National Institute of Economic and Social Research

FINANCIAL DEVELOPMENT, CYCLES AND INCOME INEQUALITY IN A MODEL WITH GOOD AND BAD PROJECTS.

Spiros Bougheas
University of Nottingham

Pasquale Commendatore
University of Naples 'Federico II'

Laura Gardini
University of Urbino 'Carlo Bo'

Ingrid Kubin
Vienna University of Economics and Business

NIESR Discussion Paper No. 545
19 December 2022

About the National Institute of Economic and Social Research

The National Institute of Economic and Social Research is Britain's longest established independent research institute, founded in 1938. The vision of our founders was to carry out research to improve understanding of the economic and social forces that affect people's lives, and the ways in which policy can bring about change. Over eighty years later, this remains central to NIESR's ethos. We continue to apply our expertise in both quantitative and qualitative methods and our understanding of economic and social issues to current debates and to influence policy. The Institute is independent of all party-political interests.

National Institute of Economic and Social Research
2 Dean Trench St
London SW1P 3HE
T: +44 (0)20 7222 7665
E: enquiries@niesr.ac.uk
www.niesr.ac.uk
Registered charity no. 306083

This paper was first published in December 2022

Competing Interest Statement: The views expressed in this paper are based on research and are not attributed to the organizations to which the researchers are affiliated. There are no conflicts of interest. The usual disclaimer applies.

Financial Development, Cycles and Income Inequality in a Model with Good and Bad Projects

Spiros Bougheas[1], Pasquale Commendatore[2], Laura Gardini[3] and Ingrid Kubin[4]

Abstract

We introduce a banking sector and heterogeneous agents in the Matsuyama et al. (2016) dynamic over-lapping generations neoclassical model with good and bad projects. The model captures the benefits and costs of an advanced banking system which can facilitate economic development when allocates resources to productive activities but can also hamper progress when invests in projects that do not contribute to capital formation. When the economy achieves higher stages of development it becomes prone to cycles. We show how the disparity of incomes across agents depends on changes in both the prices of the factors of production and the reallocation of agents across occupations.

Classification: E32, E44, G21

Keywords: Banks; Financial Innovation; Economic Development, Business Cycles; Income Inequality

The authors would like to thank participants at the 10th (2018) and 12th (2022) Workshops on Dynamic Models in Economics and Finance for helpful comments. The usual caveat applies.

[1] University of Nottingham, email: spiros.bougheas@nottingham.ac.uk
[2] University of Naples 'Federico II', email: pasquale.commendatore@unina.it
[3] University of Urbino "Carlo Bo", email: laura.gardini@uniurb.it
[4] Vienna University of Economics and Business, email: ingrid.kubin@wu.ac.at

1. Introduction

In his classic paper Diaz-Alejandro (1985) argued that financial liberalization can bring to an end periods of financial repression but also can carry the seeds for financial turmoil. The relaxation of regulatory controls by decreasing the costs of financial transactions allows capital to move more freely in the economy, encouraging economic development. The liberalization of financial markets offers incentives for financial innovation which can confer new benefits to consumers and firms by the creation of new instruments but sometimes can also have disastrous consequences as witnessed by the 2008 Global Financial Crisis.

In this paper, we capture this trade-off in a dynamic over-lapping generations neoclassical model with financial frictions and banks. The set-up is very similar to the model with good and bad projects analyzed by Matsuyama *et al.* (2016). The main difference is that in their model it is firms that have the choice between the two types of projects but in ours it is the banks. We believe that this interpretation is important for understanding what happened during the recent financial crisis. Our modeling choices are strongly motivated by banking activities that have taken place prior and during the crisis and now have been documented by numerous sources.

For example, the securitization of various banking assets, is an activity that has been taking place for a long-period prior to the crisis aimed to satisfy the growing demand for investment in safe assets, mainly by institutional investors. Allocating risk from those agents that are willing to bear it to those that prefer to avoid it is one of the main functions of an efficient financial system. However, now it is well understood that many of the new assets that banks created to achieve this risk reallocation were mispriced. Banks have underestimated the risk of the underlying assets on which the performance of the newly created assets depended upon (Mizen, 2008). Many complementary explanations have been offered for the mispricing of assets. For example, Gennaioli and Shleifer (2018) emphasize behavioral biases that have affected the formation of beliefs by investors while Richardson and White (2009) focus on the failure of credit rating agencies to judge the quality of new assets. Our model is more closely motivated by a third explanation that is founded on the practices of 'shadow banking', in general, and, in particular, the establishment of 'Special Investment Vehicles' (SIVs).[1]

Prior to the crisis, the liabilities of the banking sector were dramatically increasing and as banks attempted to maintain high yields on their investments the average risk of their assets gradually went up. Banks pursued the securitization of these assets through off-balance-sheet SIVs to get around capital requirements. That is, by taking away from their balance sheets high risk assets banks were able to obtain better bond ratings which allowed them to boost further their leverage and extend their investment activities (Benmelech and Duglosz, 2009; Stanton and Wallace, 2010). However, according to the 'too big to fail' view banks were also willing to provide implicitly liquidity guarantees to SIVs and thus retain risks because they relied on a government bailout if things went badly (Acharya and Richardson, 2009). In our model, when the yield to bank capital drops, banks invest

[1]Shadow banking refers to financial activities occurring outside the regulated banking sector (Coval, Jurek, and Stafford, 2009; Gennaioli, Shleifer and Vishny, 2013; Gorton andMetrick, 2010,2012; Gorton and Souleles. 2006; Pozsar *et al.*, 2010; Shin, 2009).

in 'bad projects' that while allow banks to achieve a higher yield on their investments are also counter-productive by not contributing to the formation of physical capital. There is now evidence that some banks knowingly engaged in such underperforming activities (see, Griffin, 2021).

Our model also addresses the development trade-off mentioned in the opening paragraph. As Acharya and Schnabl (2009) document the banks involved with the creation of new assets were among the largest financial institutions and were located in economies with high per-capita GDP. In our model the availability of 'bad projects' is a function of economic development; put differently, it takes a threshold level of development before the benefits of financial innovation become available. Given that banking is costly in our model, financially repressed economies (high cost of banking) are below that threshold in the steady-state. In contrast, for economies with efficient banking systems (low cost of banking) their level of development exceeds the threshold. As the rate of return of bank capital declines with economic development banks begin to invest funds in unproductive bad projects. We demonstrate that depending on the rate of financial innovation (availability of bad projects) the model can produce very rich dynamics including cycles of various periods.

We also diverge from Matsuyama *et al.* (2016) in another important way. In their model all agents are *ex ante* identical. In our model agents differ in their productivity which determines the effective units of labour that they supply when young and which in turn determines their net worth when they are old. Thus, our model provides an endogenous sorting mechanism for entrepreneurs. By introducing heterogeneity in our model we are able to examine how the income distribution is affected not only in the long-run but also during economic cycles. While there is a lot of attention dedicated to understanding the long-term trends of income inequality the behavior of the latter during cycles has been studied primarily within the neoclassical real business cycle framework.[2] However, the recent work by Beaudry *et al.* (2020) suggests that business cycles respond to a strong endogenous mechanism driven by financial frictions. Our model provides then a suitable framework to study the interaction between endogenous cycles and the patterns of income inequality.

We develop our model in Section 2. Each period a new generation of young agents is born who earn wages by finding employment at the final goods sector. At birth each agent's productivity is randomly determined. When old, agents can either deposit their endowments at the banking system or can become entrepreneurs by using their endowments along with funds borrowed from the banking system to produce physical capital. The choice will depend on their level of productivity with more productive agents finding it optimal to become entrepreneurs. Final goods are produced using a CRS technology that combines labor and physical capital. Financial intermediation is costly captured by a constant unit cost in the production of new loans (good projects).[3] Banks have

[2] See, for example, Bayer *et al.* (2020), Bhandari *et al.* (2021) and Castañeda *et al.* (1998).

[3] In reality, a higher cost of financial intermediation might be the outcome of either an inefficient banking system or financial repression, where banks must use some of their deposits to buy government bonds. In our model, there is no government and we will be using the first interpretation but our results can also be applied to the case of financial repression.

an alternative technology (bad projects) that yields a positive temporary return without contribution to the formation of physical capital and, thus, to economic growth.

Initially, we examine the case where bad projects are not available and show that the model behaves in exactly the same way as the neo-classical growth model. We obtain a unique steady-state that depends negatively on the cost of financial intermediation. Then, we allow banks to invest in bad projects and, initially, we assume that the investment in bad projects is unconstrained. For economies with relatively efficient banking systems in the absence of bad projects the high capital/labor ratio and the corresponding low marginal productivity of capital would have pushed the interest rate below the return of bad projects. Therefore, when bad projects are available, banks lend less funds to entrepreneurs till the point that the lending rate is equal to the return of bad projects. Once more, the model reaches a unique steady-state but with a lower capital/labor ratio for any level of financial development (intermediation cost). One important difference with Matsuyama *et al.* (2016) is that in our model both the intensive margin (size of loans) and the extensive margin (number of entrepreneurs) adjust to changes in rates of return resulting in a qualitativlely different phase map for the evolution of the capital/labor ratio. Lastly, we introduce constraints in the production of bad projects capturing economies of scale in their production. Now the model generates interesting dynamics that range from unique steady-states to regular cycles of various periodicity to chaotic attractors.

One set of predictions that our model delivers are cross-sectional relating the degrees of a country's financial development and exogenous technological progress to its economic stability. We find that economies with either/both higher level of financial innovation, implying lower intermediation costs and lower production costs of bad projects, or/and technological progress are more prone to experience the ups and downs of economic cycles. Because of the heterogeneity of agents, our model also makes interesting predictions about the behavior of income inequality along the business cycle. We find that although the marginal productivity of capital falls during expansions, entrepreneurs gain more during such periods relative to workers. Within each group inequality also increases during expansions. We also find that during recessions there are fewer entrepreneurs but those remaining earn higher profits. Thus, our work highlights the impact of cyclical fluctuations on the distribution of income complementing research that focuses on long-run trends. Lastly, in those states where banks allocate some of the funds for the production of bad projects, we find that the share of labor drops below that which would have predicted by the CRS technology in their absence. This is because the aggregate production function now takes a hybrid form that allows for the production of both types of final goods.

Related Literature Following the seminal work by Bernanke and Gertler (1999), there has been a huge literature on the relationship between financial frictions and macroeconomic fluctuations that follows the real business cycles paradigm that views cycles as generated by exogenous shocks.[4] In this review we restrict our attention on those studies, like the present one, where cycles arise endogenously.

The most closely related paper is Matsuyama (2013). Agents have the choice of investing either in good projects that indirectly enhance the future productivity of the economy

[4]For a review se Brunnermeier *et al.* (2013).

or in bad projects that are financially constrained. During booms agents net worth is sufficiently high to allow them to invest in bad projects which are more profitable at the detriment of future growth. In contrast, during slumps, agents' low net worth does not allow agents to invest in bad projects. By investing in good projects they provide the fuel for the next cycle. In our paper it is financial intermediaries that have the choice of allocating their funds between good and bad projects. We also allow for heterogeneity across agents that enable us to characterize the dynamics of income distributions along cycles.

Other theoretical work extending the Matsuyama (2013) model, include Matsuyama *et al.* (2016) that studies in more detail the stability properties of a slight simpler model and Kubin and Zörner (2021) that augments the model by introducing human capital.

Our paper is also related to the general literature on endogenous cycles arising in economies with financial frictions due to agency problems. In one strand of the literature, where the present paper also belongs, frictions are affecting the intermediation process. In Myerson (2012) there is financial moral hazard rooted at the ability of large financial institutions to abuse their power emanated in their decisions how to invest huge amounts of other people's wealth. In particular, the success of projects depends on whether or not bankers exert effort in supervising them. Optimal solutions require that bankers are remunerated on the basis of long-term performance, however, this implies that there are times when trusted managers are scarce and aggregate investment declines. In Dong and Xu (2020) there is misallocation of resources due to the ability of banks to divert funds borrowed in the interbank market. To alleviate the moral hazard problem, lending banks impose credit restrictions on borrowing banks. In a dynamic setup the interaction of credit constraints and the allocation of capital among projects with different productivities can generate shifts between an equilibrium where the interbank market is well-functioning and another where it crashes. Lastly, in Sunaga (2017) economic cycles are generated as the intensity of entrepreurial innovation interacts with the endogenous ability of banks to screen good ideas.

