities. World flows of property income balance because all assets are matched by liabilities, revaluations of liabilities match those of assets and income flows match payments.

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