In the other strand of the literature, frictions are present on the demand-side of funds. In Azariadis and Smith (1999) cycles arise as the economy moves between a Walrasian regime with no binding constraints to one with credit rationing introduced to solve self-selection problems due to adverse selection. In contrast, moral hazard is the friction in Suarez and Sussman (1997) where lenders need to incentivize firms to exert effort and increase the probability of success of their projects. During booms prices and, thus, liquidity drops, increasing the demand for external funds. This leads to excessive risk-taking resulting in low returns and a recession. As quantities slump, prices increase reducing the demand for liquidity and level of risk taking. In Kiyotaki and Moore (1997) verifiability concerns imply that lenders are unable to force borrowers to repay their loans and, thus, only provide loans that are secured. In such an economy, assets provide a dual role as (a) productive capital and (b) collateral and the dynamic interaction between asset prices and credit limits give rise to amplified and persistent fluctuations in aggregate output. Lastly, Gu *et al.* (2013) examine the role of limited commitment whereby borrowers are unable to commit not to divert the funds lent to them into alternative uses. The endogeneity of debt limits imposed by lenders to reduce diversion incentives interacts with investor beliefs to give rise to cycles.

2. The Model

Time is discrete and extends ($t = 0, 1, 2...$). At each date a generation of unit mass is born and lives for two periods. Agents of each generation are distinguished by their type z. The distribution of types $G(z)$ is continuous, time invariant with support on $[\underline{z}, \bar{z}]$ and corresponding density function $g(z)$. Agents only consume when they are old. Young agents born at t, are endowed with z units of (effective) labor that they supply inelastically for the production of the final good. We normalize the aggregate supply of labor to 1, thus, $\hat{z} \equiv \int_{\underline{z}}^{\bar{z}} zg(z)dz = 1$.

There is one final good that can either be consumed or invested. The final good can be produced by a constant returns-to-scale technology (CRS), $y_t = f(k_t)$ where y_t denotes per capita income and k_t denotes the capital/ labor ratio at t. For all t, $f^{'}(k_t) > 0 > f''(k_t)$, $f(0) = 0$ and $f'(0) = \infty$. Physical capital fully depreciates in one period. Both factor markets are competitive. Then, the reward to physical capital is equal to $\rho_t = f^{'}(k_t)$, which is decreasing in k_t and reward to labor is equal to $w_t = f(k_t) - k_t f^{'}(k_t) \equiv W(k_t) > 0$, which is increasing in k_t. Thus a type z agent's wage income is equal to $zW(k_t)$.

Old agents have two options. They can either deposit their endowment in the perfectly competitive banking system or they can borrow from the banking system and use the loan together with their endowment to produce physical capital which is then invested in the CRS technology to produce the final good. Those agents that become depositors enjoy a utility benefit u (measured in units of the final good). Physical capital is produced by using the final good as input. The return of the investment per unit of final good is equal to R units of physical capital. The amount of the final good that agents can borrow from the banking system also depends on their type. In particular, a type z agent will be able to borrow a maximum amount of $(m-1)zw_t$ units of the final good from the bank at the gross interest rate r^b_{t+1} and therefore invest a total amount of mzw_t. The same type will earn income $f^{'}(k_{t+1})Rzmw_t$, repay $r^b_{t+1}(m-1)zw_t$ to the bank and consume the difference.[5] Agents of the same type who choose to deposit their labor income in the banking system will consume $r^d_{t+1}zw_t$. Thus, as long as,

$$f^{'}(k_{t+1})Rzmw_t - r^b_{t+1}(m-1)zw_t \geqslant r^d_{t+1}zw_t + u \tag{1}$$

[5] We have made the supposition that the amount each entrepreneur can borrow from the banking system is proportional to her wage income. This is a direct application of the variable investment version of the Holmström and Tirole (1997) model. To show this claim we convert the technology into a stochastic one. Thus, let now each project succeed with probability $p \in \{p_h, p_l\}$ where ($p_h > p_l$), in which case it yields a return $\hat{R}$ per unit of investment, otherwise it fails and yields nothing. The probability of success depends on the effort level that the entrepreneur will exert on the project. The entrepreneur has two options. She can exert effort in which case the project succeeds with probability p_h or can choose not to exert effort in which case the project succeeds with probability p_l but the entrepreneur also derives a private benefit B per unit of investment. Then, Holmström and Tirole (1997) show that, as long some restrictions on the parameters of the model hold, the optimal amount of the bank loan is proportional to the entrepreneurs net worth, which in our case is equal to zw_t. By setting, $p_h\hat{R} = R$ the equivalence between the two models is established.

agents of type z will invest in the physical capital technology.

Financial Intermediaries In the above set-up financial intermediaries can enhance welfare.[6] In the absence of a banking system all agents can either invest their endowments and become entrepreneurs or can gain utility u. This utility can be either considered as obtained from leisure or from an alternative low productivity technology of the final good. Clearly, agents with low endowments would prefer the second option. Financial intermediation allows them to earn some income on their endowments through deposits and also enjoy utility, u.

Banks have two options about how to invest their deposits. They can either offer loans to agents who invest in the physical capital technology or they can invest in a one-period (Bad) technology that yields B units of the final good at $t+1$ for each unit of the final good invested at t. Investment in bad projects is unproductive as it does not contribute to the stock of physical capital. For example, during the 2008 Global Financial Crisis, investment in CDOs has temporary enhanced bank profits, however, at the expense of lending to firms and, hence, creation of productive capital.

Banking services are costly. The cost per unit of loan is equal to γ units of the final good. Clearly if $r_{t+1}^{b} < B(1+\gamma)$ banks will choose to invest in the bad technology. Let V_{t+1} denote the total investment in the bad technology. We assume that investment in bad projects is constrained by the size of the economy. The idea is that there are some economies of scale involved with financial innovation and only large institutions can afford it, as it happened before the onset of the crisis. In particular we assume that the maximum amount $\hat{V}$ that can be invested depends on the level of the capital/labor ratio which is a good proxy for the size of aggregate deposits and, hence, the size of the banking system; $\hat{V} = V(k_t)$ where $V' > 0$.[7] The generation of cyclical fluctuations in our model depends on the first derivative. With economies of scale arising from either/both the demand side (e.g. the demand of derivatives depends on the size of the banking sector) or/and the supply side (economies of scale in financial innovation) bad projects become available relatively fast and - as we will see below - cycles become possible.

2.1. The Bad Technology is not Available, $V_{t+1} = 0$

For the moment, consider the case where the banking system does not invest in the bad technology, $V_{t+1} = 0$. Given that the left-hand side of (1) is increasing in z, agents with low initial endowments prefer to become depositors and those with high initial endowments prefer to become entrepreneurs. Market clearing requires that aggregate amount of loans

[6] In Appendix 1 we derive welfare under both the presence and the absence of a banking system. All our numerical results satisfy the constraint that banks increase welfare.

[7] For example, economies of scale are important for securitization that relies on large number of assets for diversification. The function captures in reduced form a decreasing unit cost in the production of bad projects. At low levels of development diversification is limited so the unit cost is very high. As the economy expands the unit cost declines, eventually reaching a lower bound. Such economies of scale are consistent with our reduced form presentation.

is equal to aggregate deposits,

$$\int_{z^*_{t+1}}^{\bar{z}} (1+\gamma)(m-1)\, z w_t g(z) dz = \int_{\underline{z}}^{z^*_{t+1}} z w_t g(z) dz,$$

On the left hand side we have the allocation of funds between loans and expenditure to cover the cost of banking services. On the right-hand side we have aggregate deposits. The above condition implies that $z^*_{t+1} = z^*$ for every t and is the solution of

$$\hat{z} = 1 = (m + \gamma m - \gamma) \int_{z^*}^{\bar{z}} z g(z) dz \tag{2}$$

The banking system's zero-profit condition when $V_{t+1} = 0$ is given by

$$r^b_{t+1} \int_{z^*_{t+1}}^{\bar{z}} (m-1)\, z w_t g(z) dz = r^d_{t+1} \int_{\underline{z}}^{z^*_{t+1}} z w_t g(z) dz.$$

Using the market clearing condition the zero-profit condition implies that

$$r^b_{t+1} = r^d_{t+1}(1+\gamma) \quad \text{for every } t \text{ such that} \quad V_{t+1} = 0 \tag{3}$$

Using (2),

$$k_{t+1} = \frac{Rm}{m + \gamma m - \gamma} W(k_t). \tag{4}$$

Each entrepreneur invests her endowment $z w_t$ plus her funds borrowed from banks, $(m-1) z w_t$ to produce R units of physical capital. Lastly, substituting (3) and (2) in (1) determines implicitly the equilibrium interest rates, r^d_{t+1} and r^b_{t+1}, such that an agent of type z^* is indifferent between investing and saving (depositing).

Example 1 *Let* $f(k) = Ak^{\frac{1}{2}}$, $f'(k) = \frac{1}{2}Ak^{-\frac{1}{2}}$ *and* $W(k) = \frac{1}{2}Ak^{\frac{1}{2}}$.[8] *Further let* z *to be uniformly distributed on* $[0,2]$. *Then, for the case when* $V_{t+1} = 0$, *from (2) we get*

$$\begin{aligned} 1 &= (m+\gamma m - \gamma) \int_{z^*}^{2} z g(z) dz = (m + \gamma m - \gamma) \int_{z^*}^{2} z \frac{1}{2} dz \\ &\Rightarrow \frac{1}{m + \gamma m - \gamma} = \frac{1}{4}\left[4 - (z^*)^2\right] \Rightarrow \end{aligned}$$

$$z^* = \left(4 - \frac{4}{m + \gamma m - \gamma}\right)^{\frac{1}{2}} = \left(4 - \frac{4}{m + \gamma(m-1)}\right)^{\frac{1}{2}} \tag{5}$$

From (4) we get

$$k_{t+1} = F(k_t) = \frac{1}{2}\frac{RmA}{m + \gamma m - \gamma} k_t^{\frac{1}{2}} \tag{6}$$

Then, from (1) we get

$$\frac{1}{4} A k_{t+1}^{-\frac{1}{2}} R z^* m A k_t^{\frac{1}{2}} - \frac{1}{2} r^d_{t+1} (m + \gamma m - \gamma) z^* A k_t^{\frac{1}{2}} = u \Rightarrow$$

[8] We have set the factor shares equal because it reduces a lot the complexity of the problem although the capital share is typically set between 30% and 40%.

$$r^d_{t+1} = \frac{\frac{1}{2}Ak_{t+1}^{-\frac{1}{2}}Rm}{m+\gamma m-\gamma} - \frac{u}{\frac{1}{2}\left(m+\gamma m-\gamma\right)z^{*}Ak_t^{\frac{1}{2}}} \tag{7}$$

We can also solve for the steady-state capital/labor ratio by setting $k_{t+1} = k_t = k^*$ *in (6):*

$$k^* = \left(\frac{\frac{1}{2}RmA}{m+\gamma m-\gamma}\right)^{\frac{1}{2}} = \left(\frac{\frac{1}{2}RA}{1+\gamma(1-\frac{1}{m})}\right)^{\frac{1}{2}} \tag{8}$$

The following proposition summarizes how the fixed point solutions depend upon the parameters:

Proposition 1 *Comparative Statics for the fixed point solutions on* $F(k_t)$ *(No bad projects)*

	k^*	z^*
$R\uparrow$	$\uparrow$	0
$m\uparrow$	$\downarrow$	$\uparrow$
$\gamma\uparrow$	$\downarrow$	$\uparrow$
$u\uparrow$	0	0

Proof The proposition directly follows from inspecting equatios (5) and (8).

Interestingly, z^* does neither depend on R nor on u. The economic intuition is as follows: An increase in the rate of return of the technology that produces capital goods, R, increases the incentives for the marginal old agent to become entrepreneur. However, financial constraints limit the amount that each entrepreneur can borrow which keeps fixed the marginal agent. Thus, market clearing requires that the interest rate goes up. Similariy, an increase in u requires the interest rate to drop so that marginal old agent stays indifferent. Notice that k^* is decreasing in γ (see the figure below). Economies with high intermediation costs are financially repressed. A large fraction of potentially productive capital is absorbed by operational and administrative costs incurred in the process of granting new loans. k^* is also decreasing in m. At first glance, this looks counterintuitive. If the ability of firms to borrow more is usually associated with a reduction in financial frictions we should expect an overall improvement in the economy's performance. The puzzle is resolved when we concentrate on welfare and not on per capita income. As m increases it offers the opportunity to a larger fraction of low income agents (low z) to become lenders. In the absence of a financial system, $m = 0$, these agents would have to choose between either becoming entrepreneurs in which case their income would be low or do nothing and gain utility u. When m is positive they can become lenders and obtain a low income from the bank and also gain utility u. However, as the amount of aggregate lending increases the stock of productive capital decreases because some of the output is used to cover the cost of intermediation. Welfare is increasing in m for relatively low values of m and then decreasing.

2.2. Investment in Bad Projects: The Unconstrained Case

The above derivation will be the equilibrium for $t+1$ given $W(k_t)$ as long as $r^b_{t+1} \geqslant B(1+\gamma)$. However, if $r^b_{t+1} < B(1+\gamma)$ the banking system will invest some of the deposits

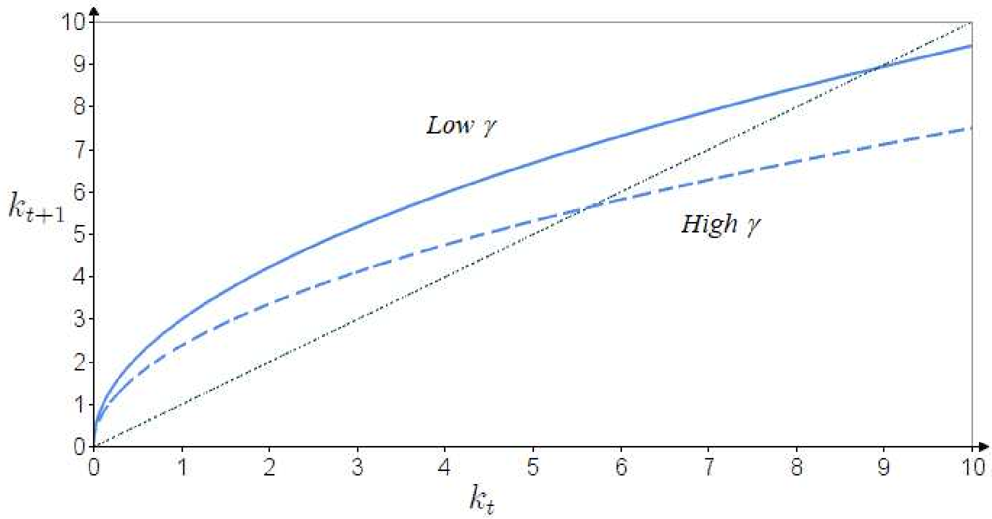

Figure 2.1: Financial Repression

in the bad technology, $V_{t+1} > 0$. For the moment we concentrate on the case where the investment in bad projects is not constrained; put differently, $\hat{V} \approx \infty$. Let V^*_{t+1} denote the solution.

The market clearing condition now is given by

$$\int_{z^*_{t+1}}^{\bar{z}} (1+\gamma)(m-1)\, z w_t g(z) dz + V^*_{t+1} = \int_{\underline{z}}^{z^*_{t+1}} z w_t g(z) dz \tag{9}$$

where now we have added on the left-hand side, which shows the use of funds by banks, the investment in bad projects. For the banking system to be indifferent between lending and investing in the bad technology we must have $r^b_{t+1} = B(1+\gamma)$. The banking system's zero-profit condition when $V_{t+1} = V^*_{t+1} > 0$, given by $r^b_{t+1} \int_{z^*_{t+1}}^{\bar{z}} (m-1)\, z w_t g(z) dz + B V^*_{t+1} = r^d_{t+1} \int_{\underline{z}}^{z^*_{t+1}} z w_t g(z) dz$ implies that

$$r^b_{t+1} = B(1+\gamma) > r^d_{t+1} = B \quad \text{for every } t \text{ such that} \quad V_{t+1} = V^*_{t+1} > 0 \tag{10}$$

The stock of physical capital at k_{t+1} when $V_{t+1} = V^*_{t+1} > 0$ is given by

$$k_{t+1} = \int_{z^*_{t+1}}^{\bar{z}} RmzW(k_t) g(z) dz \tag{11}$$

Given that in equilibrium an agent of type z^*_{t+1} must be indifferent between investing and saving (depositing) (1) implies that the following equality must hold:

$$z^*_{t+1} W(k_t) \left(f'(k_{t+1}) Rm - B(m + \gamma m - \gamma) \right) = u \tag{12}$$

Then, for given $W(k_t)$, we can solve (11) and (12) together for k_{t+1} and z^*_{t+1} and then by substituting the solution for z^*_{t+1} in (9) we can solve for V^*_{t+1}.

Example 2 *We use the same production function and the same distribution for z as in the example above.*

For the case when $V_{t+1} > 0$, we use (12) to solve for a cut-off value z^*_{t+1} such that an agent with endowment of effective units of labor equal to z^*_{t+1} is indifferent between being a borrower and a saver.

$$z^*_{t+1} = \frac{2u}{Ak_t^{\frac{1}{2}}\left(\frac{A}{2}k_{t+1}^{-\frac{1}{2}}Rm - B(m+\gamma m-\gamma)\right)} \tag{13}$$

and from (11) we get

$$k_{t+1} = \frac{1}{4}RmAk_t^{\frac{1}{2}}\int_{z^*_{t+1}}^{2} zdz = \frac{1}{8}RmAk_t^{\frac{1}{2}}\left(4-\left(z^*_{t+1}\right)^2\right) \tag{14}$$

The dynamic equation for the capital/labor ratio in the case where investment in bad projects is unrestricted, that is $V_{t+1} = V^*_{t+1} > 0$, is implicitly defined by the following fourth degree polynomial (insert (13) in (14))[9]

$$\Phi_4 y^4 + \Phi_3 y^3 + \Phi_2 y^2 + \Phi_1 y + \Phi_0 = 0 \tag{15}$$

where $y = \sqrt{k_{t+1}}$, $\Phi_4 \equiv B^2(m+\gamma m-\gamma)^2 k_t^{\frac{1}{2}}$, $\Phi_3 \equiv -ARmB(m+\gamma m-\gamma)k_t^{\frac{1}{2}}$, $\Phi_2 \equiv \frac{(ARm)^2}{4}k_t^{\frac{1}{2}} - B^2(m+\gamma m-\gamma)^2\frac{ARm}{2}k_t + \frac{Rmu^2}{2A}$, $\Phi_1 \equiv \frac{(ARm)^2}{2}B\left(m+\gamma m-\gamma\right)k_t$ *and* $\Phi_0 \equiv -\frac{(ARm)^3}{8}k_t$.

Because of the complexity of (15) it is difficult to get analytical results for the unconstrained case. In all our numerical examples there was only one positive real solution such that the corresponding value of the threshold, z^*_{t+1}, takes values in the interval $(0,2)$. We denote this solution as $H(k_t)$ and will use it in our numerical analysis below.

To derive the steady state level of capital/labor ratio we set $k_{t+1} = k_t = k^*$ in (15). We find that $y^* = \sqrt{k^*}$ is determined by the following third degree polynomial:[10]

$$\Psi_3\left(y^*\right)^3 + \Psi_2\left(y^*\right)^2 + \Psi_1 y^* + \Psi_0 = 0 \tag{16}$$

where $\Psi_3 \equiv B^2(m+\gamma m-\gamma)^2$, $\Psi_2 \equiv -mRAB(m+\gamma m-\gamma)\left(1+\frac{1}{2}(m+\gamma m-\gamma)B\right)$, $\Psi_1 \equiv m^2R^2A^2\left(\frac{1}{4}+\frac{1}{2}(m+\gamma m-\gamma)B\right)$ *and* $\Psi_0 \equiv -Rm\left(\frac{1}{8}m^2R^2A^3 - \frac{1}{2A}u^2\right)$.

In Figure 2.2 we compare - for a fixed γ - the evolution of the capital/labor ratio k_{t+1} when bad projects are not available (the higher line depicting $k_{t+1} = F(k_t)$) with the case when bad projects are available and investment in bad projects is not constrained, $k_{t+1} = H(k_t)$. For future reference, we denote the point of intersection k^{FH}. For $k_t \geq k^{FH}$ we have $r^b_{t+1} = B\left(1+\gamma\right)$ so that the banking system is indifferent between offering loans to entrepreneurs and investing in bad projects. Banks start to invest in bad projects; $H(k_t) < F(k_t)$ holds, i.e. investment in bad projects implies that the capital/labor ratio k_{t+1} is below the one that would be obtained if such investment was not available.

[9] See Appendix 2 for details of this derivation.

[10] In all numerical examples there is only one real positive root for values of z^* within the admissible range.

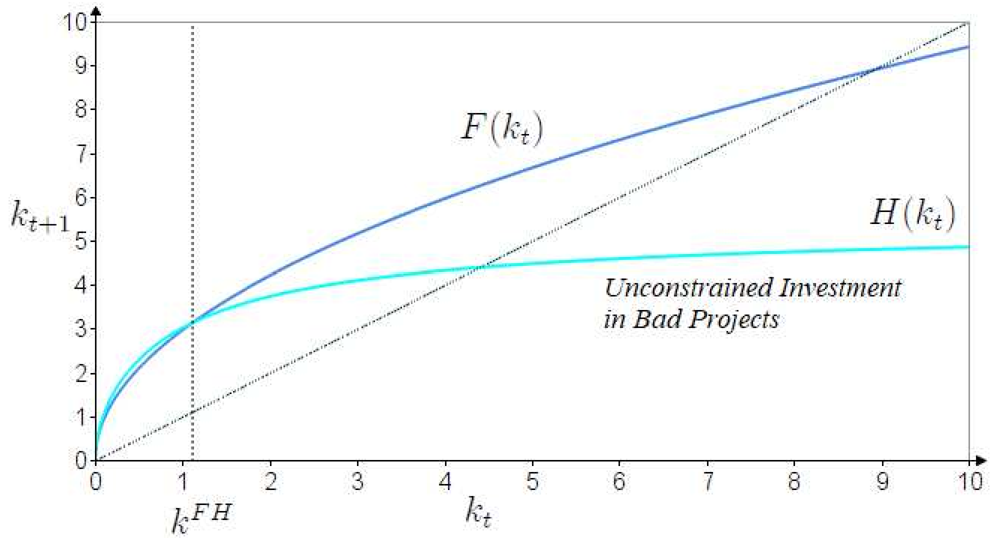

Figure 2.2: Unconstrained Investment in Bad Projects

Importantly, $H(k_t)$ is upward sloping, which is an important contrast to Matsuyama *et al.* (2016), where the corresponding branch of the function is flat. In Matsuyama *et al.* (2016) there is a representative agent so that there is no distinction between the intensive margin, how much capital is produced by each type of entrepreneur, and the extensive margin, the mass of entrepreneurs. As soon as k_t grows to the level where the return to good projects is equal to the return of the bad projects all additional savings are invested in bad projects so that the two returns stay equal. Thus, in Matsuyama *et al.* (2016), the fuction corresponding to $H(k_t)$, that is the implicit solution of (15) in our model, is flat. In our model, agents are heterogeneous and therefore adjustments can happen on both margins. Our simulations show that even with investments in Bad projects there is still a positive relationship between k_t and k_{t+1} (on $H(k_t)$) and that as k_t and k_{t+1} increase the threshold z^*_{t+1} increases, that is, the mass of entrepreneurs decreases.

Note that this is also different from the movement on $F(k_t)$. In the absence of bad projects as k_t increases the return to capital, and hence the two interest rates, decline. In fact the decline in the interest rate just compensates for the increase in the capital/labor ratio so that the marginal agent is still indifferent between becoming an entrepreneur or depositing her wages in the bank, so there is no adjustment on the extensive margin and z^*_{t+1} remains constant. However, on $H(k_t)$ with investments in bad projects the interest rate on deposits is equal to the return of the bad projects (adjusted for the cost of loans) and does not drop as some of the extra capital is invested in bad projects. The marginal old agent, because of the relatively high deposit rate, prefers to be a depositor and as a result the threshold increases, and the mass of entrepreneurs decreases.

Finally, comparing the evolution of the capital/labor ratio in the case when bad projects are not available given by (6) with the one corresponding to the case when investment in bad projects is unconstrained given implicitly by (15), we find that the latter is indirectly affected by changes in the utility benefit, u, and the return of the bad projects B, through their effects on the threshold level z^*_{t+1}, while these variables do not affect (6). This observation is also important for understanding the main methodological contribution of our paper relative to Matsuyama *et al.* (2016).

Given that there is no closed form solution for $H(k_t)$, the following comparative static results for this fixed point are obtained using simulations:[11]

Result 1 *Comparative Statics (unconstrained case)*

	k^*	z^*
$R\uparrow$	$\uparrow$	$\downarrow$
$m\uparrow$	$\uparrow$	$\downarrow$
$\gamma\uparrow$	$\downarrow$	$\uparrow$
$u\uparrow$	$\downarrow$	$\uparrow$
$B\uparrow$	$\downarrow$	$\uparrow$

Any increase in u or B will increase z^*, which in turn will have a negative effect on k^*. The intuition behind these effects is that both encourage further investment in bad projects, which in turn has a negative impact on physical capital accumulation. This is because an increase in u makes more attractive to agents to deposit their endowments at the bank, thus, increasing the supply of funds and depressing the interest rate, favoring investment in bad projects which can also be the result of a direct increase in the return of bad projects. Therefore, either of the above two changes will shift downwards $H(k_t)$ moving the intersection with the 45^o line to the left and below the original intersection and as a result k^* will decrease (see Figure 2.2). These observations will prove useful for the derivation of some analytical results below.

2.3. Investment in Bad Projects: The Constrained Case

Next, consider the case when the investment in bad projects is constrained. The following two conditions must hold: (a) $r^b_{t+1} < B\,(1+\gamma)$ when $V_{t+1} = 0$, and (b) $V^*_{t+1} > V\,(k_t)$. The second condition states that the investment in bad projects is below the level the banks would have chosen if more bad projects had been available. Exactly, because this investment is constrained the borrowing rate and the return to bad projects are not equalized. Now market clearing implies that $\int_{\hat{z}_{t+1}}^{\bar{z}} (1+\gamma)\,(m-1)\,zw_t g(z)dz + V\,(k_t) = \int_{\underline{z}}^{\hat{z}_{t+1}} zw_t g(z)dz$, which can be written as:

$$\hat{z} = (m+\gamma m-\gamma)\int_{\hat{z}_{t+1}}^{\bar{z}} zg(z)dz + \frac{V\,(k_t)}{W\,(k_t)} \tag{17}$$

and the stock of physical capital is given by

$$k_{t+1} = \int_{\hat{z}_{t+1}}^{\bar{z}} Rmzw_t g(z)dz = \int_{\hat{z}_{t+1}}^{\bar{z}} RmzW(k_t)g(z)dz \tag{18}$$

[11] For the exact parameter values, see the details provided in the following section. The reported comparative static results are robust to changes in the parameter values of the model.

From (17), assuming once again that the distribution of endowments is uniform on $[0,2]$, we find that for the constrained case the cut-off value is given by:

$$z^*_{t+1} = \left(4 - \frac{4\left(1 - \frac{V(k_t)}{W(k_t)}\right)}{m + \gamma m - \gamma}\right)^{\frac{1}{2}} \tag{19}$$

and together with (18) we find that $k_{t+1}$is given by

$$k_{t+1} = \frac{Rm}{m + \gamma m - \gamma}\left(W(k_t) - V(k_t)\right) \tag{20}$$

Notice that $\frac{\partial k_{t+1}}{\partial k_t} = \frac{Rm}{m+\gamma m-\gamma}\left(W' - V'\right) \gtrless 0$. The dynamics of the model depend on this slope, i.e. how fast bad projects become available. The slope may be positive, but it has to be flatter than the $F(k_t)$ function at the intersection point, so that beyond that point some of the funds are invested in bad projects. For the rest of the paper, we focus on the case with a negative slope, i.e. on $V' > W'$. What matters for the existence of cycles is that at the point where this function crosses the 45° line its slope slope is less than -1. The exact form of (20) would depend on the economies of scale in financial innovation and the demand for its products.

Without any strong priors about its exact form we simplify the analysis of the model by postulating a linear form. Thus, we specify the function as

$$k_{t+1} = G(k_t) = \delta\left(k_t - k^{FG}\right) + F\left(k^{FG}\right) = \delta\left(k_t - k^{FG}\right) + \frac{RmA}{m + \gamma m - \gamma}\frac{\sqrt{k^{FG}}}{2}. \tag{21}$$

This specification ensures that it passes through the point $\left(k^{FG}, F\left(k^{FG}\right)\right)$, where k^{FG} indicates the threshold value of capital beyond which bad projects become available. We ensure that it has a negative slope by assuming that $\delta < 0$. Equations (20) and (21) imply that[12]

$$V(k_t) = \frac{A}{2}\left(\sqrt{k_t} - \sqrt{k^{FG}}\right) - \frac{(m + \gamma m - \gamma)\,\delta}{Rm}\left(k_t - k^{FG}\right)$$

which we need in order to determine z^*_{t+1} explicitly (using (19)):

$$z^*_{t+1} = \left(4 - \frac{4\left(1 - \frac{V(k_t)}{W(k_t)}\right)}{m + \gamma m - \gamma}\right)^{\frac{1}{2}} = \left(4 - \frac{4\left(\sqrt{\frac{k^{FG}}{k_t}} + \frac{k_t - k^{FG}}{\sqrt{k_t}}\frac{2\delta(m+\gamma m-\gamma)}{RmA}\right)}{m + \gamma m - \gamma}\right)^{\frac{1}{2}} \tag{22}$$

We can solve for the steady-state capital/labor ratio by setting $k_{t+1} = k_t = k^*$ in (21):

$$k^* = \frac{1}{1-\delta}\left(-\delta k^{FG} + \frac{RA}{1 + \gamma(1 - \frac{1}{m})}\frac{\sqrt{k^{FG}}}{2}\right) \tag{23}$$

[12] (21) implies that $V(k_t) = W(k_t) - W(k^{FG}) - \delta\frac{m+\gamma m-\gamma}{Rm}\left(k_t - k^{FG}\right)$ with $V(k_t) > (<) 0$ for $k_t > (<) k^{FG}$ and $V' = W' - \delta\frac{m+\gamma m-\gamma}{Rm} > W'$ as long as $\delta < 0$.

In addition, inserting (22) in (23) the steady state $z^*_{t+1} = z^*_t = z^*$ can be determined as

$$z^* = 2\sqrt{1 - 2\frac{\sqrt{k^*}}{RmA}} \tag{24}$$

Proposition 2 *Comparative Statics (Constrained case)*

	k^*	z^*
$R\uparrow$	$\uparrow$	$\uparrow$
$m\uparrow$	$\downarrow$	$\uparrow$
$\gamma\uparrow$	$\downarrow$	$\uparrow$
$u\uparrow$	0	0
$k^{FG}\uparrow$	$\uparrow$	$\downarrow$
$\delta\uparrow$	$\uparrow$	$\downarrow$

Proof The first column follows from (23) and from $sign\left(\frac{\partial k^*}{\partial \delta}\right) = sign\left(\frac{F(k^{FG})-k^{FG}}{(\delta-1)^2}\right)$ and $F(k^{FG}) - k^{FG} > 0$, if a fixed point exists on $G(k)$. For the second column observe that $sign\left(\frac{\partial z^*}{\partial R}\right) = sign\left(\frac{\frac{-\delta}{1-\delta}k^{FG}+k^*}{AR^2m\sqrt{k^*}}\right)$ and $sign\left(\frac{\partial z^*}{\partial m}\right) = sign\left(\frac{2k^*-m\frac{\partial k^*}{\partial m}}{ARm^2\sqrt{k^*}}\right)$.

Remember an increase in δ (decrease in absolute terms) implies that financial innovation (production of bad projects) is more costly. Thus, there are more funds available for investment in good projects while the interest rate drops to encourage more agents to become entrepreneurs.

3. Dynamics and Cycles

In the graph below we show the complete phase map of the capital/labor ratio. The dark blue line, defined in (6) as $F(k_t)$, corresponds to that part of the map of the capital/labor ratio when there is no investment in bad projects. Banks would like to invest in bad projects when the light blue line falls below the dark blue line**,** i.e. for $k_t > k^{FH}$. Beyond that point investment in bad projects offers a higher return to the banks than lending to entrepreneurs. However, they cannot do so because such investment is not available, as the level of economic development does not support financial innovation. As soon as the dark blue line intersects the red line, defined in (21) as $G(k_t)$, banks begin to invest in bad projects which become available as a result of financial innovation. We denote the intersection of the above two segments, $F(k_t) = G(k_t)$, as k^{FG}. As long as the red line is above the light blue line, which is defined by the implicit solution of (15) as $H(k_t)$, the investment in bad projects is constrained. As soon as $H(k_t)$ intersects $G(k_t)$ investment in bad projects is unconstrained and the evolution of the capital/labor ration is captured by the light blue line. We denote the intersection of the last two segments of the map at $G(k_t) = H(k_t)$ as k^{GH}.

Then, we can define the whole dynamic map as:

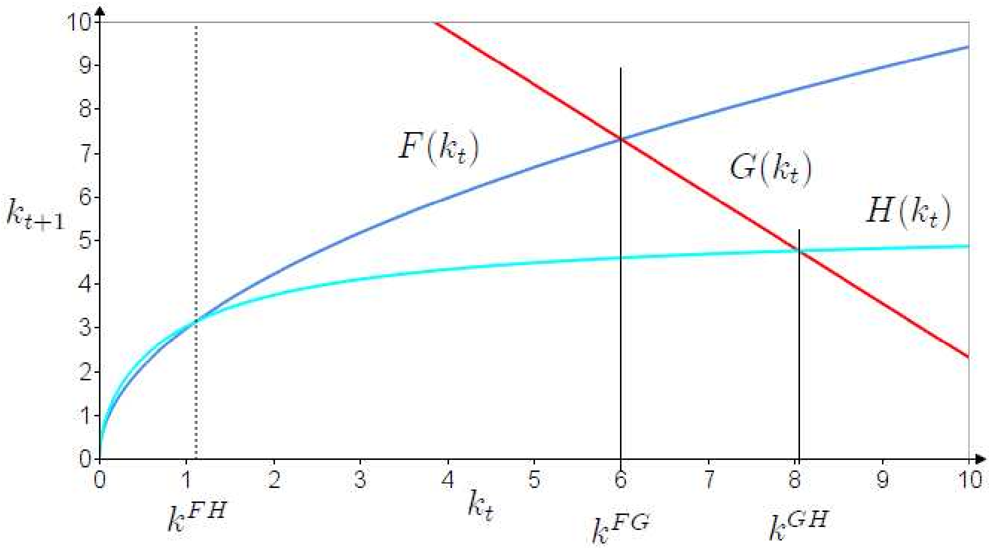

Figure 3.1: Constrained Investment in Bad Projects and Full Map

$$k_{t+1} = \begin{cases} F(k_t) & \text{if} \quad k_t \leq k^{FG} \\ G(k_t) & \text{if} \quad k_t^{FG} < k_t < k^{GH} \\ H(k_t) & \text{if} \quad k^{GH} \leq k_t \end{cases} \tag{25}$$

The phase map of the evolution of the capital/labor ratio, and, hence, its dynamic path, is very similar to the one obtained by Matsuyama *et al.* (2016). However, there is an added complexity due to the heterogeneity of agents according to their initial endowments. In this model, physical capital adjusts along two margins. Borrowing terminology from the international trade literature, over time the capital/labor ratio adjusts (a) along the *intensive* margin as entrepreneurs of the same type adjusts their output levels, and (b) along the *extensive* margin as the proportion of entrepreneurs changes.

One of the implications of this added complexity is that the part of the map that corresponds to the case when investment in bad projects is unconstrained has always a positive slope while the corresponding part in the Matsuyama *et al.* (2016) is flat. The reason is that in the Matsuyama *et al.* (2016) model as long as the capital/labor ratio is sufficiently high so that the interest on borrowing (which in turn depends on the marginal product of physical capital) is less than the return on bad investment projects next period's capital/labor ratio will drop to ensure that the two returns are equal. However, in our paper there is a second margin of adjustment. The interest rate that banks charge to entrepreneurs will still be equal to the return of the bad projects but not the marginal productivity of capital which also depends on the fraction of entrepreneurs. While there are fewer entrepreneurs those who remain invest more.

This added complexity limits our ability to study analytically the dynamic model and, therefore, to explore its properties we mostly rely on numerical analysis. However, in the following proposition we are able to make explicit some of its properties which hold when the third branch is not involved, that is, assuming a negative slope for G, $\delta < 0$, and $F(k_{FG})) < k_{GH}$ so that the dynamics in the interval $[0, k_{GH}]$ involve only the first two branches F and G.

Proposition 3 *Let the map (25) be restricted only to the F and G branches in the interval $[0, k_{GH}]$ (assuming $\delta < 0$, and $F(k_{FG})) < k_{GH}$), then the following results apply.*

Property 1. *When $F(k_{FG}) \leq k_{FG}$, so that the fixed point solution belongs only to the first branch, $k^* \leq k_{FG}$, this solution is attracting in the interval $[0, k_{GH}]$.*

Property 2a. *When $F(k_{FG}) > k_{FG}$, so that the fixed point does not belong to the first branch, then the interval $J = [G(F(k_{FG})), F(k_{FG})]$ is absorbing for $k_t \in [0, k_{GH}]$. In fact, any point $k_t < G(F(k_{FG}))$ has an increasing trajectory entering J in a finite number of iterations, and being $F(k_{FG})$ a local maximum, also any point $F(k_{FG}) < k_t \leq k_{GH}$ has a trajectory entering J in a finite number of iterations.*

Property 2b. *When $F(k_{FG}) > k_{FG}$ then the fixed point solution belonging to the second branch may be stable or unstable. When it is larger than but sufficiently close to k_{FG}, the specific attracting set inside the interval J depends on the type of transition occurring from $k^* = k_{FG}$ to $k^* > k_{FG}$ and this can be understood looking only at the slopes of the two branches at the point $k^* = k_{FG}$, occuring when the intersection point of the F and G branches is an equilibrium solution for both branches. Regular attracting cycles and chaotic dynamics in cyclical intervals are possible, the specific cyclicity depending on δ.*

Proof Property 1 can be easily verified considering that the derivative of F evaluated at the fixed point given in (7) is $F'(k^*) = 0.5$ and it does not depend on the parameters of the model. A trajectory starting from any point $k_{FG} \leq k_t \leq k_{GH}$ is necessarily mapped in one iteration into a point smaller that k_{FG} and any point $k_t < k_{FG}$ has a monotone trajectory converging to k^*.

Properties 2a and 2b can be shown drawing from the dynamical systems literature (it is related to the properties of the well studied skew-tent map that can be applied to the map (25) close to the bifurcation point[13] $k^* = k_{FG}$ under our assumptions) (see Sushko et al., 2015, and Avrutin et al., 2019). In our map we have $F'(k^*) = F'(k_{FG}) = 0.5$ and $G'(k^*) = G'(k_{FG}) = \delta$, so that the dynamics only depend on the value of δ. Clearly, when $-1 < \delta < 0$ then the fixed point on the G branch is attracting. Denoting by F^nG the symbolic sequence of cycles having n points belonging to F and one point to G, from Avrutin et al. (2019, page 296, equation (5.42)), it can be stated that stable cycles with sequence F^nG for $n = 1, 2, 3, 4$ exist when

$$-2^n < \delta < 1 - 2^n$$

so that an attracting 2-cycle ($n = 1$) exists for $-2 < \delta < -1$, a 3-cycle ($n = 2$) for $-4 < \delta < -3$, a 4-cycle ($n = 3$) for $-8 < \delta < -7$, a 5-cycle ($n = 4$) for

[13] This type of bifurcation called, "border collision bifurcation", involves the fixed point hitting one of the borders that define the branches of the map.

$-16 < \delta < -15$. Moreover, any other value of $\delta < -1$ leads to chaotic intervals (a unique interval, or $2n$ or n cyclical intervals, for $n = 1, 2, 3, 4$). □

Remark 1 *Since we do not have an explicit solution for the third branch, we cannot derive explicit boundaries when this part of the map (25) becomes relevant. From numerical simulations (some of them provided below), we can show that interesting and even complex behavior may occur.*

It is not surprising that the numerical analysis below confirms that, broadly, the dynamic behavior of our model is similar to the one in Matsuyama et al. (2016) although the introduction of agent heterogeneity has added some, from an economic point of view, interesting complexities. In particular, the inclusion of both margins in our model offers a much richer picture of the behavior of income inequality over the business cycle.

3.1. Numerical Analysis: Typology of Long-Run Equilibrium Dynamics

The model is consistent with a great variety of equilibrium dynamic paths that depend on the values of particular parameters. There are cases where such paths converge to a unique steady-state. But more often we find that the paths converge to cycles that are either regular or irregular and of various degrees of periodicity including chaotic dynamics. Our model is too stylized to attempt an exact calibration and therefore we study its behavior under a wide range of possible parameterizations. For the benchmark case we let $R = 2.4$, $A = 2.5$, $m = 1.5$, $\gamma \in 0.02$, $\alpha = 0.5$, $\delta = -1.25$, $u = 1$, $B = 1.25$, $k_{FG} = 6$ and $z \sim U[0, 2]$. The debt to total assets ratio for many firms is between 0 and 1/2 with corresponding m values of 1 and 2. What matters for the determination of equilibrium interest rates are the values of γ and B with γ also determining the resource cost of banking. With that in mind we have fixed the value of B and allow γ to take a wide range of values in our simulations. The exact value of u does not play a significant role other than its effect on the interest rate and we have normalized it to 1. Allowing both for the TFP factor, A, and the return R offers some flexibility with our computations.

What gives rise to cyclical fluctuations in our model is the availability of bad projects. With that in mind, in this section, we fix all other parameters at their benchmark values and let δ vary to produce the bifurcation diagram in Figure 3.2. The diagram shows the values of the capital/labor ratio approached asymptotically (fixed points, periodic cycles, or chaotic attractors) as we vary δ. We also provide a detailed description of the economic mechanism behind these dynamics.

Given that δ takes negative values it is more convenient to describe what happens as its values decrease (increase in absolute terms). For very high values of δ (close to zero) we have convergence to a unique steady-state with a high capital/labor ratio. The cost of creation of bad projects is too high and therefore banks use all deposits to lend funds to entrepreneurs. In terms of Figure 3.1, as δ increases the red line gets flatter by rotating counterclockwise where eventually cuts the 45° line above its intersection with the dark blue line.

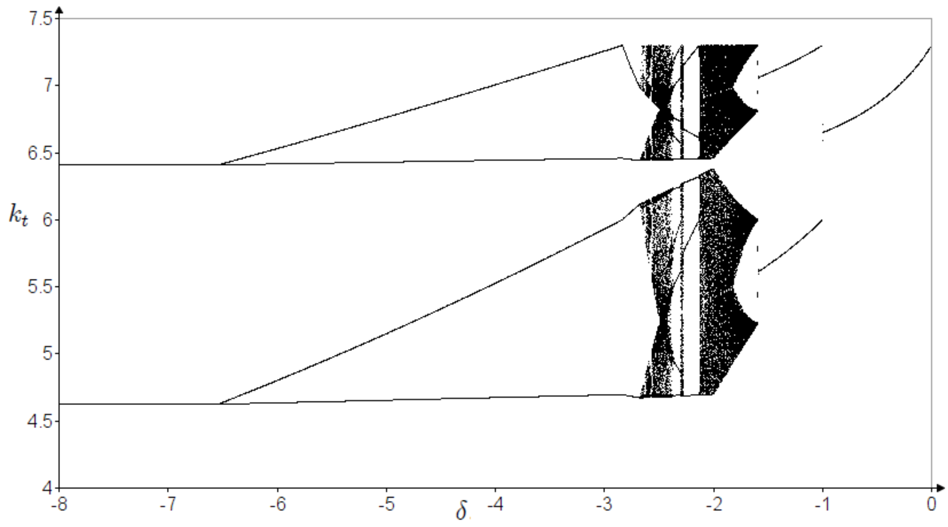

Figure 3.2: Bifurcation Diagram for δ. The parameter values used are $R = 2.4$, $A = 2.5$, $m = 1.5$, $\gamma = 0.02$, $u = 1$, $B = 1.25$, $k^{FG} = 6$ and $-8 \leqslant \delta \leqslant 0$.

For slightly lower values of δ the fixed point loses stability and we obtain stable period-2 cycles as shown in Figure 3.3. In this case, the economy rotates between a high capital/labor ratio state with no investment in bad projects, which we denote by A, and a low capital/labor ratio state with constrained investment in bad projects, denoted by B. We refer to this period-2 cycle as Type 1. Now the red line in Figure 3.1 gets steeper and cuts the 45° line below its intersection with the dark blue line. In this case the cost of creation of bad projects has declined sufficiently for banks to invest some of the deposited funds in bad projects but is not low enough given the level of economic development for this investment to be unconstrained. When in some period $t - 1$ the capital/labor ratio and, hence, wages are high, at period t there will be a lot of funds available and banks will invest some of them in bad projects (but not as much as they would if the availability of bad projects was unconstrained). The reason is that if all funds were lent to entrepreneurs the marginal productivity of capital, and hence the equilibrium borrowing interest rate would drop well below the return to bad projects. However, in the new equilibrium the interest rate will still be below the return of bad projects given that the level of financial innovation is not extensive enough to create a quantity of bad projects consistent with the equalization of the two rates. As some funds are invested in bad projects the new equilibrium capital/labor in period t, and hence the wage rate, is below that of period $t - 1$. Then, at $t + 1$ the new level of deposits would be too low to support financial innovation and all funds will now be lent to entrepreneurs. The new higher capital/labor ratio marks the beginning of a new cycle.

For still lower values of δ we enter a region where high periodicity cycles appear including chaotic attractors. But eventually the economy settles into a period-4 cycle (see Figure 3.4). The economy now moves from a high capital/labor ratio state to a low capital/labor ratio state where the investment in bad projects is constrained to a very high

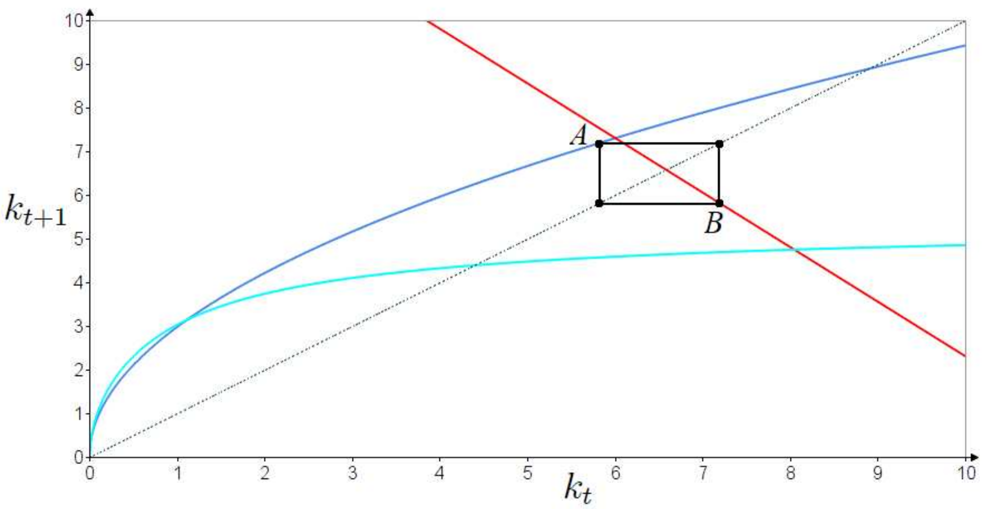

Figure 3.3: Type 1 Period-2 Cycle. The parameter values used are $R = 2.4$, $A = 2.5$, $m = 1.5$, $\gamma = 0.02$, $u = 1$, $B = 1.25$, $k^{FG} = 6$ and $\delta = -1.25$.

capital/labor ratio state to a very low capital labor ratio state where the investment in bad projects is unconstrained. We refer to this cycle as a Mixed Period-4 Cycle. Suppose that at $t-1$ the economy has reached the high capital/labor ratio state. Next period the level of deposits will allow some investment in bad projects but the level of financial innovation constrains that investment. Because at t the investment in bad projects was constrained the capital/labor ratio and, thus, wages did not drop as much as they would have done had the investment in bad projects being unconstrained. Thus, at $t+1$ the economy moves to a very high capital/labor ratio. This is because (a) last period the investment in bad projects was constrained thus allowing for more lending to entrepreneurs, and (b) this period there is no investment in bad projects. Next, the good performance of the economy in period $t+1$ means that at $t+2$ there are plenty of deposits, intensive financial innovation and unconstrained investment in bad projects which, in turn, implies that the economy has moved to the very low capital/labor ratio state. Next, period there will not be any investment in bad projects and the economy enters a new cycle.

Lastly, for very low values of δ, financial innovation is very efficient and the economy follows a new period-2 cycle shown in Figure 3.5. We will refer to this cycle as Type 2 Period-2 Cycle. In terms of Figure 3.1 the red line is much steeper. As the economy moves from the high capital/labor ratio state (denoted by C) to the low capital/labor ratio state (denoted by D), thus, bringing forward a lot of savings due to high wages, and given that availability of bad projects is unconstrained, the investment in bad projects is such that the interest rate banks receive on their loans, after adjusting for the costs of creating new loans, is equal to the return on bad projects.

Diaz-Alejandro (1985) identified two phases of financial development. One corresponds to financial repression where the high costs of financial intermediation, captured by the parameter γ in our model, limits funds available for productive investments and, thus,

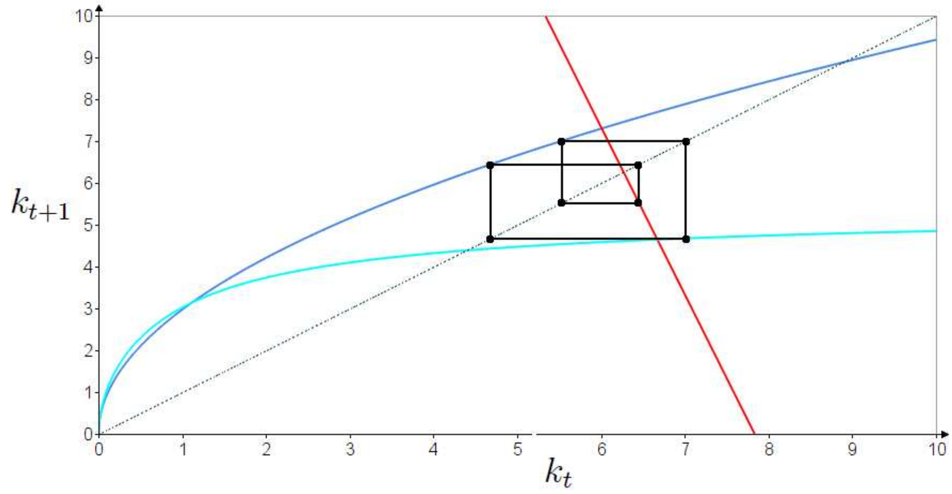

Figure 3.4: Period-4 Mixed Cycle. The parameter values used are $R = 2.4$, $A = 2.5$, $m = 1.5$, $\gamma = 0.02$, $u = 1$, $B = 1.25$, $k^{FG} = 6$ and $\delta = -4$.

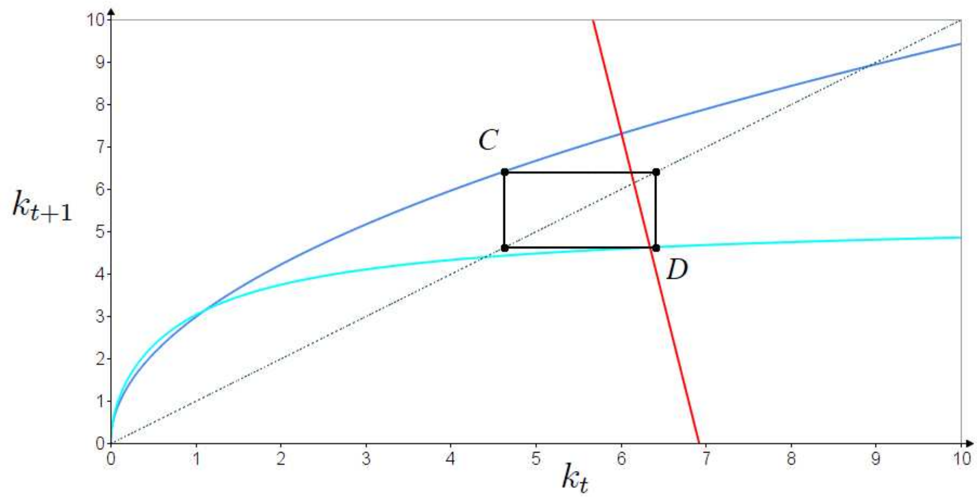

Figure 3.5: Type 2 Period-2 Cycle. The parameter values used are $R = 2.4$, $A = 2.5$, $m = 1.5$, $\gamma = 0.02$, $u = 1$, $B = 1.25$, $k^{FG} = 6$ and $\delta = -8$.

restraining the development of the economy. The second phase reflects the consequences of financial liberalization which encourages economic growth but also makes the economy more prone to crises. In this paper, we argue that this two-phase process can be accounted by two distinct aspects of financial innovation, one positive and one negative. On the positive side, financial innovation reduces intermediation costs and, thus, makes more funds available for productive investments. On the negative side, financial innovation can also introduce new products that while enhance the short-term profitability of financial institutions they do not contribute to capital formation. In this section, thus far, we have concentrated on variations in the parameter δ, which captures the second type of innovation and is responsible for the main cause of instability in our model.

In the next section, we show how the dynamics of the model are affected as we vary two other parameters of the model. In particular, we consider how changes in the rate of return of good projects, R, and the cost of bank loans, γ, affect the dynamics of the model. Variations in R are interesting because the parameter captures the level of technological progress. In the next section, we are going to take a closer look at the impact of technological progress on the relationship between business cycles and income inequality. We also analyze variations in γ to get a complete picture of the impact of financial innovation on the economy. Crucially, the bifurcation diagrams obtained display very similar patters.

3.2. Numerical Analysis: Technology and Cost of Banking

Variations in R Figure 3.6 shows the bifurcation diagram for R where there are 5 distinct regions.[14] For very low values of R we have a unique steady-state. Given that the productivity of good projects is low, the capital/labor ratio and wages are also low, and therefore the size of the banking sector is too small to support financial innovation and there is no investment in bad projects. As R increases we enter a region of Type 1 Period-2 Cycles. When there is no investment in bad projects the capital/labor ratio and, thus, the wages are sufficiently high to support financial innovation and, hence, investment in bad projects in following period, however not high enough for this investment to be unconstrained. Further increases in R bring the economy into a region of Type 2 Period-2 Cycles. The productivity of good projects is sufficiently high so that in periods where there is no investment in bad projects wages are high enough so that the following period the size of the banking system can support a level of financial innovation so that the investment in bad projects is unconstrained. Even higher increases in R bring the economy in a new type of region where it rotates between a state where the investment in bad projects is constrained to another state where such investment is unconstrained. We refer to this type of cycle as Type 3 Period-2 Cycle. Now even in periods where the investment in bad projects is unconstrained the performance of the economy is strong enough so that the following period can still support financial innovation though now at a lower level. Lastly, for very high values of R we have again a region where the economy converges to a unique steady-state where the investment in bad projects is always unconstrained. The reason

[14]We will refer to the points denoted by capital letters in our analysis of the dynamics of income distribution below.

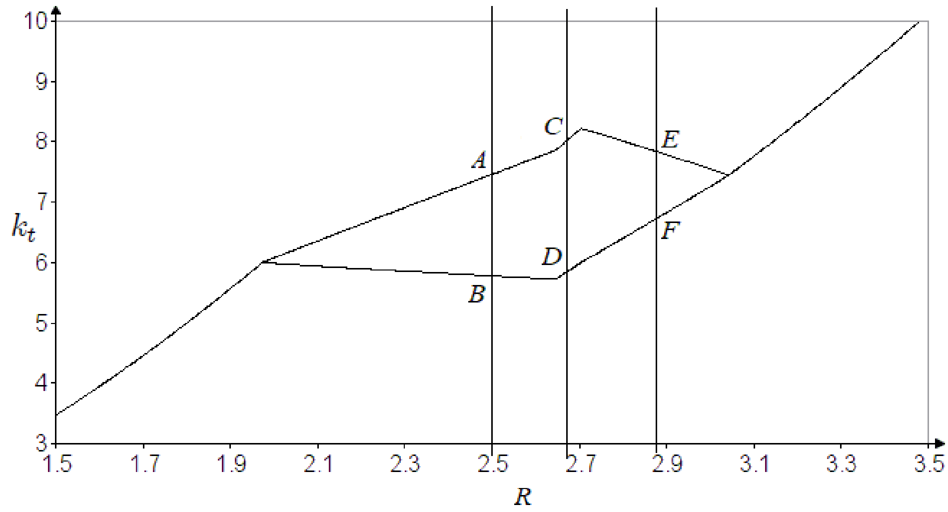

Figure 3.6: Bifurcation Diagram for R. The parameter values used are $1.5 \leqslant R \leqslant 3.5$, $A = 2.5$, $m = 1.5$, $\gamma = 0.02$, $u = 1$, $B = 1.25$, $k^{FG} = 6$ and $\delta = -1.25$.

is that despite the large amount of funds invested in bad projects the capital/labor ratio remains at a sufficiently high level, because of the high productivity of good projects, to support the level of financial innovation needed to set the equilibrium interest rate equal to the return of bad projects.

Variations in γ Figure 3.7 shows the bifurcation diagram for γ. These patterns on this diagram are almost the reverse of the diagram obtained above. When the provision of loans by the banking system is very efficient (low γ), even if the investment in bad projects is unconstrained there are still enough funds left to keep the level of lending to entrepreneurs sufficiently high to support a high wage economy. As the cost of lending increases the economy enters a region of Type 3 Period-2 Cycles. As in the previous case, every period there is investment in bad projects but now it alternates between a constrained state and an unconstrained state. Following an unconstrained state where the funds made available to entrepreneurs were lower than in the case above because of the higher cost of lending, the funds available to the banking system in the next period will be lower. As a result the level of financial innovation will decline. With fewer bad projects available there will be more funds available to entrepreneurs. The resulting higher wages will bring the economy back to the unconstrained state the following period starting the cycle all over again. Further increases in the cost of lending will bring the economy in a region with Type 2 Period-2 Cycles where now following the unconstrained state the level of economic performance is too low to support any financial innovation the following period. Still higher increases in the cost of lending will now bring the economy in a region with Type 1 Period-2 Cycles. Even after a period where there is no investment in bad projects the level of economic performance will not be strong enough to support sufficient financial innovation for the investment in bad projects to be unconstrained. The reason

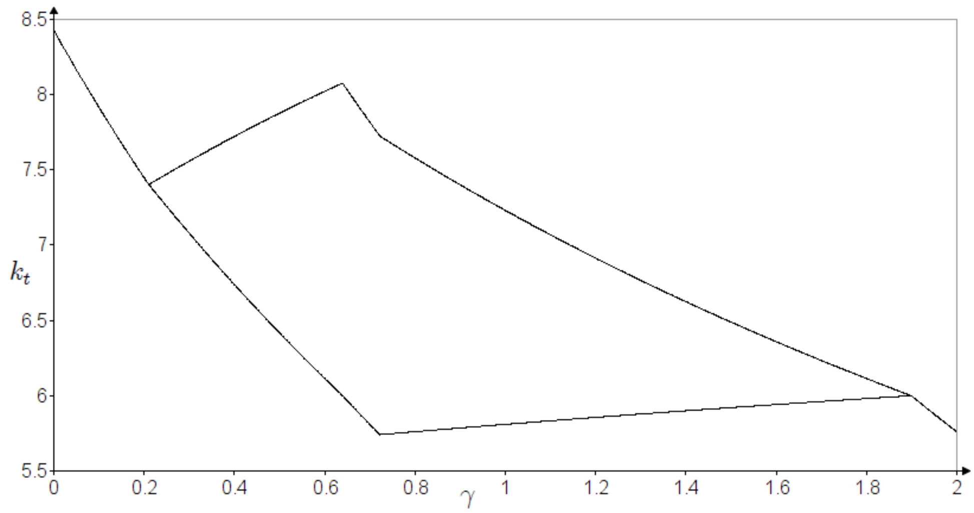

Figure 3.7: Bifurcation Diagram for γ. The parameter values used are $R = 3.2$, $A = 2.5$, $m = 1.5$, $0 \leqslant \gamma \leqslant 2$, $u = 1$, $B = 1.25$, $k^{FG} = 6$ and $\delta = -1.25$.

is that too many resources are absorbed by the cost of providing loans to entrepreneurs. Lastly, for very high values of the cost of lending the economy always converges to a unique steady state where there is no financial innovation and no investment in bad projects. The banking system is very inefficient spending too many resources to support its operations rather than offering funds to entrepreneurs.

3.3. Inequality and Cycles

Above we have identified a great variety of equilibrium dynamic paths that arise as the parameters of the model vary. Some of these paths are simple converging to a unique steady-state, other paths are converging to regular finite order cycles and still other more complex paths are characterized by chaotic long-run cyclical dynamics. In this section, we study the behavior of income inequality along those cycles. In particular we are going to analyze what happens to the steady-state dynamics of the economy as we vary the technological parameter R. The purpose of this exercise is to try to understand the effects of technological progress on the relationship between business cycles and the income distribution.[15]

For the moment, we focus the discussion on a Type-1 cycle but we provide numerical analysis of income distribution dynamics along any Type-2 cycle. Along a Type 1 cycle the economy keeps rotating between a state where aggregate deposits are low and there is no investment in bad projects (point B in Figure 3.6) to a state where aggregate deposits are high and banks now invest some of their deposits into bad projects but this investment is still constrained (point A in Figure 3.6). The equilibrium dynamics of the economy along this path are as follows. Suppose at time $t-1$, the capital/labor ratio and wages are low.

[15] A more complete picture requires that we endogenise technological progress, which would allow us to investigate the interaction between growth and cycles, but this is beyond the scope of this work.

Then, at t bad projects will not be available for investment and all deposits will be lent to entrepreneurs. By comparing (2) and (19) we find that in this state z^* is relatively low and thus the proportion of old agents who become entrepreneurs will be relatively high. The high capital/labor ratio at time t implies that the marginal productivity of capital and the interest rates in the same period are low. Notice that (7) implies that the interest rate is decreasing in the current capital/labor ratio, increasing in the lagged capital labor ratio and increasing in the threshold z^*. Because the interest rates are low, banks would have liked to invest some of their deposits in bad projects, however, the latter are not available. Because of the high capital/labor ratio wages at t are also high. At $t+1$ there are more funds available for investment but now bad projects become available. As a result the funds available to entrepreneurs are low, higher z^*, and the corresponding capital/labor ratio drops. Because of the increase in the marginal productivity of capital interest rates rise but are still below the return of bad projects. This is because the availability of bad projects is constrained. Wages now drop and the cycle is completed.

Next, consider the income of each type of agent along the cycle. At any given date t there are two generations of equal size alive. We need to consider the incomes of three types of agents:

1. The average income of young agents (half of the population) born at t is equal to w_t (remember endowments are distributed on $[0,2]$ and the mass of young agents is equal to 1).

2. A proportion $\frac{2-z_t^*}{2}$ of old agents (born at $t-1$ and with a mass of 1) borrow from the banks and become entrepreneurs. As a group, they earn

$$\int_{z_t^*}^{2}\left(f'(k_t)Rmzw_{t-1}g(z) - r_t^b(m-1)zw_{t-1}g(z)\right)dz =$$
$$= \frac{4-(z_t^*)^2}{4}\left(f'(k_t)Rm - r_t^b(m-1)\right)w_{t-1}$$

   Dividing by $\frac{2-z_t^*}{2}$ results in the average entrepreneurial income:

$$\frac{2+z_t^*}{2}\left(f'(k_t)Rm - r_t^b(m-1)\right)w_{t-1}$$

   Note that $\frac{2+z_t^*}{2}$ is the average endowment of one entrepreneur.

3. A proportion $\frac{z_t^*}{2}$ of old agents become depositors. As a group, they earn income equal to

$$\int_{0}^{z_t^*} r_t^d z w_{t-1} g(z)\,dz = \frac{(z_t^*)^2}{4} r_t^d w_{t-1}$$

   The average depositor income is equal to $\frac{z_t^*}{2} r_t^d w_{t-1}$. Note that $\frac{z_t^*}{2}$ is the average endowment of one depositor.

Clearly the income of workers (young agents) is procyclical given that wages are increasing with the capital/labor ratio. Entrepreneurial income of a given type is countercyclical because of the marginal productivity of capital and the fact that along the cycle k_t and w_{t-1} move in the same direction. Part of the decline in the marginal productivity of capital during booms is compensated by the decline in the borrowing interest rate. Nevertheless, total entrepreneurial income and, thus final good output, increases because the mass of entrepreneurs has increased. The income of a depositor is also countercyclical because of the fall in the interest rates when the capital/labor ratio rises.

The increase in the threshold during recessions exacerbates the increase in inequality between the three groups. There are fewer entrepreneurs each earning more. Lastly, given that all incomes are linearly increasing with z, any increase in the marginal payoff of a group also increases the income dispersion within that group.

Numerical Analysis We keep all parameters constant except R.[16] Table 1 below shows average income for each type of agent and total income of all agents of a given type along the two phases of each of the three types of Period-2 cycles identified above. The purpose of this exercise is to show how the cycle affects inequality across the three types of agents. We make two observations. Firstly, the table obscures what happens to inequality within each group. However, the effects of the cycle on this type of inequality are easy to identify. Given that all incomes are (linearly) increasing in the endowments any increase in the aggregate income of any of these groups would also increase the inequality within that group and vice versa. With that in mind we focus our attention to the effects of the cycle on the inequality across groups. Secondly, we focus on relative inequality across the cycle, that is who gains and who loses. This is important because we have assumed that only young agents can work and thus the population of workers is artificially fixed to be the same as that of old agents.

Referring to Figure 3.6, we find that for low values of R we have a Type 1 Period-2 cycle along which the economy rotates between a high capital/labor ratio state with no investment in bad projects and a low capital/labor ratio state with constrained investment in bad projects; as R increases we move to a Type 2 Period-2 cycle along which the economy now rotates between a state of no investment in bad projects to one where such investment is unconstrained. For still higher values of R we move to a Type 3 Period-2 cycle where there is always investment in bad projects, however, it is constrained in the low state. The table also shows the proportions of entrepreneurs and depositors in the population of old agents which are important given that the cutoff point changes from phase to phase.

[16] The other parameter values were set equal to those used for the baseline calculations, that is $A = 2.5$, $R = 2.4$, $m = 1.5$, $\delta = -1.25$, $u = 1$, $B = 1.25$, $\gamma = 0.02$, $k_{FG} = 6$.

	Type 1 ($R = 2.5$)		Type 2 ($R = 2.65$)		Type 3 ($R = 2.9$)	
State	Point A	Point B	Point C	Point D	Point E	Point F
k_{t+1}	7.461	5.777	7.886	5.744	7.785	6.829
z^*	1.162	1.482	1.162	1.534	1.345	1.483
r^d	0.947	1.161	0.982	1.25	1.14	1.25
Number of Depositors (D)	0.581	0.741	0.581	0.767	0.672	0.742
Number of Entrepreneurs (E)	0.419	0.259	0.419	0.233	0.328	0.258
Average D Endowment	0.581	0.741	0.581	0.767	0.672	0.742
Average E Endowment	1.581	1.741	1.581	1.767	1.672	1.742
Total D Income	0.961	2.175	0.993	2.582	1.683	2.397
Total E Income	2.454	2.093	2.517	2.075	2.447	2.266
Average D Income	1.653	2.936	1.709	3.366	2.503	3.233
Average E Income	5.858	8.074	6.01	8.905	7.47	8.767
Average Worker Income	3.414	3.004	3.51	2.996	3.488	3.266

Table 1: Income Distribution Dynamics

In all three types of cycles the proportion of entrepreneurs increases as the economy moves from the low states (points B, D and F) to the high states (A, C and E) because investment in bad projects decreases and thus banks lend more funds to entrepreneurs. Those who gain in high states are workers because the higher capital/labor ratio implies that the marginal productivity of labor increases. Entrepreneurs income is countercyclical because of the decline in the marginal productivity of capital during booms. However, as a group they earn more because their numbers have significantly increased. The decline in interest rates in the high state implies that the income of depositors is countercyclical. Depositors gain in states where there is a high investment in bad projects as the interest rates increase to attract funds in the banking system. However, these funds are allocated to bad projects and the ensuing low capital/labor ratio boosts the marginal productivity of capital and hence the interest rates. The highest fluctuations of income among the three types of cycles for all three types of agents are observed along a Type 2 cycle where in the high state there is no investment in bad projects while in the low state this investment is unconstrained.

What happens to relative inequality as the economy grows? We view increases in R as the outcome of technological progress (similar results are obtained by varying A). We compare the evolution of the income of entrepreneurs relative to that of workers as a result of technological progress (moving from a Type 1 to a Type 3 cycle). Comparing recessions (low states) we find that the ratio remain unchanged at 2.7. In contrast, comparing booms (high states) we find that the ratio increases from 1.7 to 2.1. In relative terms it is the entrepreneurs who benefited more from technological progress.

Table 2 below shows the income shares of each group of agent along the two phases of the cycle for the same values of technological progress as above.

	Type 1 ($R = 2.5$)		Type 2 ($R = 2.65$)		Type 3 ($R = 2.9$)	
State	Point A	Point B	Point C	Point D	Point E	Point F
Depositors	0.14	0.30	0.14	0.34	0.22	0.30
Entrepreneurs	0.36	0.29	0.36	0.27	0.32	0.29
Workers	0.50	0.41	0.50	0.39	0.46	0.41

Table 2: Income Shares

Comparing the entries of Table 2 with those of Table 1 we find that entrepreneurial shares are procyclical while average entrepreneurial income is countercyclical. This is because the higher proportion of entrepreneurs during periods of expansion more than compensates for the decline in productivity. We have set the share factor in the Cobb-Douglas function equal to 0.5 and this is reflected at points A and C but not in the other phases of the three cycles. The explanation is that only at points A and C the banks lend all funds to entrepreneurs. In all other states, some funds are allocated to bad projects and, thus, the aggregate production function is a hybrid of Cobb-Douglas and the technology producing bad projects. In fact, when banks invest in bad projects both workers and entrepreneurs who are involved in the production of good projects find that their income shares decline. In contrast, depositors gain from the higher interest rates required to attract their funds.

4. Concluding Comments

Thomas Piketty's (2014) seminal work has generated a lot of interest in the relationship between growth and income inequality. Research in this area has focused on the high concentration of wealth at the top of the income distribution. However, income disparities can be also affected by the state of the economy a subject that is relatively neglected in the economics literature. The limited work in this area builds on the real business cycles framework; e.g. Bhandari *et al.* (2021). However, Beaudry *et al.* (2020) provide evidence that cycles are generated by an endogenous mechanism related to financial market frictions. In this paper, we have analyzed a theoretical model of such a mechanism that builds on the work of Matsuyama *et al.* (2016). There are two main differences between the two models. Firstly, in our model there is a banking system through which all credit is intermediated. The decision of allocating funds between good projects (funding entrepreneurs) and bad projects (investments that do not contribute to capital formation) are taken by banks. Our intent has been to capture in broad brushes some of the activities taking place within the financial system prior to the 2009 Global Financial Crisis. Secondly, in our model agents are heterogenous which allows us to address the impact of cycles not only on the level of incomes of different occupations but also on occupational choice.

We have found that as the economy goes through its booms and busts phases relative prices fluctuate affecting differentially the incomes of entrepreneurs, workers and savers. Among the three groups workers suffer the highest income losses during recessions because the larger share of funds misallocated by banks to unproductive activities result to further reductions in labor productivity.

The introduction of heterogeneity has significantly complicated the model thus limiting its scope. In the remaining of this section, we discuss some of these limitations. In our model all funds are intermediated by banks the only financial institution. We have concentrated on banks given that they have played such a major role during the 2009 Global Financial Crisis but doing so we have ignored the important role played by capital markets (equity and bond financing), investment banking, private equity, hedge funds, etc. It is impossible to understand the sheer complexity of the financial instruments (derivatives, CDOs, etc) and their corresponding transactions without taking a more holistic approach to financial markets. Nevertheless, we believe that what we have referred to as bad projects in our model captures that part of the activities of financial markets that are unproductive. We have also ignored the role of the central bank that plays an important part as regulator and provider of liquidity. In fact, one key aspect of the economy around the time of the crisis, that of very low interest rates, is not captured by our model. Furthermore, our discussion of the effects of technological progress on the income distribution is limited by the fact growth is also endogenous and can be affected by what happens during cycles.

Despite the above limitations we believe that our model captures some of the dynamics of the real economy during the crisis caused by financial frictions and the impact of cycles on the distribution of income. Along the way, it has provided a possible explanation for the questions posed by Diaz-Alejandro (1985) on the relationship between financial development and financial crises.

Appendix 1: Welfare Benefits of Banking

We derive conditions such that welfare (expressed in terms of consumption of the final good) under banking is higher than under an economy without a banking system. Given that agents do not consume when they are young for welfare comparisons we aggregate the utilities of the two types of old agents. Under banking this would include the consumption of the final good by entrepreneurs, the consumption of the final good by depositors plus the additional non-transferrable utility that depositors obtain from their engagement with the alternative activity. In contrast, in the absence of banking system the utility of those agents who do not become entrepreneurs is limited to that obtained from their engagement with the alternative activity. Let $z^*_{B,t}$ denote the cutoff level of endowment obtained under banking and $z^*_{N,t}$ the corresponding cutoff obtained in the absence of a banking system. Then at t, welfare under banking, W_t^B, is given by[17]

$$W_t^B = \frac{\left(z^*_{B,t}\right)^2}{4}\left(u + r_t^d w_{t-1}\right) + \left(1 - \frac{\left(z^*_{B,t}\right)^2}{4}\right)\left(f'(k_t)Rm - r_t^b(m-1)\right) w_{t-1}$$

To find the level of welfare in the absence of a banking system, W_t^N, we set $m = 1$, to get

$$W_t^N = \frac{z^*_{N,t}}{2}u + \left(1 - \frac{\left(z^*_{N,t}\right)^2}{4}\right) f'(k_t) R w_{t-1}.$$

Then, as long as $W_t^B > W_t^N$ welfare under banking is higher. The parameter values in all our numerical examples satisfy this inequality.

[17] For the derivation of the expression please refer to the section on Inequality and Cycles.

Appendix 2: Derivation of equation (14) and (15)

From (12) and (13) we get:

$$k_{t+1} = \frac{1}{4} RmAk_t^{\frac{1}{2}} \int_{z^*_{t+1}}^{2} zdz \Rightarrow$$

$$k_{t+1} = \frac{1}{8} RmAk_t^{\frac{1}{2}} \left(4 - \frac{u^2}{\frac{A^2}{4} k_t \left(\frac{A}{2} k_{t+1}^{-\frac{1}{2}} Rm - B(m+\gamma m - \gamma) \right)^2} \right) \Rightarrow$$

$$k_{t+1} \left(\frac{A^2}{4} k_t \left(\frac{A}{2} k_{t+1}^{-\frac{1}{2}} Rm - B(m+\gamma m-\gamma) \right)^2 \right) =$$

$$\frac{1}{8} RmAk_t^{\frac{1}{2}} \left(A^2 k_t \left(\frac{A}{2} k_{t+1}^{-\frac{1}{2}} Rm - B(m+\gamma m-\gamma) \right)^2 - u^2 \right) \Rightarrow$$

$$A^2 k_{t+1} k_t \left(\frac{A}{2} k_{t+1}^{-\frac{1}{2}} Rm - B(m+\gamma m-\gamma) \right)^2 =$$

$$\frac{1}{2} RmAk_t^{\frac{1}{2}} \left(A^2 k_t \left(\frac{A}{2} k_{t+1}^{-\frac{1}{2}} Rm - B(m+\gamma m-\gamma) \right)^2 - u^2 \right) \Rightarrow$$

$$A^2 k_{t+1} k_t \left(\frac{A^2}{4} k_{t+1}^{-1} R^2 m^2 + B^2 (m+\gamma m-\gamma)^2 - Ak_{t+1}^{-\frac{1}{2}} RmB(m+\gamma m-\gamma) \right) =$$

$$\frac{1}{2} RmAk_t^{\frac{1}{2}} \left(A^2 k_t \left(\frac{A^2}{4} k_{t+1}^{-1} R^2 m^2 + B^2 (m+\gamma m-\gamma)^2 - Ak_{t+1}^{-\frac{1}{2}} RmB(m+\gamma m-\gamma) \right) - u^2 \right) \Rightarrow$$

$$\frac{A^4}{4} k_t m^2 R^2 + A^2 k_{t+1} k_t B^2 (m+\gamma m-\gamma)^2 - A^3 k_{t+1}^{\frac{1}{2}} k_t mRB(m+\gamma m-\gamma) =$$

$$\frac{1}{8} k_{t+1}^{-1} k_t^{\frac{3}{2}} R^3 m^3 A^5 + \frac{1}{2} RmA^3 k_t^{\frac{3}{2}} B^2 (m+\gamma m-\gamma)^2 - \frac{1}{2} R^2 m^2 (m+\gamma m-\gamma) A^4 k_t^{\frac{3}{2}} k_{t+1}^{-\frac{1}{2}} B - \frac{1}{2} RmAk_t^{\frac{1}{2}} u^2$$

Multiplying by $k_{t+1} k_t^{\frac{-1}{2}} A^{-2}$ results in equation (14).

To calculate the steady-state capital/labor ratio we set $k_{t+1} = k_t = k^*$:

$$\frac{A^4}{4} k^* m^2 R^2 + A^2 (k^*)^2 B^2 (m+\gamma m-\gamma)^2 - A^3 (k^*)^{\frac{3}{2}} mRB(m+\gamma m-\gamma) - \frac{A^5}{8} (k^*)^{\frac{1}{2}} R^3 m^3 -$$

$$\frac{1}{2} RmA^3 (k^*)^{\frac{3}{2}} B^2 (m+\gamma m-\gamma)^2 + \frac{1}{2} R^2 m^2 (m+\gamma m-\gamma) A^4 k^* B + \frac{1}{2} RmA (k^*)^{\frac{1}{2}} u^2 = 0 \Rightarrow$$

$$B^2 (m+\gamma m-\gamma)^2 (k^*)^2 - RmBA(m+\gamma m-\gamma) \left(1 + \frac{1}{2} (m+\gamma m-\gamma) B \right) (k^*)^{\frac{3}{2}}$$

$$+ m^2 R^2 A^2 \left(\frac{1}{4} + \frac{1}{2} (m+\gamma m-\gamma) B \right) k^* - Rm \left(\frac{1}{8} R^2 m^2 A^3 - \frac{1}{2A} u^2 \right) (k^*)^{\frac{1}{2}} = 0$$

Dividing by $(k^*)^{\frac{1}{2}}$ results in equation (15).

References

[1] Acharya, V., Richardson, M., (eds.) 2009. Restoring Financial Stability: How to Repair a Failed System. John Wiley & Sons, Inc., Hoboken, NJ

[2] Acharya, V., Schnabl, P., 2009. Do global banks spread global imbalances? The case of asset-backed commercial paper during the Financial Crisis of 2007–09. Paper presented at the 10th Jacques Polak Annual Research Conference Hosted by the International Monetary Fund Washington, DC, November 5–6, 2009

[3] Azariadis, C., Smith, B., 1998. Financial Intermediation and regime switching in business cycles. *American Economic Review* 88, 516-536

[4] Avrutin, V., Gardini, L., Sushko, I., Tramontana, F., 2019. Continuous and Discontinuous Piecewise Smooth One-Dimensional Maps: Invariant Sets and Bifurcation Structures. World Scientific, Singapore

[5] Bayer, C., Born, B., Luetticke, R., 2020. Shocks, frictions, and inequality in US business cycles. Available at SSRN: https://ssrn.com/abstract=3535449

[6] Beaudry, P., Galizia, D., Portier, F., 2020. Putting the cycle back into business cycle analysis. *American Economic Review* 110, 1-47

[7] Bernanke, B., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework. In Taylor, J., Woodford, M., (eds.) Handbook of Macroeconomics. Elsevier, North Holland, 1341-1393

[8] Benmelech, E., Dlugosz, J., 2009. The credit ratings crisis, in Acemoglu, D., Rogoff, K., Woodford, M., (eds.) NBER Macro Annual, National Bureau of Economic Research, Cambridge, MA

[9] Bhandari, A., Evans, D., Golosov, M., Sargent, T., 2021. Inequality, business cycles, and monetary-fiscal policy. *Econometrica* 89, 2559-2599

[10] Brunnermeier, M., Eisenbach, T., Sannikov, Y., 2013. Macroeconomics with financial frictions: A Survey, in D. Acemoglu, M. Arellano and E. Dekel (eds.). Advances in Economics and Econometrics, Tenth World Congress of the Econometric Society, Vol. II: Applied Economics, Cambridge University Press, New York, 4-94

[11] Castañeda, A., Diaz-Gimenez, D., Rios-Rull, J.-V., 1998. Exploring the income distribution business cycle dynamics. *Journal of Monetary Economics* 42, 93-130

[12] Coval, J., Jurek, J., Stafford, E., 2009. The economics of structured finance. *Journal of Economic Perspectives* 23, 3–26

[13] Diaz-Alejandro, C., 1985. Good-bye financial repression, hello financial crash. *Journal of Development Economics* 19, 1-24

[14] Donfg, F., Zhiwei, Xu., 2020. Cycles of credit expansion and misallocation: The good, the bad and the ugly. *Journal of Economic Theory* 186. Available at: https://doi.org/10.1016/j.jet.2020.104994

[15] Gennaioli, N., Shleifer, A., 2018. A Crisis of Beliefs. Princeton University Press, Princeton, NJ

[16] Gennaioli, N., Shleifer, A., Vishny, R., 2013. A model of shadow banking. *Journal of Finance* 68, 1331-1363

[17] Gorton, G., and Metrick, A., 2012. Securitized banking and the run on repo. *Journal of Financial Economics* 104, 425–451

[18] Gorton, G., and Souleles, N., 2006. Special purpose vehicles and securitization, in Stulz, R., Carey, M., (eds.) The Risk of Financial Institutions. NBER, University of Chicago Press, Chicago, IL

[19] Griffin, J., 2021. The years of evidence: Was fraud a force in the financial crisis? *Journal of Economic Literature* 59, 1293-1321

[20] Gu, C., Mattesini, F., Monnet, C., Wright, R., 2013. Endogenous credit cycles. *Journal of Political Economy* 121, 940–965

[21] Holmstrőm, B., Tirole, J., 1997. Financial intermediation, loanable funds, and the real sector. *Quarterly Journal of Economics* 112, 663-691

[22] Kiyotaki, N., Moore, J., 1997. Credit cycles. *Journal of Political Economy* 105, 211-248

[23] Kubin, I., Zörner, T., 2021. Credit cycles, human capital and the distribution of income. *Journal of Economic Behavior and Organization* 183, 954-975

[24] Matsuyama, K., 2013. The good, the bad, and the ugly: An inquiry into the causes and nature of credit cycles. *Theoretical Economics* 8, 623-651

[25] Matsuyama, K., Sushko, I., Gardini, L., 2016. Revisiting the model of credit cycles with good and bad projects. *Journal of Economic Theory* 163, 525-556

[26] Mizen, P., 2008. The Credit Crunch of 2007-2008: A Discussion of the Background, Market Reactions, and Policy Responses. *Federal Reserve Bank of St. Louis Review* 90, 531-567

[27] Myerson, R., 2012. A model of moral-hazard credit cycles. *Journal of Political Economy* 5, 847-878

[28] Piketty, T., 2014. Capital. Harvard University Press, Cambridge, MA

[29] Pozsar, Z., Tobias, A., Ashcraft, A., Hayley B., 2010. Shadow banking. Working Paper, Federal Reserve Bank of New York

[30] Richardson, M., White, L., 2009. The rating agencies: Is regulation the answer? in Acharya,V., Richardson, M., (eds.) Restoring Financial Stability: How to Repair a Failed System. John Wiley & Sons, Inc., Hoboken, NJ

[31] Shin, H., 2009. Reflections on Northern Rock: The bank run that heralded the global financial crisis. *Journal of Economic Perspectives* 23, 101–120

[32] Stanton, R., Wallace, N., 2010. CMBS subordination, ratings inflation, and regulatory-capital arbitrage. NBER Working Paper 16206

[33] Suarez, J., Sussman, O., 1997. Endogenous cycles in a Stiglitz-Weiss economy. *Journal of Economic Theory* 76, 47-71

[34] Sunaga, M., 2017. Endogenous growth cycles with financial intermediaries and entrepreneurial innovation. *Journal of Macroeconomics* 53, 191-206

[35] Sushko, I., Avrutin, V., Gardini, L., 2016. Bifurcation structure in the skew tent map and its application as a border collision normal form. *Journal of Difference Equations and Applications* 22, 1040–1087